Compulsory versus Voluntary Savings as Incentive
Mechanism in Microlending Contracts

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by

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Abstract
This paper investigates the incentive mechanism of individual microlending contracts focusing particularly on microsavings. We built a model to show the role of compulsory and voluntary microsavings in addressing problems of information asymmetries. We show that voluntary savings can serve as a complementary tool in repayment enforcement at the same time inducing borrowers to reveal abilities of their projects.

Keywords: Microfinance, Compulsory microsavings, Voluntary microsavings, Incentive mechanism, Repayment enforcement, Screening

1. Introduction

Microfinance is seen as one of various segments of the financial system. Microfinance programmes have twofold objectives: to fill the gap left by (larger) conventional institutions in the provision of financial services to disadvantaged sections of society, and to contribute to their social inclusion. Nowadays, microfinance institutions (MFIs) offer more inclusive range of financial services – loans, savings, insurance, remittances and joint-lending, which are tailored to better meet needs of the poor. In doing so, one of the main challenges the MFIs face is the vulnerability of their clientele who typically is unemployed, lacks documented credit histories and pledgeable collateral. Given that MFIs operate within a variety of principal-agent relations, most financial institutions likewise, they have to face key concepts of the imperfect information paradigm which are applied to credit markets: adverse selection and moral hazard. How to make those borrowers exhibit responsible behavior (and repay their loan) and how to acquire information absent when the loan endorsement is given?

Drawing on the major success stories, researchers are attempting to unfold different mechanisms of repayment enforcement and screening. Numerous studies tackle information asymmetry problems for both group lending and individual microfinance programmes in diverse context throughout different techniques of dynamic incentives. As a general point, dynamic incentives provide a future incentive (e.g. larger or cheaper loans) in exchange for a change in behavior now (e.g. lower moral hazard, self-commitment). Almost all MFIs reply on dynamic incentives. Existing mechanisms have mainly been based on joint liability in group lending (see the seminal papers of Stiglitz (1990), Varian (1990), Morduch (1999a) and Ghatak (1999, 2000)) or on the use of social sanctions, collaterals and progressive lending to individuals (Armendáriz and Morduch, 2000; Egli, 2004; Tedeschi, 2006 or Bhole and Ogden, 2010).

The main objective of this paper goes in line with the above mentioned literature as we want to address both issues of repayment enforcement and borrowers’ screening.

However, we suggest another answer to the key questions of how to frame and time the incentives in microlending contracts so as to be socially optimal and revealing by studying the role of microsavings in the incentive mechanism.
Over the years, the empirical evidence has witnessed the significant role of microsavings in the service portfolio of the development finance. The large demand for financial savings, particularly in developing countries, has been empirically demonstrated (Vogel, 1984; Martin, Hulme and Rutherford, 2002; Robinson, 2001; Dowla and Alamgir, 2003). This argument is raised from the evidence that the poor know the value of saving and that many households already use (informally or formally) a variety of financial and non-financial savings mechanisms (Glisovic, El-Zoghbi, and Forster, 2010). The possibility to save gives options to the poor to have a longer vision on their activity planning, to acquire certain stability and risk protection, to increase their working productivity (Ledgerwood, 1999; Wright, 2003; Deshpande and Glisovic-Mezieres, 2007). This saving behavior traditionally is viewed as an opportunity for MFIs to develop convenient and appropriate savings services in order to meet the existing demand and fulfill their social mission. For MFIs, deposit-taking can indeed be the most stable and affordable funding source that also strengthens their self-sufficiency and reduces their dependency on external funds in long-term (Mata, 2009, Gadway and O'Donnell, 1996; Otero, 1989; Jackelen and Rhyne, 1991).

In this paper, we aim to demonstrate that attracting microsavings into joint-lending contracts can also serve as a tool for repayment enforcement by inducing 'good behaviour' of borrowers and allowing MFIs to better screen loan applicants.

On the whole, microsavings are grouped into two main types: compulsory and voluntary. Compulsory savings (also known as forced savings) is defined as the minimum amount of savings which conditions borrowers' access to loans, and can be seen as a collateral substitute imposing a positive inducement for repayment. The regulation of MFIs typically does not allow to withdraw and use those savings until the loan is repaid. Voluntary savings (also known as flexible savings) allows borrowers as well as non-borrowers to deposit or withdraw according to their needs (Montgomery, 1996). Voluntary savings assumes that savings and credit are integral components of financial intermediation and that savers already know why and how to save (Robinson, 1994).

Consequently, compulsory savings perceives savings as an integral part of loans; savers learn financial discipline and qualify for credit by a convincing savings record. The main objective of mobilizing compulsory savings is the belief that a process of small, regular payments will contribute to repayment performance by borrowers (Wisniwski, 1999; Ledgerwood, 1999; Armendariz and Morduch, 2005). As stated by Fiebig, Hannig and Wisniwski (1999) many microcredit programmes have used forced savings as part of their financial technology. However, the other key challenge of MFIs, which consists in discriminating abilities of loan applicants, stays uncovered by the compulsory savings mechanism. Several authors have argued that this mechanism, in fact, does not serve as an indicator about borrower's "quality" and potential of successfully undertaking the project. The core argument of our study boils down to investigating whether the introduction of voluntary savings into individual microlending contracts can serve as a screening device and selection criterion revealing information about borrowers’ abilities. It is an important matter for MFIs as in general the probability of project success depends on borrower's ability.

Our contribution is thus related to the literature on repayment enforcement and incentive mechanism. We, particularly, built on arguments of Armendáriz and Morduch (2000), which are in favor of turning to savings as deposit mobilization in addressing adverse selection and moral hazard problems.

We develop a model of asymmetric information with hidden information where microlending is joined with microsavings. We consider two classes of agents – micro-entrepreneurs and a Microfinance Institution (hereafter MFI) and assume that the population of micro-entrepreneurs is divided into two types: "high-ability" and "low-ability" with two
different success probabilities (high and low). Micro-entrepreneurs need external funds in order to invest in a risky project but because of their poor characteristics, they do not have access to the capital market and/or bank loans and must demand financing to the MFI. We assume that the MFI faces two informational problems in its relationships with micro-entrepreneurs. On the one hand, the MFI is not capable of discriminating between "high" and "low" ability micro-entrepreneurs. On the other hand, we retain the Costly State Verification framework as we assume that micro-entrepreneurs have better information on the profitability of their projects and they can divert a part of the cash from the MFI.

First, we construct an equilibrium contract between micro-entrepreneurs and the MFI, where the repayment incentive is based on the requirement of a compulsory savings. We show that this kind of contract allows the financing of projects with positive social value but is not optimal since it does not allow to discriminate between "high" and "low-ability" micro-entrepreneurs.

Second, we show that the introduction of voluntary savings in our framework allows the MFI to discriminate between "high" and "low-ability" micro-entrepreneurs and restore optimality. We construct a separating equilibrium in which all "high-ability" micro-entrepreneurs will choose a financing contract with voluntary savings whereas "low-ability" micro-entrepreneurs will choose a financing contract with compulsory savings only.

This complete enforcement and revealing mechanism can be seen as a new approach as compared to the existing incentive methods designed to face enforcement and screening issues in individual lending microfinance contracts.

The rest of this paper is organized as follows. In Section 2 we present the emerging importance of microsavings. A basic three-period lending model designed for joint-lending contract is specified in Section 3. In Section 4 we derive the equilibrium contract employing compulsory savings as enforcement to reimburse whereas Section 5 deals with the selection enforcement created by voluntary savings. In Section 6 we present concluding remarks. Technical details involving the proofs of all propositions are presented in the Appendix.

2. Emerging importance of microsavings

For many years, the policymakers and bankers have been taught to believe that “(...) the poor do not save, cannot save, do not trust in financial institutions and prefer non-financial forms of savings (...)” (Robinson, 2001: 228). It kept the microfinance practitioners away from mobilization of savings from the poor. This view was broken with the increasing awareness about numerous informal savings schemes, which have been successful in mobilizing savings from low-income people.

Additionally, MFIs offering both savings and credit products in many countries are argued to have more savers than borrowers, and to have a higher volume of savings than volume of loans (Mata, 2009). Consequently, the demand of microsavings as well as socially and financially benefits of its mobilization is now commonly acknowledged. Contrary to the common believe that the poor spend all their income, which is not even enough to cover their needs, they are inclined to save in order to face survival uncertainties (Karlan and Morduch, 2010; Matin, Hulme and Rutherford, 2002, to name just a few).

A number of studies providing insightful empirical evidence of demand in microsavings was carried out in Bangladesh. Montgomery (1996) conducted case studies of MFIs in Bangladesh and Sri Lanka by providing an evidence of how savings among the variety of other protective mechanisms can help and protect the poor. As opposed to the BRAC approach, SANASA cooperatives in Sri Lanka instead of practicing exclusion of poorer members due to repayment problems, introduced a range of coping mechanisms such
as flexible repayment schedules, open access savings and instant consumption loans. This approach of adapting savings and credit facilities to the poor resulted in a decreased perception of individual risk, more encouragement of mutual trust and support.

After having analyzed six deposit-taking MFIs, Wisniowski (1998) concludes that “...individual and voluntary savings have proven to be the most successful savings products in terms of number of accounts and volume of savings”.

Referring to a series of research, Meyer (2001) stresses the fact that a larger number of poor people, apart from loans, have a great demand for savings, insurance, leasing and other financial services. Evidence from an experimental project implemented by SafeSave (in Bangladesh) since 1996 comes to demonstrate that strong demand exists for voluntary open-access savings among the very poor, and when they are given the possibility they are motivated and capable of saving (Meyer, 2001; Morduch, 1999b).

Dowla and Alamgir (2003) describe how the microfinance industry in Bangladesh has evolved from its main focus on standardized loan products and collecting compulsory savings to the development of flexible savings products. The authors argue that poor people are willing to save once the means of depositing savings are available. Their study demonstrates the process how mandatory savings are used as a precondition for microcredit and how it ensures certain discipline among the members. Furthermore, referring to Morduch (2009), one should outline that “the pattern of borrowing while saving is a regularity in the ‘financial diaries’ collected over a year in villages and poor urban neighborhoods in India, South Africa, and Bangladesh.”

More recent studies, such as Glisovic, El-Zoghbi, and Forster (2010), argue that the “demand for convenient, safe, and affordable savings services” is evidenced in the growing number of savers in deposit-taking institutions over the last decade. They base their arguments on MIXMarket data to show the examples of Grameen Bank and Equity Bank Kenya. Between 2005 and 2008 Grameen Bank added over 2 million new savers and currently, deposits amount to 147 percent of its outstanding portfolio. In the same vein, Equity Bank Kenya, which served over 3 million savers in 2008, has added on average 550,000 new deposit clients per year over the past five years. The main and the most essential expectations for the poor who are willing to save in a financial institution are: convenience (an easy access to savings services), liquidity (an access to savings whenever needed) and security (safety of the savings and stability of the institution that collects them) (Ledgerwood, 1999; Wright, 2003; Deshpande and Glisovic-Mezieres, 2007).

From the MFIs perspective, savings represents long-term financial interest. The evolution of microfinance financing has led the industry to the current situation where regulated financial institutions can access clients’ savings in most jurisdictions. Client deposits have been a dominating source of funding for traditional financial institutions, and they have started to be introduced into MFIs as well (Fehr and Hishiguren, 2006). As mentioned earlier, attracting savings requires certain conditions in order for MFIs to offer those services. These conditions are: enabling macro-economy and some political stability, appropriate regulatory environment; public supervision of MFIs; accountable ownership, effective governance, and consistently good management of its funds; and allocation of high-level management resources to the institution’s microfinance effort (Robinson, 2001). Initially mobilizing highly liquid and small voluntary savings implies increasing operational costs and requires more sophisticated management skills (Ledgerwood, 1999; Fiebig, Hannig and Wisniwski, 1999). However, in long-term deposit taking can be beneficial for MFIs: first, financial self-sufficiency of the institution is fortified - savings represents a relatively stable and less expensive source of funds as compared to the cost of debt; second, savings allows to reduce external dependency; third, savings mobilization allows MFIs to create savings records
of clients which later can serve for relationship management with borrowers (Mata, 2009; Gadway and O’Donnell, 1996; Otero, 1989; Jackelen and Rhyne, 1991). Furthermore, Rutherford (2000) argues that the essence of microfinance credit contracts is that they clearly provide a way to substitute for imperfect savings vehicles.

All of the above mentioned empirical evidence creates solid basis to argue that the ability to mobilize savings can contribute both to meeting demands of the poor and to enhancing MFIs’ long-term sustainability by also reducing their dependency on subsidies (Ledgerwood, 1999; Robinson, 2001). Thus, the claim is clear: encouraging savings is empirically justified, and its development challenge lies in designing a financial technology to serve the specific needs of borrowers and lenders.

This evidence allows us to take another perspective of microsavings for our study. It consists in developing a novel approach to microsavings mobilization, which is based on employing both compulsory and voluntary savings as enforcement and screening mechanism in individual microlending contracts. We show in our model that compulsory savings, which can serve as an enforcement mechanism, lacks means of revealing potential of borrowers in individual lending contracts. To complete this gap, voluntary savings is introduced into the incentive mechanism, thus allowing MFIs to discriminate abilities of loan applicants. At the end of the day, abilities of borrowers boil down to conditioning the probability of projects’ success.

3. The Model

We consider two classes of agents – micro-entrepreneurs and a Microfinance Institution (hereafter MFI) – and three periods. In the first period, micro-entrepreneurs need external funds in order to invest in a risky project. We assume that, because of their poor characteristics, micro-entrepreneurs do not have access to the capital market and/or bank loans and must demand financing to the MFI. In this period, financial contracts are signed between micro-entrepreneurs and the MFI and investment decisions are made. In the two following periods, payoffs on investment are realized and micro-entrepreneurs have to pay for their external funds or can be liquidated. We assume that all parties are risk neutral and protected by limited liability.

3.1 Micro-entrepreneurs’ behaviors

At the initial period \((t = 0)\), micro-entrepreneurs can either choose to borrow one unit of capital (microcredit) in order to undertake a risky project or to yield income from other sources of activities (payroll employment). Let us denote \(Y_t\) with \(t = 1,2\) the value of the income generated by these activities at period 1 and 2. We also take for granted that micro-entrepreneurs have an initial level of income inherited from previous activity and labeled \(Y_0 < 1\), so they lack capital in order to invest.

We assume that, if they decide to invest, all micro-entrepreneurs have access to the same risky project that generates a stochastic payoff equal to 0 (in the case of project failure) or \(R_t\) (if the project is successful) with \(t = 1,2\), the two periods of investment. Project success probability, \(p_i\), depends on the "quality" of the micro-entrepreneur who runs it. Actually, the population of micro-entrepreneurs is divided into two types: "high-ability" and "low-ability" respectively in proportion \(\theta\) and \((1 - \theta)\). If the micro-entrepreneur is of "high-ability", the success probability of his project is equal to \(p_h\) whereas it is equal to \(p_l\) in case
of "low-ability", with $p_h > p_l$. Finally, we define $\gamma > 1$ as the opportunity cost of the fund which is defined as the riskless interest rate of the economy plus the initial value of the investment.

Figure 1 resumes the various possible payoffs of an investment and the corresponding probabilities.

Assumption 1. At period 0, investment projects generate a positive net expected value for period 2 whatever the characteristic of micro-entrepreneurs ("high" or "low-ability") and

$$p_h(R_1 + R_2) > p_l(R_1 + R_2) > \gamma^2$$

(1)

Equation (1) is easily comprehensible. On the one side, project yields payoff $R_1$ in period 1 and $R_2$ in period 2 with probability $p_l$. Thus $(R_1 + R_2)$ represents the total value of the project whereas $p_l(R_1 + R_2)$ is the total expected value at period 0 of a project for period 2. On the other side, $\gamma^2$ measures the total opportunity cost to undertake a project of size 1 for two periods. Consequently, we assume that, at period 0 the expected social value of a risky project undertaken by micro-entrepreneurs is always positive.

Assumption 2. In case of project failure at period 1, only investment projects undertaken by "high-ability" micro-entrepreneurs will generate a positive net expected value at the second period and

$$p_h R_2 > \gamma^2 > p_l R_2$$

(2)

Equation (2) means that a project that fails to generate a payoff at period 1 may have a positive net expected value at period 2 if it is run by "high-ability" micro-entrepreneurs. Consequently, it is efficient from the entire society point of view to allow "high-ability" micro-entrepreneurs to continue their projects even if they failed to generate a positive payoff at period 1. This is not the case for projects run by "low-ability" micro-entrepreneurs.

Finally, because of their poor characteristics and their information opacity, micro-entrepreneurs that decide to invest do not have access to the traditional financial services and must demand financing from a Microfinance Institution in order to obtain a microcredit.
3.2. Microfinance Institution

We assume that the MFI does not have equity capital and raises funds at cost $\gamma$ per period from local or external financial institutions in order to provide small-scale financial services to micro-entrepreneurs. The MFI requires a payoff $r_t \leq R_t$ with $t = 1, 2$ from all micro-entrepreneurs it is financing.

We assume that the MFI faces two types of informational problems in its relationship with micro-entrepreneurs.

On the one hand, we assume that the MFI is not capable to discriminate between "high" and "low" ability micro-entrepreneurs. However, since at period 0 all investment projects have a positive net expected value for period 2 (Assumption 1), this *ex-ante* informational problem does not prevent the MFI to finance micro-entrepreneurs. Nevertheless, problems may occur at period 1 if the financed project fails to generate a positive payoff. In that case, indeed, because of Assumption 2, projects run by "low ability" micro-entrepreneurs must be ended as their net expected value for period 2 is now negative. However, it is still efficient to allow "high ability" micro-entrepreneurs to continue their projects even if they do not generate positive payoff at period 1. Consequently, as the MFI is not able to discriminate between the two types of micro-entrepreneurs, inefficient situations are possible.

On the other hand, we retain a Costly State Verification framework as we assume that micro-entrepreneurs have better information on the profitability of their projects and they can divert a part of the cash from the MFI. More precisely, micro-entrepreneurs may announce $\tilde{R}_t = 0$, $\forall t = 1, 2$ in order not to pay back $r_t$ to the MFI whereas the true value of the payoff is $R_t > 0$. In the literature, it is usually assumed that lenders can conduct costly deterministic audit that reveals the value of the current payoff. However, because of the very specificity of microcredit (low amount, high opacity) we assume that the verification cost is too high to be profitable for the MFI.

Actually, the repayment incentive is based on a new mechanism, namely the requirement of a compulsory savings $\bar{S}_0$ which serves also as a kind of collateral for the MFI. It means that micro-entrepreneurs must save an amount $\bar{S}_0 < Y_0$ prior to signing the lending contract. We assume micro-entrepreneurs cannot withdraw this amount until they totally pay back their loan at the end of period 2. In this case, they receive the initial amount of their savings plus interest which means $\gamma^2 \bar{S}_0$. On the contrary, if micro-entrepreneurs announce $\tilde{R}_0 = 0$ and do not reimburse $r_1$ to the MFI at period 1, we assume that they can be liquidated and the MFI keeps their compulsory saving for an amount $\bar{S}_0 \gamma$ and reimburses $\gamma$ to the market. Liquidation is costless and generates residual value neither for micro-entrepreneurs nor for the MFI. Finally, if micro-entrepreneurs announce $\tilde{R}_0 = 0$ and do not reimburse $r_2$ to the MFI at period 2, we assume that they will lose only their initial savings plus interest. It means that liquidation is only possible at period 1.

Figure 2 resumes the various possible cases and the respective payoff for the micro-entrepreneurs.

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1 See Townsend (1979) for instance
3.3 Incentive mechanisms

We deal with the main incentive conditions that are required in order for the MFI to finance micro-entrepreneurs and for micro-entrepreneurs to undertake a risky project. Let us first begin with incentive for micro-entrepreneurs (with "high" or "low-ability") to announce the true payoff at each period in case they have obtained a loan. We solve for the incentive constrains using backward induction. Recall that \( \hat{R}_t \) is defined as micro-entrepreneurs' payoff announcement to the MFI at period \( t \), and that this announcement may be different from the true payoff \( R_t \).

Lemma 1. If \( S_0 = \frac{r_2}{\gamma^2} < 1 \) as \( r_2 < \gamma^2 \), a micro-entrepreneur that succeeded in period 1 and announced the true payoff \( \hat{R}_1 = R_1 \) will also announce the true payoff \( \hat{R}_2 = R_2 \) and pay back \( r_2 \) to the MFI if his project succeeds in period 2.

Proof of Lemma 1 is the following. Just recall that micro-entrepreneurs must save an amount \( S_0 \) (compulsory saving) prior to obtaining a loan. This savings is invested by the MFI at rate \( \gamma \) and reimbursed at the end of period 2 if micro-entrepreneurs have paid back \( r_1 \) and \( r_2 \). Assume that the project of a micro-entrepreneur succeeded in period 1 and that he announced the true payoff \( \hat{R}_1 = R_1 \) and paid \( r_1 \) to the MFI. In period 2, if his project succeeds, this micro-entrepreneur has two choices. First, he announces the true payoff \( \hat{R}_2 = R_2 \), reimburses \( r_2 \) and receives the initial amount of his savings plus interest \( \gamma^2 S_0 \). Second, he announces the false payoff \( \hat{R}_2 = 0 \), retains the total payoff \( R_2 \) for himself, does not reimburses \( r_2 \) and loses the amount of his saving. The micro-entrepreneurs will always announce the true payoff and reimburse the MFI at period 2 if \( (R_2 - r_2) + S_0 \gamma^2 \geq R_2 \) which leads to \( S_0 \geq \frac{r_2}{\gamma^2} < 1 \) as \( r_2 < \gamma^2 \) (see appendix 1). Actually, the MFI imposes the minimum incentive value for the compulsory saving at the initial period and we have \( S_0 = \frac{r_2}{\gamma^2} \).
According to Lemma 1, a minimum amount $S_0$ of compulsory savings creates an incentive for micro-entrepreneurs to announce the true payoff at period 2.

**Lemma 2.** If $r_i \leq \frac{p_i R_2 - Y_2}{\gamma}$ with $p_i R_2 \geq Y_2$, a micro-entrepreneur that succeeds in period 1 declares $R_t = R_i$, pays back $r_i$ to the MFI and he is encouraged to continue his project in period 2.

The proof of Lemma 2 is in two parts.

First, assume that a micro-entrepreneur (with "high" or "low-ability") succeeds and pays back $r_i$ to the MFI. In that case, he can continue to run his project which gives rise to an expected present value of $\frac{p_i}{\gamma} \left( R_2 - r_i + S_0 \gamma^2 \right)$, $\forall i = h, l$. Note that this value is conditional on the announcement of the true payoff at period 2 in case of success of the project. It means that, at the initial period $t = 0$, the MFI imposes the minimum incentive value for the compulsory saving $S_0 = \frac{r_2}{\gamma^2}$ and $\frac{p_i}{\gamma} \left( R_2 - r_i + S_0 \gamma^2 \right) = \frac{p_i}{\gamma} R_2$. The micro-entrepreneur can also drop his project and find a job in order to receive the present value $\frac{Y_2}{\gamma}$ at period 2. Consequently, he will prefer to continue if $\frac{p_i}{\gamma} R_2 \geq \frac{Y_2}{\gamma}$ or $p_i R_2 \geq Y_2$. As this value must be an incentive even for the "low-ability" micro-entrepreneurs, we must have $p_i R_2 \geq Y_2$ since $p_i < p_h$.

Now, we must prove that a micro-entrepreneur that succeeds in period 1 declares $R_t = R_i$ and pays back $r_i < R_i$ to the MFI. Similar to the previous situation, in period 1, a micro-entrepreneur that succeeds has two choices. First, he announces the true payoff $R_t = R_i$, reimburses $r_i$ and continues his project in order to obtain $R_2$ with a probability $p_i$ at period 2. The expected present value of this action is given by

$$\left( R_i - r_i \right) + \frac{1}{\gamma} p_i \left( R_2 - r_i + S_0 \gamma^2 \right)$$

(3)

Note that this value is still conditional on the true declaration at period 2 in case of success of the project. As in the earlier situation we set $S_0 = \frac{r_2}{\gamma^2}$ and equation (3) becomes

$$\left( R_i - r_i \right) + \frac{1}{\gamma} p_i R_2.$$  

Second, if the micro-entrepreneur announces the false payoff $R_t = 0$, he retains the total payoff $R_t$ for himself, he does not reimburse $r_i$, loses the amount of his saving and finds a job in order to receive $Y_2$ at period 2. The expected present value of this action is given by
\[ R_i + \frac{Y_2}{\gamma} \]. Consequently, the micro-entrepreneur will always announce the true payoff and reimburse the MFI at period 1 if \( (R_i - r_i) + \frac{1}{\gamma} p_i R_2 \geq R_i + \frac{Y_2}{\gamma} \) or \( r_i \leq \frac{p_i R_2 - Y_2}{\gamma} \).

According to Lemma 2, in order to prevent micro-entrepreneurs to drop the project at period 1, risky projects must generate in expectation at period 2 a higher payoff that the payroll employment for the same period.

Finally, we must find the condition under which a micro-entrepreneur, no matter what his ability is, prefers to invest rather than having payroll employment.

**Lemma 3.** \( \forall Y_0, Y_1, Y_2, \) if \( p_i \frac{1}{\gamma} (R_i - r_i) + p_i^2 \frac{1}{\gamma^2} R_2 \geq \frac{1}{\gamma} Y_i + p_i \frac{1}{\gamma^2} Y_2 + S_0 \), at period 0, the micro-entrepreneur prefers to invest in a risky project than to have payroll employment \( \blacksquare \)

At period 0, the micro-entrepreneur may decide to keep his payroll employment and the present expected value of this choice is given by \( Y_0 + \frac{1}{\gamma} Y_1 + \frac{1}{\gamma^2} Y_2 \).

He may also decide to undertake a risky project and to borrow a microcredit from the MFI. In that case, according to the previous incentive conditions, the present expected value of this choice is given by

\[
(Y_0 - S_0) + \frac{1}{\gamma} \left[ (1 - p_i) \frac{1}{\gamma} Y_2 \right] + \frac{1}{\gamma} p_i \left[ (R_i - r_i) + \frac{1}{\gamma} R_2 \right] \quad (4)
\]

Taking into consideration the fact that according to Lemma 1 we have \( S_0 = \frac{r_2}{\gamma^2} \), a micro-entrepreneur decides to undertake a risky project if 
\[ p_i \frac{1}{\gamma} (R_i - r_i) + p_i^2 \frac{1}{\gamma^2} R_2 \geq \frac{1}{\gamma} Y_i + p_i \frac{1}{\gamma^2} Y_2 + S_0. \]

According to Lemma 3, in order to encourage micro-entrepreneurs to undertake a risky project at period 0, this project must generate a higher expected payoff at period 0 than the payroll employment for the same period.

In respect to the conditions stated by Lemma 1 to 3, it is now possible to present the equilibrium contract between micro-entrepreneurs and the MFI that allows the financing of their risky projects. We show that a contract with compulsory savings only is sub-optimal for the whole economy as it prevents "high ability" micro-entrepreneurs to continue their activity in case of project failure at period 1 even if their projects still have a net present value for the economy at period 2.

4. Financial equilibrium contracts with compulsory savings

We assume that the objective of the MFI is to maximize micro-entrepreneurs access to micro-credit, since all projects have a positive net expected value at period 0. However, the MFI must also ensure its financial equilibrium. Consequently, we assume that it behaves
competitively and fixes \( r_1 \) and \( r_2 \) according to a nonprofit condition. Proposition 1 gives the main characteristic of the equilibrium contract between micro-entrepreneurs and the MFI.

**Proposition 1.** Under conditions exposed in lemma 1 to 3 and assuming that 
\[
R_2 > \frac{1}{p_t} \left( \frac{\gamma^2}{2} + Y_2 \right),
\]
there is an equilibrium contract \([S_0^*, r_1^*, r_2^*]\) between micro-entrepreneurs and the MFI with:
\[
S_0^* = \left( 1 - \frac{1}{2} (\theta p_h + (1 - \theta) p_t) \right) < 1, \quad r_1^* = \frac{\gamma}{2}, \quad r_2^* = \gamma^2 \left( 1 - \frac{1}{2} (\theta p_h + (1 - \theta) p_t) \right) < \gamma^2
\]
such that:

a. All micro-entrepreneurs ask for a microcredit at period 0 and are financed;

b. It is always optimal for the MFI to liquidate a project that fails to generate a positive payoff at period 1 whereas all micro-entrepreneurs have an incentive to continue;

c. The MFI is at equilibrium and realizes the nonprofit condition.

Proof: see appendix 1.

Proposition 1 states that compulsory savings can create the incentive condition required in order to allow all micro-entrepreneurs to be financed by the MFI. Moreover, this equilibrium contract maximizes the net expected payoff of micro-entrepreneurs since the MFI just received the minimum payoffs \( (r_2^*, r_1^*) \) that ensure its financial equilibrium. Nevertheless, this equilibrium contract cannot solve the ex-ante informational problem faced by the MFI and it is still impossible for it to discriminate between "high" and "low-ability" micro-entrepreneurs. Consequently, it is always optimal for the MFI, in order to minimize its losses, to liquidate a project that fails to generate a positive payoff at period 1.

**Proposition 2.** The equilibrium contract \([S_0^*, r_2^*, r_1^*]\) is socially non-optimal since a fraction \( \theta (1 - p_h) \) of positive net expected value projects are liquidated in period 1 whereas their social value is positive.

Proof of Proposition 2 is obvious since there is a proportion \( \theta \) of "high-ability" micro-entrepreneurs that are financed in the initial period and that fails to reimburse \( r_1^* = \frac{\gamma}{2} \) at period 1 with a probability \( (1 - p_h) \). According to the proposition 1, the projects of these micro-entrepreneurs are liquidated by the MFI even if their social value is positive. Consequently, there is a net expected loss of \( \theta (1 - p_h) (p_h R_2 - \gamma^2) \) for the whole society.

Proposition 1 and 2 allow us to derive two main results. Firstly, there is an equilibrium contract between micro-entrepreneurs and the MFI which permits to give an equal access to all types of loan applicants. This contract is based on compulsory saving as an equilibrium solution for the lender to induce the repayment by the borrower when attributing the loan. This equilibrium contract also enlarges the number of financed projects emphasizing the social outreach of micro lending. Secondly, we show that this equilibrium contract is non-optimal since it cannot prevent the MFI to liquidate efficient project at period 1. Consequently, the question is what kind of mechanism could be put into practice to help to discriminate borrowers’ ability and continue to finance "promising" projects? We further demonstrate that in order to reestablish optimality we can introduce voluntary savings to
incentivize non-delinquent and "high-ability" micro-entrepreneurs to declare their abilities. These more complete lending contracts including voluntary savings need an adjustment of interest rates for the sake of creating a positive social value and not making "high-ability" micro-entrepreneurs suffer from the burden of "low-ability" borrowers.

5. Financial equilibrium contracts with compulsory and voluntary savings

We now consider equilibrium contracts with compulsory and voluntary savings. We assume that the MFI offers two different contracts to micro-entrepreneurs: one with compulsory savings only (hereafter CSC for Compulsory Savings Contract) and the other with compulsory and voluntary savings (hereafter VSC for Voluntary Savings Contract). In fact, many different saving mechanisms have evolved directly in the field itself. First, we retain compulsory savings as the integral part of the credit, which is required as part of loan terms. Second, following the literature on savings services for the poor (Glisovic, El-Zoghbi, and Forster, 2010; Hirschland, 2005), we defining voluntary savings as time/certificate deposit practiced, in particular, by regulated MFIs such as credit unions or savings banks. This savings service allows micro-entrepreneurs to decide on voluntary basis to make a single deposit that cannot be withdrawn for a specific period of time. At the appointed time, the micro-entrepreneur can withdraw the saved amount.

5.1. Structure of the contracts

The two contracts (CSC and VSC) have the following characteristics:

If a micro-entrepreneur chooses a CSC, he must provide an amount $S_{\text{CSC}}$ of compulsory savings in order to be financed at period 0 and he will be liquidated if he fails to reimburse to the MFI at period 1. On the contrary, if the micro-entrepreneur pays back at period 1, he can continue his project for period 2. At that period, he pays back to the MFI in case of success or losses the amount of his savings. This contract is thus similar to the one we have just described in the previous sections;

If a micro-entrepreneur chooses a VSC, he must provide an amount $S_{\text{VSC}} + S_v$ of savings respectively composed of compulsory and voluntary savings in order to be financed at period 0. He can continue his project for period 2 no matter he pays back or not to the MFI at period 1. However, in the case he does not pay back to the MFI at period 1, the MFI will retain the voluntary part of his savings ($S_v$) at period 2. At that period, he pays back to the MFI in case of success or losses the amount of his compulsory savings.

In order to be optimal for micro-entrepreneurs and for the MFI, contracts are designed in such a way that CSC will be chosen by "low-ability" micro-entrepreneurs (whose projects have a negative expected value for period 2 if they fail at period 1) whereas VSC will be chosen by "high-ability" micro-entrepreneurs (whose projects have a positive expected value for period 2 even if they fail at period 1).

As before, the compulsory part of the savings is used by the MFI in the two contracts in order to encourage micro-entrepreneurs to reimburse their loan plus interest at the end of period 2, and the conditions stated in Lemma 1 still hold. Moreover, we assume that the amount of compulsory savings will be the same between the two contracts, the difference between contracts being linked with the voluntary part of the savings.
If we name \( r_{2, \text{CSC}} \) and \( r_{2, \text{VSC}} \) respectively the interest rate charges by the MFI at period 2 on CSC and VSC, we have \( S_{\text{CSC}} = \frac{r_{2, \text{CSC}}}{\gamma^2} \) and \( S_{\text{VSC}} = \frac{r_{2, \text{VSC}}}{\gamma^2} \) the compulsory part of the savings of the two contracts and at equilibrium we must have \( S_{\text{CSC}} = S_{\text{VSC}} = \frac{r_{2, \text{VSC}}}{\gamma^2} \) which means \( r_{2, \text{VSC}}^* = r_{2, \text{CSC}}^* \).

In the same vein, the MFI will charge the minimum interest rate at period 1 that is consistent with the incentive constraint to pay back at that period. For a CSV, this incentive constraint is similar to the one stated by Lemma 2 and we have \( r_{1, \text{CSC}}^* = \frac{\gamma}{2} \). For a VSC, we must find the incentive value \( r_{1, \text{VSC}}^* \) that encourages micro-entrepreneurs to pay back to the MFI in case of success at period 1.

Lemma 4. If \( r_{1, \text{VSC}}^* = \gamma p \mu S_v \) a micro-entrepreneur that succeeds in period 1 declares \( \tilde{R}_1 = R_1 \), pays back \( r_{1, \text{VSC}}^* \) to the MFI and he is encouraged to continue his project in period 2 ■

|Proof| see appendix 2.|

Finally, the voluntary part of the savings will be designed in such a way that only "high-ability" micro-entrepreneurs will choose a VSC at the initial period. This equilibrium level of voluntary savings is given by Proposition 3.

**Proposition 3.** Recall that \( S_{\text{CSC}} = S_{\text{VSC}} \) and assume that \( R_2 > \frac{Y_2 + \gamma^2}{p_h + p_l - 1} \), if the level of voluntary saving \( S_v^* \) is such that:

\[
p_h + (1 - p_h) \left( \frac{R_2 p_h - Y_2}{\gamma^2} \right) > S_v^* > p_l + (1 - p_l) \left( \frac{R_2 p_l - Y_2}{\gamma^2} \right)
\]

"High-ability" micro-entrepreneurs will choose a VSC whereas "low-ability" micro-entrepreneurs will prefer a CSC ■

**Proof of Proposition 3:** see appendix 3.

5.2. Financial equilibrium contracts with compulsory and voluntary savings

As before, we assume that the objective of the MFI is to maximize micro-entrepreneurs’ access to microcredit and that the MFI behaves competitively and fixes the various interest rates according to a nonprofit condition. Proposition 4 gives the main characteristic of the two equilibrium contracts between micro-entrepreneurs and the MFI.

**Proposition 4.** Under conditions exposed in Lemma 1 to 4 and Proposition 3, and assuming that \( p_h R_2 - \frac{p_h p_l \gamma^2}{2(1 - p_h (1 - p_l))} < Y_2 < p_h R_2 - \left( \frac{p_h [1 - p_h (1 - p_l)] - p_l}{2(1 - p_h (1 - p_l))(1 - p_h)} \right) \gamma^2 \), the MFI will offer two equilibrium contracts:
a. A Compulsory Savings Contract (CSC) such that $[\bar{S}_{CSC}^*, r_{1CSC}^*, r_{2CSC}^*]$ with

$$S_{CSC}^* = \left(1 - \frac{1}{2}p_i\right), \quad r_{1CSC}^* = \frac{\gamma}{2} \text{ and } r_{2CSC}^* = \gamma^2 \left(1 - \frac{1}{2}p_i\right),$$

that will be chosen by all "low-ability" micro-entrepreneurs.

b. A Voluntary Savings Contract (VSC) such that $[\bar{S}_{VSC}^*, S_v^*, r_{1VSC}^*, r_{2VSC}^*]$ with

$$S_{VSC}^* = \left(1 - \frac{1}{2}p_i\right), \quad S_v^* = \frac{p_i}{2(1 - p_h + p_h p_i)}, \quad r_{1VSC}^* = \gamma \left(\frac{p_h^2}{1 - p_h + p_h p_i}\right), \quad \text{and}$$

$$r_{2VSC}^* = \gamma^2 \left(1 - \frac{1}{2}p_i\right),$$

that will be chosen by all "high-ability" micro-entrepreneurs.

These two equilibrium contracts are optimal since all "high-ability" micro-entrepreneurs, with a positive net expected value projects at period 1 are not liquidated.

In such a separating equilibrium, on one hand "high-ability" micro-entrepreneurs will choose the VSC. They are encouraged to pay back the MFI in case of success of their project but are not liquidated in case of failure (the MFI just keep the voluntary part of the saving at the end of period 2). Consequently, projects undertaken by "high-ability" micro-entrepreneurs, with a positive net expected value at period 1, can be financed for the second period. Note that this equilibrium contract is less costly for "high-ability" micro-entrepreneurs than equilibrium contract with compulsory savings only since the MFI can discriminate between micro-entrepreneurs.

On the other hand, "low-ability" micro-entrepreneurs will choose the CSC. If they fail to pay back to the MFI at period 1, they are liquidated. On the contrary, if their projects succeed in period 1, they can continue and reimburse the MFI at period 2 in case of success. In all cases, the MFI will keep the compulsory part of the project in case of failure at period 2. Finally, note that the VSC enables MFIs to discriminate types of micro-entrepreneurs and consequently, projects of "high-ability" micro-entrepreneurs with positive net expected value at period 1 are not liquidated any longer. Hence, these new contracts come to solve the inefficiency of the incentive mechanisms offered in contracts with compulsory savings only. The reason of that inefficiency is explained by the fact that in case of repayment default at period 1 projects of "high-ability" micro-entrepreneurs, which potentially could have a profitable outcome, are ended. Thus, the results of this section show that the new contracts based on both compulsory and voluntary microsavings go hand in hand with the twofold mission of MFIs: firstly, providing the poor with an equal access to financial intermediation and secondly, which is even more important, increasing the social value of lending from the entire society point of view.

6. Concluding remarks

Microfinance evolution has led to establishment of novel lending methods to poor households and small-scale entrepreneurs. Usual incentive mechanisms have been mainly expressed by joint liability in group lending, progressive lending and regular repayment schedules in individual lending contracts. Both in practice and academic writings it remains evident that savings gains important interest in microfinance programmes and constitutes a significant pillar of MFIs’ service portfolio (Robinson, 2001; Meyer, 2001; Dowla and Alamgir, 2003; Matin, Hulme and Rutherford, 2002). This study aims to suggest a more complete incentive
mechanism based on coupled lending contracts which employ compulsory and voluntary savings.

Our model shows that introducing voluntary savings into the incentive mechanism enables discriminating abilities of borrowers and addressing screening problems in microfinance programmes. Voluntary savings allows borrowers to have a possibility of revealing the potential of their projects and to continue undertaking the project even if they have not paid at the end of the first period. The beauty of the model consists in giving all borrowers equal chances of obtaining a loan once they agree to invest compulsory savings in the initial stage. This serves as high certainty about the enforceability of repayments. The dark side consists of bringing this mechanism closer to traditional bank loans which require obligatory collateral. However, it does not allow MFIs to control for probability of projects’ success.

A related question is what the optimal microlending contract is. For instance, how can the borrower be induced to reveal its ability after the first installment has or has not been done? In this model we have shown that there is an equilibrium contract when borrowers are required to invest compulsory savings in order to access the loan. Though, as mentioned above this guarantee is not optimal as it does not allow to discriminate abilities of borrowers. As shown, voluntary savings can complete this picture and together with compulsory savings can serve as an effective approach for MFIs to face adverse selection and moral hazard issues.

By acknowledging certain limits of requiring savings we argue that it can raise a number of additional issues such as coping with country regulations, providing both convenience and security, finding profitable reinvestment opportunities, etc. (Morduch, 1999b).

To conclude, the theoretical literature on individual lending incentive mechanisms to which this paper contributes, has proposed many different ways in which those mechanisms can enable MFIs to solve enforcement and screening issues. Researchers increasingly are interested in exploring savings as dynamic incentives. However, empirical work testing the effect of the specific instrument of combining compulsory and voluntary savings in a microlending contract has lagged behind theoretical work on the topic. Apart from pure academic interest, such evidence could help to gain a further understanding of the importance and necessity of current use of savings in developing more enhanced incentive mechanisms that can help achieve the full promise of microfinance programmes. An interesting implication of our model is that by employing both compulsory and voluntary savings as an incentive mechanism MFIs can generate positive social value even when carrying on projects which do not perform enough well in the beginning.

An important next step will be to investigate to what type of MFIs’ ownership structure this enforcement and screening mechanism will contribute the best. As here we are not in the frame of credit-only programmes further research can collect data that allow careful quantification of the roles of these two savings in risk and liquidity management issues and internal control mechanisms of MFIs.
Appendix

Appendix 1: proof of Proposition 1.

Part a. of Proposition 1 is obvious as under incentives conditions state by Lemma 1 to 3, all micro-entrepreneurs decide to ask for a microcredit at period 0.

Proof of part b.

The net expected value for the MFI to end a project that fails to give a positive payoff at period 1 is given by

$$\bar{S}_0 \gamma - \gamma = \frac{r^*_2}{\gamma} - \gamma < 0 \quad \text{with} \quad \bar{S}_0 = \frac{r^*_2}{\gamma} \quad \text{and} \quad r^*_2 < \gamma^2$$

It means that the MFI keeps the savings plus interest of the micro-entrepreneurs and reimburses the fund.

The net expected value for the MFI to continue a project that fails to give a positive payoff at period 1 is given by

$$\theta \left[ p_h r^*_2 + (1 - p_h) \bar{S}_0 \gamma^2 \right] + (1 - \theta) \left[ p_h r^*_2 + (1 - p_h) \bar{S}_0 \gamma^2 \right] - \gamma^2$$

as the MFI cannot make a distinction between "high" and "low" ability micro-entrepreneurs.

Finally, as \( \bar{S}_0 = \frac{r^*_2}{\gamma^2} \), we obtain

$$r^*_2 - \gamma^2 < 0$$

Note that \( \left| r^*_2 - \gamma^2 \right| > \left| \frac{r^*_2}{\gamma} - \gamma^2 \right| \) for \( \gamma > 1 \) and the loss of the MFI is lower in the first case than in the second one. Consequently, in order to minimize its loss, a MFI will still liquidate a project that fails to give a positive payoff at period 1.

Proof of part c.

Under the conditions stated by Lemma 1 to 3, the MFI will choose \( r^*_2 \) in order to ensure its financial equilibrium (non profit condition).

The condition of a zero net expected profit for the MFI is given by

$$\Pi_{MFI} = \theta \left[ \frac{1}{\gamma} (1 - p_h) \left( \bar{S}_0 \gamma - \gamma \right) + \frac{1}{\gamma} p_h \left( r^*_1 + \frac{1}{\gamma} \left[ (1 - p_h) \bar{S}_0 \gamma^2 + p_h r^*_2 - \gamma^2 \right] \right) \right]$$

$$+ (1 - \theta) \left[ \frac{1}{\gamma} (1 - p_h) \left( \bar{S}_0 \gamma - \gamma \right) + \frac{1}{\gamma} p_h \left( r^*_1 + \frac{1}{\gamma} \left[ \left( (1 - p_h) \bar{S}_0 \gamma^2 + p_h r^*_2 - \gamma^2 \right) \right] \right) \right] = 0$$

Substituting \( \bar{S}_0 = \frac{r^*_2}{\gamma^2} \) in the previous equation we obtain after simplification \( \Pi_{MFI} = 0 \) for...
\[ r_2^* = \gamma^2 - r_1^* \gamma \left[ (\theta p_h + (1 - \theta) p_l) \right] \leq \gamma^2 \] and \( r_2^* > 0 \)

As \( 0 < (\theta p_h + (1 - \theta) p_l) < 1 \) for \( \theta < 1, 0 < p_h, p_l < 1 \)

Note that the optimal value of the reimbursement at period 2 (payoff of the MFI) depends on the value charged by the MFI at period 1. We assume here that the MFI will smooth the total charge of the funds on the two periods and \( r_1^* = \frac{\gamma}{2} \) and

\[ r_2^* = \gamma^2 \left[ 1 - \frac{1}{2} (\theta p_h + (1 - \theta) p_l) \right] < \gamma^2. \]

Moreover, according to Lemma 2, we must have

\[ r_1 \leq \frac{p_i R_2 - Y_2}{\gamma} \]

which means that \( R_2 > \frac{1}{p_i} \left( \frac{\gamma^2}{2} + Y_2 \right) \). Finally,

\[ \bar{S}_0 = \frac{r_2^*}{\gamma^2} \leq \left[ 1 - \frac{1}{2} (\theta p_h + (1 - \theta) p_l) \right] < 1. \]

The proof of Proposition 1 is completed ■

Appendix 2: proof of Lemma 4.

If a micro-entrepreneur (with "high" or "low" ability) chooses a VSC, he will be certain to continue his project even if he does not pay back to the MFI at period 1. The expected value of a project at period 1 for a micro-entrepreneur that decides to pay back \( r_{1,\text{VSC}}^* \) to the MFI if he succeeds is given by

\[ \left( R_i - r_{1,\text{VSC}}^* \right) + \frac{1}{\gamma} p_i \left( R_2 - r_{2,\text{VSC}} + \gamma^2 \left( \bar{S}_{\text{VSC}} + S_\gamma \right) \right) \text{ with } i = h, l \]

On the contrary, the expected value of a project at period 1 for a micro-entrepreneur that decides not to pay back \( r_{1,\text{VSC}}^* \) to the MFI if he succeeds is given by

\[ R_i + \frac{1}{\gamma} p_i \left( R_2 - r_{2,\text{VSC}} + \gamma^2 \bar{S}_{\text{VSC}} \right) \] (Recall that in this case, the MFI will retain the voluntary part of the savings).

Consequently, a micro-entrepreneur will pay back if

\[ \left( R_i - r_{1,\text{VSC}}^* \right) + \frac{1}{\gamma} p_i \left( R_2 - r_{2,\text{VSC}} + \gamma^2 \left( \bar{S}_{\text{VSC}} + S_\gamma \right) \right) \geq \left( R_i + \frac{1}{\gamma} p_i \left( R_2 - r_{2,\text{VSC}} + \gamma^2 \bar{S}_{\text{VSC}} \right) \right) \]

Using the fact that \( \bar{S}_{\text{CSC}} = \frac{r_{2,\text{CSC}}}{\gamma^2} \) and \( \bar{S}_{\text{VSC}} = \frac{r_{2,\text{VSC}}}{\gamma^2} \), means that we must have \( S_\gamma p_i \geq r_{1,\text{VSC}}^* \). As this condition must be an incentive for "high" and "low" ability micro-entrepreneurs we must have \( S_\gamma p_i \geq r_{1,\text{VSC}}^* \) for \( p_h > p_l \). Consequently, the MFI will choose the value of \( r_{1,\text{VSC}}^* = S_\gamma p_i \) that creates an incentive for both the "high" and "low-ability" micro-entrepreneurs ■

Appendix 3: proof of Proposition 3.
The net expected value of a project undertaken by "low-ability" micro-entrepreneurs and financed by a VSC is given by

\[ (Y_0 - \overline{S}_{VSC} - S_e) + p_l \left( \frac{1}{\gamma} \left( R_1 - r^*_{1,VSC} \right) + p_l \frac{1}{\gamma^2} \left( R_2 - r_{2,VSC} + (\overline{S}_{VSC} + S_e) \right)^2 \right) + \\
\left( 1 - p_l \right) \left[ p_l \frac{1}{\gamma^2} \left( R_2 - r_{2,VSC} + \overline{S}_{VSC} \gamma^2 \right) \right] \]

This gives after simplification

\[ (Y_0 - \overline{S}_{VSC} - S_e) + p_l \left( \frac{1}{\gamma} \left( R_1 - \frac{\gamma}{2} \right) + p_l \frac{1}{\gamma^2} \left( R_2 - r_{2,VSC} + \overline{S}_{VSC} \gamma^2 \right) \right) + \left( 1 - p_l \right) \frac{1}{\gamma^2} Y_2 \]

Similarly, the net expected value of a project undertaken by "low-ability" micro-entrepreneurs and financed by a CSC is given by (using \( r^*_{CSC} = \frac{\gamma}{2} \))

\[ (Y_0 - \overline{S}_{CSC}) + p_l \left( \frac{1}{\gamma} \left( R_1 - \frac{\gamma}{2} \right) + p_l \frac{1}{\gamma^2} \left( R_2 - r_{2,CSC} + \overline{S}_{CSC} \gamma^2 \right) \right) + \left( 1 - p_l \right) \frac{1}{\gamma^2} Y_2 \]

This gives after simplification

\[ (Y_0 - \overline{S}_{CSC}) + p_l \left( \frac{1}{\gamma} \left( R_1 - \frac{\gamma}{2} \right) + p_l^2 \frac{R_2}{\gamma} + \left( 1 - p_l \right) \frac{1}{\gamma^2} Y_2 \right) \]

Consequently, a "low-ability" micro-entrepreneur will choose a CSC contract at period 0 if

\[ (Y_0 - \overline{S}_{CSC}) + p_l \left( \frac{1}{\gamma} \left( R_1 - \frac{\gamma}{2} \right) + p_l \frac{1}{\gamma^2} \left( R_2 - r_{2,CSC} + \overline{S}_{CSC} \gamma^2 \right) \right) + \left( 1 - p_l \right) \frac{1}{\gamma^2} Y_2 > \]

And using the fact that \( r^*_{1,VSC} = S_e \gamma p_l \), we obtain

\[ S_v > S_{e_{\min}} \equiv \frac{1}{2} p_l + \left( 1 - p_l \right) \frac{\left( R_2 p_l - Y_2 \right)}{\gamma^2} \]

Using the same reasoning, we find that a "high-ability" micro-entrepreneur will choose a VSC at period 0 if

\[ S_v < S_{e_{\max}} = \frac{1}{2} p_h + \left( 1 - p_h \right) \frac{\left( R_2 p_h - Y_2 \right)}{\gamma^2} \]

Finally, we must have \( S_{e_{\max}} > S_{e_{\min}} \) which is true if \( R_2 > \frac{Y_2 + \gamma^2}{p_h + p_l - 1} \) and this condition is always realized when \( R_2 > \frac{1}{p_l} \left( \frac{\gamma^2}{2} + Y_2 \right) \) as required in proposition 2.

Consequently, the MFI must fix the level of the voluntary savings such that

\[ S_{e_{\max}} > S_v > S_{e_{\min}} \]
Appendix 4: proof of Proposition 4.

Let us begin by CSC. The MFI knows that "low-ability" micro-entrepreneurs will choose a CSC. They will reimburse $r_{1,\text{CSC}}^* = \frac{\gamma}{2}$ in case of success at period 1 or they will be liquidated.

If they are not liquidated, they will continue their project and will pay back $r_{2,\text{CSC}}^*$ to the MFI at the end of period 2 in case of success. Otherwise, in case of failure of the project at period 2, the MFI will retain the level of the compulsory savings.

The net expected profit linked with that contract is given by

$$\Pi_{\text{MFI,CSC}} = \frac{1}{\gamma} (1 - p_1) \left( S_{\text{CSC}} \gamma - \gamma \right) + \frac{1}{\gamma} p_1 \left( r_{1,\text{CSC}}^* + \frac{1}{\gamma} \left( (1 - p_1) S_{\text{CSC}} \gamma^2 + p_1 r_{2,\text{CSC}}^* - \gamma^2 \right) \right)$$

The MFI will choose $r_{2,\text{CSC}}^*$ such that $\Pi_{\text{MFI,CSC}} = 0$ and $r_{2,\text{CSC}}^* = \gamma \left( 1 - \frac{1}{2} p_1 \right) < \gamma^2$.

Consequently, we have $S_{\text{CSC}}^* = \frac{r_{2,\text{CSC}}^*}{\gamma^2} = \left( 1 - \frac{1}{2} p_1 \right)$.

Let us turn our attention to VSC. According to Proposition 3, the MFI will choose a level of voluntary savings such that $S_{\text{Vmax}} > S_v > S_{\text{Vmin}}$ in order to be sure that "high-ability" micro-entrepreneurs will choose this contract whereas "low-ability" micro-entrepreneurs will choose the CSC. We will name this equilibrium value of the voluntary savings $S_v^*$.

The MFI knows that "high-ability" micro-entrepreneurs will choose VSC, and pay back respectively $r_{1,\text{VSC}}^*$ and $r_{2,\text{VSC}}^*$ if their projects succeed at period 1 and period 2. In that case, the MFI will give back to micro-entrepreneurs the compulsory and the voluntary part of their savings. However, in case of failure of the project at period 1, the project is not liquidated but the MFI will retain the voluntary part of the savings even if the project succeeds at period 2. Finally, in case of failure of the project at period 2, the MFI will retain the total level of the savings (compulsory plus voluntary). The net expected profit associated with that contract is given by

$$\Pi_{\text{MFI,VSC}} = \frac{1}{\gamma} \left( 1 - p_h \right) \left[ r_{1,\text{VSC}}^* + \frac{1}{\gamma} \left( p_h r_{2,\text{VSC}}^* + (1 - p_h) S_{\text{VSC}}^* \gamma^2 \right) \right] + \frac{1}{\gamma} (1 - p_h) \left[ \frac{1}{\gamma} p_h \left( r_{2,\text{VSC}}^* + S_{\text{VSC}}^* \gamma^2 \right) + \frac{1}{\gamma} (1 - p_h) \left( S_{\text{VSC}}^* + S_v^* \right) \gamma^2 \right] - 1$$

The MFI will choose $r_{2,\text{VSC}}^*$ such that $\Pi_{\text{MFI,VSC}} = 0$ and $r_{2,\text{VSC}}^* = \gamma \left( 1 - S_v^* (1 - p_h) - \frac{p_h r_{1,\text{VSC}}^*}{\gamma} \right) < \gamma^2$.

Note that the equilibrium value of the interest rate at period 2 is still dependent of the value of the rate fixed by the MFI at period 1.

Substituting the equilibrium value of $r_{1,\text{VSC}}^*$ in $r_{2,\text{VSC}}^*$ we obtain

$$r_{2,\text{VSC}}^* = \gamma^2 \left( 1 - S_v^* (1 - p_h) - S_v^* p_h p_1 \right) < \gamma^2$$
Consequently, we have \( S_{VSC}^* = \frac{r_{LVSC}^*}{\gamma^2} = 1 - S_v^*(1 - p_h) - S_v^* p_h p_t \). Since the compulsory part of the savings is identical in the two contracts we must have \( S_{VSC}^* = 1 - S_v^*(1 - p_h) - S_v^* p_h p_t = \left(1 - \frac{1}{2} p_t\right) < 1 \) which gives the equilibrium value of the voluntary part of the savings \( S_v^* = \frac{p_t}{2(1 - p_h + p_h p_t)} < 1 \) for \((1 - p_t)(1 - p_h) > 0\) which is always true.

Substituting this equilibrium value in \( r_{LVSC}^* \) we obtain \( r_{LVSC}^* = \frac{\gamma}{2} \left(\frac{p_t^2}{(1 - p_h + p_h p_t)}\right) < \frac{\gamma}{2} = r_{LCSC}^* \)

The total amount of savings linked with the VSC is given by \( S_{VSC}^* + S_v^* = \left(1 - \frac{1}{2} p_t\right) + S_v^* < 1 \) if \( p_h (1 - p_t) > 0 \) which is always true.

Finally, we must have \( S_{v_{min}}^* < S_v^* = \frac{p_t}{2(1 - p_h + p_h p_t)} < S_{v_{max}}^* \) which is true if

\[
p_t R_2 - \frac{p_t p_h \gamma^2}{2(1 - p_h (1 - p_t))} < Y_2 < p_t R_2 - \frac{\gamma^2}{2(1 - p_h (1 - p_t)) (1 - p_h)}
\]

The proof is completed \( \blacksquare \)
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