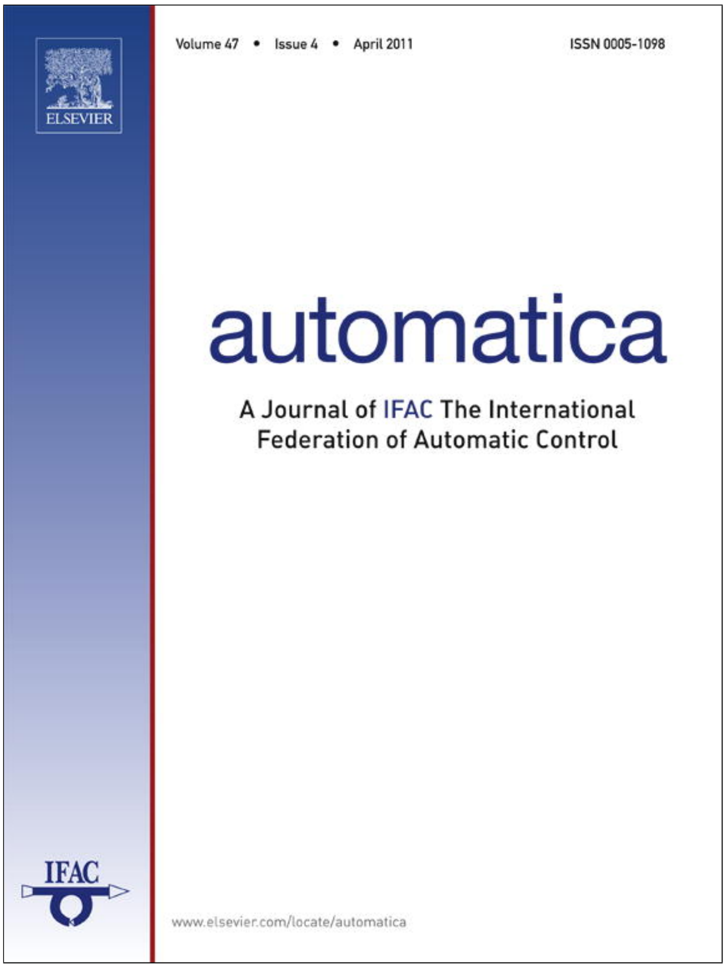
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Brief paper

Formation control using range-only measurements☆

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ABSTRACT

This paper proposes algorithms to coordinate a formation of mobile agents when the agents are *not* able to measure the relative positions of their neighbors, but only the distances to their respective neighbors. In this sense, less information is available to agents than is normally assumed in formation stabilization or station keeping problems. To control the shape of the formation, the solution advanced in the paper involves subsets of non-neighbor agents cyclically localizing the relative positions of their respective neighbor agents while these are held stationary, and then moving to reduce the value of a cost function which is nonnegative and assumes the zero value precisely when the formation has correct distances. The movement schedule is obtained by a novel vertex-coloring algorithm whose computation time is linear in the number of agents when implemented on the graphs of minimally rigid formations. Since in some formations, it may be that an agent is never allowed to be stationary (e.g. it is a fixed-wing aircraft), or because formations may be required to move as a whole in a certain direction, the results are extended to allow for cyclic localization of agents in this case. The tool used is the Cayley–Menger determinant.

1. Introduction

In recent years, teams of sensor-equipped autonomous robotic platforms have been utilized in a wide range of applications, such as infrastructure security, environment and habitat monitoring, industrial sensing, traffic control and so on Chong and Kumar (2003). It has been demonstrated that, when coordinating robotic agents to cooperatively execute a task, it is sometimes advantageous to have them maneuvering in a formation (Leonard et al., 2007). However, it is challenging to coordinate a formation of mobile agents when each agent can *only* measure ranges (distances) to their neighbors, but *not* their neighbors' relative positions. Even though in most of the previous solutions to the formation control problem in similar settings, control is secured through maintenance of distances, a richer measurement set is required than simply those distances (Cao, Morse, Yu, Anderson, & Dasgupta, 2011; Krick, Broucke, & Francis, 2009; Olfati-Saber & Murray, 2002; Yu, Anderson, Dasgupta, & Fidan, 2009). For example, using the popular gradient-descent approach (Cao et al., 2011; Krick et al., 2009) to implement the control law, the corresponding agent has to know exactly in real time in which directions (and at what distance) its neighbors lie.

To control the shape of a formation using range-only measurements, we follow a Lyapunov function approach. As far as agents are concerned, the key is for a particular agent to switch cyclically between periods of (a) identification (localization of the positions of other agents), (b) control, in which the agent moves to a position causing a decrease in the Lyapunov function, and (c) resting, i.e. remaining stationary while other agents engage in identification (localization) and control (motion to reduce the Lyapunov function). A novel vertex-coloring algorithm (Diestel, 1997) can thus be designed for any minimally rigid formation (Anderson, Yu, Fidan, & Hendrickx, 2008) to obtain an efficient movement schedule.

In some applications, it may be that agents in a formation are never allowed to be stationary, or that the formation itself is supposed to acquire a certain shape and at the same time move as a formation with a certain velocity v. When no active

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communications are allowed and the information about v is not known by some agents, we then need to design algorithms using range-only measurements for such agents to compute v. For both of these situations, we extend the localization idea to allow one agent to repeatedly measure its distance to a neighbor agent, possibly a leader, at discrete time instants, and then process the data using the Cayley–Menger determinant (Blumenthal, 1953; Crippen & Havel, 1988), a convenient tool from distance geometry, to both localize the neighbor/leader *and* learn its velocity.

The rest of the paper is organized as follows. In Section 2, we formulate the formation control problem with range-only measurements. In Sections 3 and 4, the motions for individual agents and the agents as a team are discussed and a cyclic stop-and-go strategy is proposed. When the agents are not allowed to be stationary or the formation is required to move with a constant velocity, we can still apply the cyclic stop-and-go strategy by attaching the agents' local coordinate systems rigidly to some reference framework that moves with a constant velocity. Then in Section 5, an algorithm is presented for a follower agent to compute a leading agent's velocity using only distance information.

2. Multi-agent formation

We assume the mobile agents can be described by kinematic points. We consider a formation in $\mathbb{R}^2$ with the underlying graph $\mathbb{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ and $\mathcal{E}$ denote the vertex set and edge set of the graph. An edge joining vertices i, j is present in $\mathcal{E}$ if and only if the desired distance to be maintained between agents i, j is specified; call this edge (i, j) and call the specified distance d_{ij}^*. We assume that the specified distances are indeed achievable by an appropriate set of vertex positions. We assume further that each agent can measure distances to its neighbors in the graph (but *not* the relative positions), and can maintain a map of its own motion into which it can insert estimates of the positions of other agents, in every case using its own local coordinate system. Each agent has a distinguishing label, and any agent is able to correctly associate each measured distance with the particular neighbor in question. When the agents are initially located at positions for which the distance constraints are not satisfied, the control task is to bring the formation to the correct distances. Unless otherwise specified, we shall assume that the underlying graph of the formation is *generically minimally rigid* and the associated formation is *minimally rigid* (Anderson et al., 2008). Note that the orientation and location of the centroid of the formation are irrelevant for this purpose; an extended version of the problem treated in Section 5, however, envisages assuring that the agents learn the velocity of a leading agent. The leader's velocity is assumed to be constant and initially unknown, and the followers take up positions while moving with the same velocity as the leader.

We shall measure the closeness of a particular formation shape to a desired formation shape by a type of Lyapunov function. For those edges (i, j) in the edge set $\mathcal{E}$, let d_{ij} denote the instantaneous distance between the associated agents. Let $p_i \in \mathbb{R}^2$ denote the coordinate vector of agent i. The Lyapunov function is defined as

$$W(p_1(t), p_2(t), \ldots, p_n(t)) = \sum_{(i,j)\in\mathcal{E}} \left(d_{ij}^2(t) - d_{ij}^{*2}\right)^2. \tag{1}$$

Actually, there is considerable freedom in specifying the Lyapunov function. For example, we can replace the individual summands in this equation by terms $\phi(d_{ij}^2, d_{ij}^{*2})$ where ϕ is a smooth and convex function of d_{ij}^2, parametrized by d_{ij}^{*2}, and with a minimum at $d_{ij} = d_{ij}^*$.

3. Motion of a single agent

One moving agent can evidently localize a second stationary agent if it has the ability to map its own motion within its own local coordinate system and the ability to repeatedly or continuously measure the distance to any other agent, and does not move just in a straight line. Consider an agent, A say, with $n_A > 0$ neighbors, the set of which is denoted by $\mathcal{N}_A$. Suppose A has just localized the agents in $\mathcal{N}_A$. Then A could at once move to a position to minimize the following agent-specific potential function, other agents remaining stationary:

$$W_A(p_1, p_2, \ldots, p_n) = \sum_{(A,j)\in\mathcal{E}} \left(d_{Aj}^2 - d_{Aj}^{*2}\right)^2. \tag{2}$$

If there are several positions that minimize the cost function, the agent randomly picks one of them and stays there until the cost function can be further reduced as a result of the movement of its neighbors. Of course, the agent does not move if the function happens to be already minimized, even if there is a second position at which the same value of the agent-specific cost function is achieved. Although each agent only minimizes its own agent-specific potential function, we want to show in the next section that it is possible to coordinate the agents in such a way so that the value of the Lyapunov function of the overall multi-agent system can be effectively reduced during the evolution of the system.

4. Motion of the set of agents

We shall first of all describe a deterministic algorithm, the implementation of which requires sharing limited further information among the agents. Then we shall indicate how it can be made random and asynchronous.

The first task is to determine a movement schedule. A *movement schedule* consists of a cyclic sequence $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \ldots, \mathcal{V}_m, \mathcal{V}_1, \ldots\}$ of subsets of agents, such that

(1) In each $\mathcal{V}_i$, no two agents are neighbors;
(2) Each agent occurs in at least one of $\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_m$ or equivalently $\mathcal{V} = \cup_{i=1}^m \mathcal{V}_i$;
(3) If agent A is in $\mathcal{V}_i$, it is not in $\mathcal{V}_{i+1}$, interpreting indices cyclically.

Between successive clock pulses, each agent of the active subset identifies and then controls, i.e. it first moves so as to localize its neighbors, whose stationarity is guaranteed by the first requirement of the three above, and then it moves to a position minimizing its potential function. Obviously, in a larger formation, more than one non-neighboring agent can move at the same time. The second requirement on the movement schedule ensures that over one cycle, all agents have an opportunity to move, and the third requirement enforces a measure of efficiency. We call the above deterministic motion algorithm the *cyclic stop-and-go strategy*.

Now we show how to construct efficiently the cyclic sequence using only a small number of subsets $\mathcal{V}_i$.

Proposition 1. *For any minimally rigid formation $\mathbb{G}$ in the plane, there always exists a movement schedule consisting of a cyclic sequence of at most four subsets $\mathcal{V}_i$, $i = 1, \ldots, 4$.*

We use the idea of vertex coloring in graph theory (Diestel, 1997) to prove this proposition. By *vertex coloring* we mean the operation to assign different colors to the vertices of the formation's graph $\mathbb{G}$ such that there is no edge in $\mathbb{G}$ connecting two vertices with the same color.

Proof. It suffices to prove that we can use at most four colors $\{c_i\}$, $i = 1, \ldots, 4$, to construct a set of vertex subsets $\mathcal{V}_i$, each of which corresponds to a different color. Consider any graph $\mathbb{G}$ that can be vertex colored by four colors. Now we look at the two Henneberg operations of vertex addition and edge splitting.

Let $\mathbb{G}'$ be any graph obtained from $\mathbb{G}$ by a vertex addition operation, namely $\mathbb{G}'$ is obtained from $\mathbb{G}$ by connecting a new vertex to two of the vertices of $\mathbb{G}$. Then it is always possible to color this new vertex using a color from $\{c_i\}$ that is different from the colors of the existing two vertices that it connects to. So the vertices of $\mathbb{G}'$ can be colored by $\{c_i\}$.

Let $\mathbb{G}''$ be any graph obtained from $\mathbb{G}$ by an edge splitting operation, namely $\mathbb{G}''$ is obtained from $\mathbb{G}$ by deleting an edge $(i, j) \in \mathbb{G}$, connecting another vertex $k \in \mathbb{G}$ to a new vertex v, and adding two new edges (v, i) and (v, j). Since the three different vertices i, j and k are colored by at most three different colors in $\{c_i\}$, it must be possible to pick a different color from $\{c_i\}$ to color v. In other words, we can color the vertices of $\mathbb{G}''$ by the colors in $\{c_i\}$.

So we have proved that starting from a graph whose vertices can be colored by $\{c_i\}$, the new graph obtained by either of the two Henneberg sequence operations can always be colored by $\{c_i\}$ as well. In view of the well-known fact that any minimally rigid graph in the plane can be generated from a single edge by the Henneberg sequence operations Tay and Whiteley (1985) and the vertices of a single edge can always be colored by two colors, we have proved that one can color the vertices of any minimally rigid graph in the plane by $\{c_i\},\ i = 1, \ldots, 4$. □

In fact the smallest number of the colors c_i to color the vertices of a graph is called the *chromatic number* of the graph. The computation of the chromatic number for an arbitrary graph is NP-hard (Brelaz, 1979), but we have just shown that the chromatic number of a minimally rigid graph is at most four. Since in the proof of Proposition 1, each graph operation involves at least three colors, it must be true that the chromatic number of a minimally rigid graph with more than two vertices is at least three. So we have proved the following result.

Corollary 1. *For any minimally rigid graph with at least three vertices in the plane, its chromatic number is either three or four.*

Note that in the proof of Proposition 1, we have also provided a sequence of steps to assign the vertices of a formation's graph $\mathbb{G}$ to a movement schedule $\{\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_m, \mathcal{V}_1, \ldots\}$ with $m \leq 4$. So in the design stage, one can follow these steps to obtain an effective movement schedule, and then every agent knows of which particular $\mathcal{V}_i$ it is a member. Suppose each agent is aware of a common clock and the initial time, at which the subset $\mathcal{V}_1$ is deemed to become active for the first time. At each clock pulse, with pulses occurring periodically, the active subset changes to the next member of the sequence defining the movement schedule. When an agent is a member of an active subset, it can move. Otherwise, it remains stationary. Therefore at every (continuous) time instant, except at the clock times when there is a discontinuity, each agent knows whether it is stationary or moving, according to whether it is in the active subset. In one cycle of the movement schedule, a given agent may move more than once. We show next how the value of the Lyapunov function changes when adopting such a strategy.

Lemma 1 (*Behavior of the Lyapunov Function*)**.** *For the formation control problem with range-only measurements under the cyclic stop-and-go strategy, the Lyapunov function W defined in (1) will decrease monotonically to a limit.*

Proof. Consider the sequence of values of the Lyapunov function obtained by sampling at each clock pulse. The Lyapunov function W in (1) contains a number of summands; when the agent pair associated with a summand involves two agents which are not moving, the summand will remain constant between those clock pulses. The summands that can change are those involving an agent which is a member of the set $\mathcal{V}_i$, the active subset for the interval in question. Let A be one such agent. Since each agent optimizes its potential function after its movement, between two clock pulses, W_A cannot increase and in general will reduce. Therefore, W can never increase, and in general will decrease, from one clock pulse to another.

Obviously then the formation distances remain bounded. For analysis purposes, regard the values of the edge lengths d_{ij} and the agents' positions as entries of a state vector, and consider the discrete-time system obtained by the mapping from their values at one clock pulse to their values at the next clock pulse. This system is a discrete-time dynamical system. Accordingly, the d_{ij} asymptotically go to the set ensuring that W at one clock pulse equals the value at the next clock pulse. This means that the asymptotic values of the d_{ij} must be such that no W_A can be made smaller by any change of the d_{ij}. This implies that W will go to a limit as time goes to infinity. □

Lemma 1 can be further strengthened when we examine the distances between the agents at the limit point of the Lyapunov function W.

Lemma 2. *For the formation control problem with range-only measurements under the cyclic stop-and-go strategy, each distance specified by the elements of the edge set $\mathcal{E}$ will reach a limit when the Lyapunov function W defined in (1) reaches a limit.*

Proof. From the proof of Lemma 1, one can see that when W reaches a limit, W_i, $i \in \{1, \ldots, n\}$, cannot be further reduced and thus reach their limit points. This implies that each d_{ij}, $(i, j) \in \mathcal{E}$, will remain fixed when W reaches its limit. In other words, the distances corresponding to the elements in $\mathcal{E}$ will reach a limit. □

Lemmas 1 and 2 describe the global behavior of the Lyapunov function and the distances between the agents under the cyclic stop-and-go formation control strategy. Note that even for the position-based formation control strategies, there is no complete result about the global convergence analysis for the system's behavior. In what follows, we give the local convergence result for the formations with range-only measurements that are perturbed from its desired shape.

Theorem 1. *For the $2n$-dimensional multi-agent system with range-only measurements starting from the neighborhood of the manifold of the desired formation, if the graph of the formation is generically minimally rigid, the stop-and-go strategy will always cause the formation to converge to a formation with desired shape where all distances are correctly attained.*

Proof. We first examine the stationary points of W because such points will be corresponding to the final shape of the multi-agent formation. Suppose agent A positioned at (x_a, y_a) has as its neighbors $A_{n1}, A_{n2}, \ldots, A_{na}$, with coordinates $(x_{a_{n1}}, y_{a_{n1}})$, $(x_{a_{n2}}, y_{a_{n2}})$, $\ldots$, $(x_{a_{na}}, y_{a_{na}})$. Then the use of the fact that (x_a, y_a) minimizes V_A yields the following necessary condition for an equilibrium point of the system:

$$\begin{bmatrix} x_a - x_{a_{n1}} & y_a - y_{a_{n1}} \\ x_a - x_{a_{n2}} & y_a - y_{a_{n2}} \\ \vdots & \vdots \\ x_a - x_{a_{na}} & y_a - y_{a_{na}} \end{bmatrix}^T \begin{bmatrix} d^2_{aa_{n1}} - d^{*2}_{aa_{n1}} \\ d^2_{aa_{n2}} - d^{*2}_{aa_{n2}} \\ \vdots \\ d^2_{aa_{na}} - d^{*2}_{aa_{na}} \end{bmatrix} = 0. \tag{3}$$

A set of equations of this form can be written for each A, and the equations can be grouped as

$$R^T E = 0 \tag{4}$$

where R is the rigidity matrix (Anderson et al., 2008) of the formation and E is a vector of errors in squared distances, a generic entry being $d_{ij}^2 - d_{ij}^{*2}$. If the formation graph is generically minimally

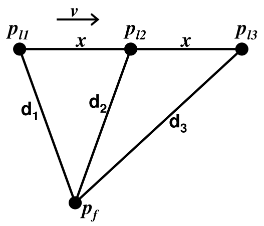

Fig. 1. The follower f measures the relative distances d_1, d_2 and d_3 at times 0, T and $2T$.

rigid, it follows that for generic positions of the formation, the kernel of R^T is trivial, and therefore the equilibrium points for such a formation are necessary such that either all distances are correctly attained, or the agents of the formation are at non-generic positions causing the formation to not be infinitesimally rigid at those positions.

In the neighborhood of the desired formation in the shape space, there is no equilibrium for which the agents are at non-generic positions. According to Lemma 1, the Lyapunov function W goes to a limit, and in view of (4) and the fact that in the neighborhood the equilibrium points always correspond to the formations with the desired shape, it must be true that W will always decrease to zero in which case all the distances within the formation are correctly attained. □

It has been shown before that using the gradient-like control laws, formations described by directed graphs behave substantially differently from their undirected counterparts (Cao et al., 2011; Krick et al., 2009). However, this is not the case for the cyclic stop-and-go strategy proposed in this paper. The key fact is that in the framework discussed in this paper, the directedness of a formation graph only affects the scheduling since neighbor relationships are now undirectional. As a result, the Lyapunov function W will still converge to a limit although it may decrease with a slower rate in the directed case than in the undirected case.

The algorithm requires all agents to have a common clock, and to have advance knowledge of a movement schedule. Now we seek to relax these assumptions. If an agent, A say, is stationary, it will know this; at the same time, it can in general detect when its neighbors are moving. Suppose A cannot detect motion of any of its neighbors. After a random but bounded waiting time that is also bounded below, if no neighbor agent has yet commenced to move, A begins to move with constant velocity. If one of its neighbors commences movement at the same time, A can detect this. Likewise, the moving neighbor can detect that A is moving. Both cease moving, and restart the random waiting time part of the process. Note that such a strategy is analogous to the medium access control protocols used for communication in wireless sensor networks (Demirkol, Ersoy, & Alagoz, 2006).

In the next section, we consider the problem of coordinating a formation of mobile agents with a neighbor moving at constant velocity under the constraint that only distance measurements are available. Such a neighbor may be a leader of a formation, with the formation required to maintain movement as a whole with a prescribed velocity which is initially only known by that leader.

5. Coordination with a moving leader

We consider a neighbor and possibly a leading agent l and a following agent f in the plane. Agent l is moving with a constant velocity v that is unknown to the follower f. Agent f cannot communicate with the leader l to acquire agent l's position and can only measure the distance $d(t)$ between itself and agent l through its range sensor. We assume that agents l and f are in generic positions; in other words, they do not coincide with each other in the beginning and agent f's initial position is not in the line that is determined by agent l's initial position and direction of movement. We also assume that agent l will be within the disk, which is centered at agent f's initial position with radius to be agent f's sensing range, for a sufficiently long time. The problem is to devise a computation algorithm for agent f that enables it to infer agent l's velocity v (and position) after a minimum number of measurements of $d(t)$. We first introduce a powerful tool in distance geometry, called the Cayley–Menger determinant.

5.1. Cayley–Menger determinants

The *Cayley–Menger Matrix* of two sequences of n points, $\{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$ and $\{\mathbf{q}_1, \ldots, \mathbf{q}_n\} \in \mathbb{R}^m$, is defined as

$$M(\mathbf{p}_1, \ldots, \mathbf{p}_n; \mathbf{q}_1, \ldots, \mathbf{q}_n) \triangleq \begin{bmatrix} d^2(\mathbf{p}_1, \mathbf{q}_1) & d^2(\mathbf{p}_1, \mathbf{q}_2) & \cdots & d^2(\mathbf{p}_1, \mathbf{q}_n) & 1 \\ d^2(\mathbf{p}_2, \mathbf{q}_1) & d^2(\mathbf{p}_2, \mathbf{q}_2) & \cdots & d^2(\mathbf{p}_2, \mathbf{q}_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d^2(\mathbf{p}_n, \mathbf{q}_1) & d^2(\mathbf{p}_n, \mathbf{q}_2) & \cdots & d^2(\mathbf{p}_n, \mathbf{q}_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \quad (5)$$

where $d(\mathbf{p}_i, \mathbf{q}_j)$, $i, j \in \{1, \ldots, n\}$ is the Euclidean distance between the points $\mathbf{p}_i$ and $\mathbf{q}_j$. The determinant of this matrix M is then called the *Cayley–Menger bideterminant* (Crippen & Havel, 1988) of these two sequences of n points. When the two sequences of points are the same, e.g. $\mathbf{p}_i = \mathbf{q}_i$, we then write the matrix M as $M(\mathbf{p}_1, \ldots, \mathbf{p}_n)$ and call the determinant the *Cayley–Menger determinant*. The following theorem is a classical result on the Cayley–Menger determinant (Blumenthal, 1953).

Theorem 2. *For an n-tuple of points $\mathbf{p}_1, \ldots, \mathbf{p}_n$ in m-dimensional space with $n \geq m + 2$, the rank of the Cayley–Menger matrix $M(\mathbf{p}_1, \ldots, \mathbf{p}_n)$ is at most $m + 2$.*

5.2. Coordination algorithm

For clarity of the discussion, we assume that the follower f's range measurements are precise. The proposed algorithm consists of two steps. The first step is to compute the speed $|v|$ of the leader; and the second step is to compute the direction of v.

5.2.1. Computation of the speed $|v|$

It is obvious that agent f cannot determine $|v|$ by just taking one range measurement. Then the questions are how many more range measurements are needed and when and how these additional measurements should be taken. By purely geometric arguments, one can easily see that it is still not sufficient to determine $|v|$ by just taking two range measurements. Now we will show that it is possible for agent f to compute $|v|$ by taking three range measurements.

As indicated in Fig. 1, we propose to let agent f measure, at a fixed position, the relative distances d_1, d_2 and d_3 with respect to agent l at times 0, T and $2T$ where T is some positive constant.

Let x denote the distance traveled by agent l over a period of time T. Then agent l's speed $|v| = \frac{x}{T}$. Let $y = x^2$, $l_1 = d_1^2$, $l_2 = d_2^2$ and $l_3 = d_3^2$. Now consider the degenerate quadrilateral shown in Fig. 1. In view of Theorem 2, we can obtain the algebraic equation describing the geometric relationships between x, d_1, d_2 and d_3 as follows:

$$\det \begin{bmatrix} 0 & y & 4y & l_1 & 1 \\ y & 0 & y & l_2 & 1 \\ 4y & y & 0 & l_3 & 1 \\ l_1 & l_2 & l_3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = 0. \quad (6)$$

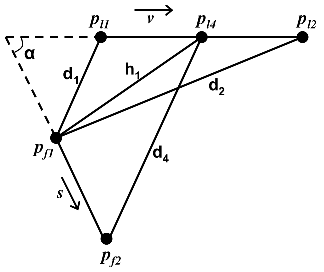

Fig. 2. Three distance measurements d_1, d_2, d_4 and one computed distance h_1.

There are three solutions to this equation, which are $y_1 = 0$ and $y_2 = y_3 = \frac{l_1 - 2l_2 + l_3}{2}$. Since x has to be positive, it must be true that $x = \sqrt{\frac{d_1^2 - 2d_2^2 + d_3^2}{2}}$. Then there is a unique solution for the speed $|v|$ which is

$$|v| = \sqrt{\frac{d_1^2 - 2d_2^2 + d_3^2}{2}} \Big/ T. \tag{7}$$

In fact, the strategy for agent f discussed in this subsection can be further generalized. In particular, it is not needed for the agent f to measure its relative distances to the leader l at equally spaced time instances 0, T and $2T$. Assume agent f takes the measurements in sequence at times T_1, T_2 and T_3. Then the distances traveled by agent l over $[T_1, T_2]$ and $[T_2, T_3]$ are $(T_2 - T_1)|v|$ and $(T_3 - T_2)|v|$ respectively. Again, let $l_1 = d_1^2$, $l_2 = d_2^2$ and $l_3 = d_3^2$ and denote $|v|^2$ by z. Then similar to (6), one can use the Cayley–Menger determinant again to write down an algebraic equation in z and then solve for $|v|$.

5.2.2. *Computation of the direction of v*

In order to determine the direction of movement of the leader, agent f has to perform some local movement and cannot remain stationary. Here we propose a possible maneuvering strategy that continues the discussion in the previous subsection where range measurements d_1, d_2 and d_3 are taken at times 0, T and $2T$.

As indicated in Fig. 2, we require agent f to start a linear motion at time 0 with a given constant velocity s, take a measurement d_4 at time $T/2$, and then return to its initial position to take the planned measurement d_2 at time T.

After $2T$, we have obtained $|v|$ using the algorithm discussed in the previous subsection. Then we can compute the distances $|\mathbf{p}_{l1}\mathbf{p}_{l4}| = |\mathbf{p}_{l4}\mathbf{p}_{l2}| = |v|T/2$. Now we examine the distances between the four points $\mathbf{p}_{f1}$, $\mathbf{p}_{l1}$, $\mathbf{p}_{l4}$ and $\mathbf{p}_{l2}$. There is only one unknown $|\mathbf{p}_{f1}\mathbf{p}_{l4}| \triangleq h_1$ which can be computed by solving the equation in the form of the Cayley–Menger determinant of the corresponding four points:

$$D(\mathbf{p}_{f1}, \mathbf{p}_{l1}, \mathbf{p}_{l4}, \mathbf{p}_{l2}) = 0.$$

One can find that this equation gives us the solution

$$h_1 = \sqrt{\frac{d_1^2 + d_2^2}{2} - \frac{|v|^2 T^2}{4}}.$$

To find the direction of the velocity v, we need to solve for the value of the angle α as indicated in Fig. 2. We first note that in the triangle formed by the points $\mathbf{p}_{f1}$, $\mathbf{p}_{f2}$ and $\mathbf{p}_{l4}$, using the law of cosines, one can compute for the angles $\angle p_{f1}p_{f2}p_{l4}$ and $\angle p_{f1}p_{l4}p_{f_2}$. Then applying the law of cosines to the triangle formed by the points $\mathbf{p}_{f1}$, $\mathbf{p}_{l1}$ and $\mathbf{p}_{l4}$, one can compute for the angle $\angle p_{f1}p_{l4}p_{l1}$. Finally, we examine the triangle formed by the points $\mathbf{p}_{f2}$, $\mathbf{p}_{l4}$ and the intersection point of the lines $\mathbf{p}_{f1}\mathbf{p}_{f2}$ and $\mathbf{p}_{l1}\mathbf{p}_{l4}$, and we have

$$\alpha = \pi - \angle p_{f1}p_{f2}p_{l4} - \angle p_{l1}p_{l4}p_{f2}. \tag{8}$$

However, with only the value of α, there is still a flip ambiguity for the direction of v with respect to the linear trajectory of the follower's movement. To get rid of the flip ambiguity, the follower can take some range measurements while making a linear motion in a different direction in the time interval $(T, 2T)$. Then there must be a unique solution which fits all the range measurement data with respect to both of the two linear motion trajectories. In fact, it is not necessary for the agent f to follow a linear trajectory in the second time interval $(T, 2T)$. Any motion that is not in the direction of s should be sufficient to get rid of the flip ambiguity in the direction of v.

Combining the discussions in the above two subsections, we have designed an algorithm for the follower f to obtain the velocity of the leader f using only range measurements without active communication. It is trivial to see that agent l can also obtain the instantaneous position of f. This algorithm is distributed because it does not need centralized coordination and only makes use of local information. Thus this algorithm can be applied to a scalable team of autonomous mobile agents when proper connectivity relationships are ensured.

6. Conclusions

In this paper, we have proposed cooperative control strategies for the formation control problem with range-only measurements. Despite the fact that each agent has less information available than what has been usually assumed, the proposed stop-and-go strategy can stabilize a generically minimally rigid formation. We have also used the Cayley–Menger determinant to enable a follower in the formation to compute the constant velocity of a neighbor, possibly a formation leader, using range-only measurements.

Currently we are carrying out analysis for the effect of measurement errors and delays on the convergence of the proposed strategy. We are also interested in implementing the strategies discussed in this paper using a mobile robotic testbed. There are also some more general developments that can be considered. The strategy of localizing individual stationary agents by a moving agent using range measurements could similarly be used if bearing measurements only were available; this is easy to see. Establishing that one can localize and determine the velocity of an agent moving with uniform velocity via bearing measurements alone is less obvious. Also, localization via range measurements with bounded sensing range constraints should be considered: this may be hard when an agent is moving with a constant unknown velocity.

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