Casimir force in the low frequency regime

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***Abstract*—For Casimir force calculations it is important to have a model for the dielectric function that correctly describes the material under study in the low frequency regime, since these frequencies have an important contribution to the Casimir force, but are experimentally not accessible. Current paper describes several dielectric models used for this extrapolation. Accurate results have been obtained using the plasma model, however this model is physically unrealistic. The often applied Drude model appears to be not very accurate. More sophisticated models like the Drude-Smith model and the frequency dependent damping Drude model appeared to be especially accurate for describing very thin films. It is argued that a physically realistic model to combine these different models might be the Extended Drude Model.**

I. Introduction

In the late fourties of the previous century Hendrik B. G. Casimir (figure 1) stunned the physics community by publishing three papers ([1], [2] and [3]) in which he showed that two parallel perfect conducting plates attract each other. The force could be explained by the radiation pressure due to the quantum fluctuations of the quantized electromagnetic field. Before this, the electromagnetic zero-point energy was merely considered as a mathematical constant that came about when solving the equations for a quantum mechanical harmonic oscillator. For calculating or measuring purposes this constant was ignored or taken out by renormalization. When Casimir told Wolfgang Pauli about his results, Pauli rejected this by saying that it is "absolute nonsense" considering the zero-point energy for radiation purposes. Casimir persisted however and eventually Pauli realized that the plates do attract each other[4]. At the moment many theorists and experimentalists are still intrigued by the Casimir effect and recent advantages in both precision measurements of the Casimir force and the downscaling of devices, has resulted in many publications and growing interest in this topic. Also more people are working on possible applications of the Casimir force. An important point in this research is changing the Casimir force by external factors. An example of this is the switching of the Casimir force by switching a sample from an amorphous to a crystalline phase of a phase change material (Torricelli et al., not yet published). Changes up to 20% have been observed at a separation of about 100 nm.

As will be shown in the next section, the formulas for the Casimir force for perfect conductors are surprisingly simple. Calculating the force for real materials, however is much more complicated. In 1956 Evgeny Lifshitz extended the theory for using finite conductivity materials [5]. Although Lifshitzs extension is proved to be successful so far, it is difficult to obtain accurate values for the force, because in the calculation the dielectric function of the material needs to be known for all frequencies. The main problem with current theory is that values for the dielectric function for wavelengths longer than 30 μm cannot yet be obtained experimentally and therefore there is no proof that the dielectric model that is used describes the behaviour of the metal correctly in the entire frequency range. In order to really understand the origin of the force, the behaviour at different separations and the influence of different materials, it is of crucial importance to be able to describe the dielectric properties of the materials correctly. Section III focuses on different models that are known to describe dielectric properties of different types of materials under different circumstances. Having introduced the problem that occurs in the long wavelength regime, the Casimir force and the existence of the different dielectric models, section IV discusses the application of the models in Casimir force calculations. The final section V will draw the conclusions and give suggestions for further research.

II. What is the Casimir force?

The discovery of the Casimir force leads back to one of the first great successes of the quantum theory in the 1920s, when

Fig. 1. *Hendrik B. G. Casimir (1909-2000). Photo taken around 1950 [4]*

Fritz London derived the experimentally known Van der Waals correction for non polar molecules. He reasoned that the origin of the attractive force between non polar molecules or atoms was due to zero-point fluctuations in the positions of their charged constituents [4]. From this derivation followed that the interaction should fall of inversely with the sixth power of the intermolecular separation. However, when Theo Overbeek and Evert Verwey, both working at the Philips Labs, tried to verify this result experimentally, they came to the conclusion that the force falls of faster than $1/R^6$ at large separations. Noting that London's derivation assumes instantaneous interaction, Overbeek approached Hendrik Casimir and Dirk Polder to solve the problem including retardation effects. By first considering a simpler system, an atom in a cavity with perfectly conducting walls, the interaction energy of the particle with the wall was calculated. At large separations the atom is attracted to the walls by a force now known as the Casimir-Polder (also, the retarded Van der Waals) force [1].

$$E = -\frac{3}{8\pi}\hbar c \frac{\alpha}{R^4} \tag{1}$$

In this formula c is the velocity of light, α the electrostatic polarizability of the atom and R the distance between the atom and the wall. Using the same approach they were able to obtain the formula that was asked for to account for the attractive potential between to identical atoms [2].

$$E = \frac{23}{4\pi}\hbar c \frac{\alpha^2}{R^7} \tag{2}$$

The simplicity of the formulas led Casimir in finding the fundamental physics behind the problem. He came to the conclusion that the force can be explained in terms of radiation pressure due to the quantum vacuum fluctuations of the quantized electromagnetic field when boundaries are present. This is illustrated by figure 2.

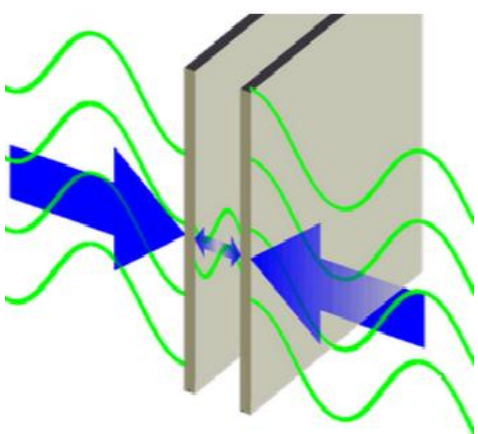

Fig. 2. *Inside the cavity less electromagnetic modes can exist than in the vacuum.*

Using this interpretation he was able to reproduce their previously obtained results (1, 2), by simply adding a zero-point energy $\hbar\omega/2$ to each cavity mode and calculating the net shift of the zero-point energy. Later on he performed, as Casimir called it, "the obvious extension" of the force between the two perfectly conducting cavity walls [3].

$$F = -\frac{\pi^2 \hbar c}{240 d^4 A} \tag{3}$$

The reason this idea became so famous is that it shows that boundary conditions affect the zero point field energy and therefore the system properties.

In 1956, Evgeny Lifshitz generalized Casimir's perfect conductor calculation for using real materials, in a true masterpiece of mathematical physics [5]. The following equation [6] gives the Casimir force for two identical parallel plates of a finite conductivity material

$$F = -\frac{\hbar c}{2\pi^2}\int_0^\infty d\zeta \int_0^\infty dq q k_0 \sum_{\mu=s,p} \frac{r_\mu^2 e^{-2ak_0}}{1 - r_\mu^2 e^{-2ak_0}} \tag{4}$$

where q is the wave vector along the plates,

$$r_s = \frac{k_0 - k_1}{k_0 + k_1},\ r_p = \frac{\epsilon(i\zeta)k_0 - k_1}{\epsilon(i\zeta)k_0 + k_1} \tag{5}$$

are the reflectivities for s and p polarized light respectively and k_0 and k_1 being the normal components of the wave vector in vacuum and metal, respectively

$$k_0 = \sqrt{\zeta^2/c^2 + q^2},\ k_1 = \sqrt{\epsilon(i\zeta)\zeta^2/c^2 + q^2}. \tag{6}$$

In the Liftshitz formalism the material properties enter via the frequency dependence of the reflectivity which can be calculated from the dielectric function. However, it depends on the dielectric function at imaginary frequencies $\epsilon(i\zeta)$. This function, of course, cannot be measured directly, but can be expressed via the observable function $\epsilon"(\omega)$ by using the Kramers-Kronig relations [7]

$$\epsilon(i\zeta) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega\epsilon"(\omega)}{\omega^2 + \zeta^2} d\omega \tag{7}$$

As can be seen from the formulation of this relation, the integral needs to be calculated over the whole frequency domain. For calculation of the Casimir force, the infrared domain is the most important. Measuring the dielectric function for wavelengths above 33 μm up to now is not possible. This implies that a suitable model for the ϵ is needed to extrapolate the data down to zero frequency. Following the work of Svetovoy et al. [7] one obtains

$$\epsilon(i\zeta) = 1 + \epsilon_{cut}(i\zeta) + \epsilon_{exp}(i\zeta), \tag{8}$$

where ϵ_{cut} needs to be calculated (using equation 7 by extrapolating the dielectric function for the frequency range in which no measurements can be performed. Traditionally, the Drude model was often used to describe the dielectric properties of the metal in this frequency range but, recent publications have shown that simply extrapolating the Drude model is not always allowed and therefore finding the right model is important for acquiring precision measurements of the Casimir force. This problem will be discussed in the next section.

III. Models for dielectric function

A. Plasma model

The easiest model for describing dielectric properties of metals is the plasma model, which assumes free electrons.

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{9}$$

Although, many features of the dielectric function are missed, this model is able to describe a general feature of metals, the real part of the dielectric function becomes negative when ω exceeds the plasma frequency. However, other very important characteristics of metals, which are the interband transitions, are not taken in to account in the model. A way to include these effects in this model is by manually adding resonances to equation 9 [8]. The more generalized model then becomes

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} + \sum_{j=1}^{K} \frac{f_j}{\omega_j^2 - \omega^2 - i g_j \omega} \tag{10}$$

Note that the term $j = 0$ ($\omega_0 = 0$) is not included. This means that only the core electrons are described and not the conductions electrons, like in the Drude model that will be explained in the next section

B. Drude model

The most famous model for the dielectric function was developed by Paul Drude in 1900 [9]. Although it predates quantum theory, it is very successful in describing transport properties of electrons in metals under influence of an electric field. A well know representation of the Drude dielectric function is

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \tag{11}$$

and this has been used widely as been the only way of describing the dielectric properties of metals, also in Casimir force measurements. However, in a number of publications (e.g. [7] it was shown that the Drude model does not always describe the optical properties of the metals very accurately. There seems to be a great dependence on the sample thickness and purity. In order to understand this discrepancy, the assumptions made in deriving the Drude model need to be studied.

For describing the moving of electrons, Drude made the following assumptions

1) Collisions between electrons and ions are instantaneous and uncorrelated.
2) The probability for colliding in a time interval dt is dt/τ, where τ is a constant and therefore does not depend on the position or momentum of the electrons.
3) In between these scattering events, electrons travel in a straight line, so that all other interactions (except applied fields) are ignored.
4) After a collision electrons move again with the velocity corresponding to the temperature of the local environment.

In order to describe the movement of the electron, one needs to find how the average momentum of the electrons evolves over time. Given that the momentum on time t is $\vec{p}(t)$, the momentum at time $t + dt$ can be found considering to cases: there has been a collision or no collision in the time period dt. If there has been a collision the electrons will have on average no momentum. If there has been no collision the momentum would have evolved as is would normally have subjected to an external field, i.e. $\vec{p}(t + dt) = \vec{p}(t) + \vec{F}(t)dt$. The probability of the electrons having a collision is dt/τ (assumption 2) and therefore the probability of no collision is $1 - dt/\tau$. This gives for the total average momentum

$$\begin{aligned} \vec{p}(t + dt) &= \frac{dt}{\tau} 0 + (1 - \frac{dt}{\tau})(\vec{p}(t) + \vec{F}(t)dt) \\ &= \vec{p}(t) + \vec{F}(t)dt - \frac{\vec{p}}{\tau}dt - \frac{\vec{F}(t)}{\tau}(dt)^2, \end{aligned} \tag{12}$$

where the last force term is very small and can be neglected. The remaining part of the equation can be rearranged to yield the equation of motion averaged over all electrons,

$$\frac{d\vec{p}(t)}{dt} + \frac{\vec{p}(t)}{\tau} = -e\vec{E}(t). \tag{13}$$

To obtain information about how the momentum evolves under the influence of an applied electromagnetic field with a frequency ω, we assume that this field drives the momentum with the same frequency, i.e.

$$\vec{E}(t) = \vec{E} e^{-i\omega t}, \vec{p}(t) = \vec{p} e^{-i\omega t} \tag{14}$$

Inserting this in equation 13 gives

$$\vec{E} = \frac{i\omega - 1/\tau}{e} \vec{p} \tag{15}$$

At this point we start using currents instead of the momentum of the electrons. The following formula converts the average momentum of all electrons to the current density,

$$\vec{J} = \frac{-Ne\vec{p}}{m}. \tag{16}$$

N is the number density of electrons in the metal, e the electron charge and m the mass of the electrons. From the current density we can easily obtain the conductivity

$$\sigma = \frac{J}{E} = \frac{Ne^2\tau}{m} \frac{1}{1 - i\omega\tau} \tag{17}$$

which leads us to the dielectric function

$$\sigma = -i\omega(\epsilon - 1) \tag{18}$$

$$\Rightarrow \epsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma} \tag{19}$$

where the relaxation time τ has been replaced by the damping constant $\gamma = 1/\tau$ and ω_p is the plasma frequency defined by

$$\omega_p^2 = \frac{Ne^2}{m} \tag{20}$$

Often, ω_p is determined experimentally using

$$\int_0^\infty \sigma_1(\omega)d\omega = \frac{\omega_p^2}{8} \tag{21}$$

C. Drude-Smith model

The Drude-Smith model is a classical generalization of the Drude formula for the optical conductivity, developed by Smith in 2001 [10]. This model is particularly powerful in describing the infrared properties of poor metals that display a minimum in the optical conductivity at zero frequency. It has been used to describe the dielectric properties of disordered metals, liquid metals, and recently also the metal-insulator transition of thin gold films [11]. This last property was used by Esquivel-Sirvent to calculate the Casimir force near the conductor-insulator transition for thin gold films [12]. The basis of the generalization of the Drude model is to include the persistence of the velocity of the electrons after a collision. To include this, Poisson statistics are used. Suppose the collisions are randomly distributed in time, but the average time interval is τ. Then the probability $p_n(0,t)$ of n collisions in the time interval $(0,t)$ is given by the Poisson distribution

$$p_n(0,t) = (t/\tau)^n e^{(-t/\tau)}/n! \tag{22}$$

Using this the current relative to the equilibrium current can be expressed as the following series

$$\frac{j(t)}{j(0)} = e^{(t/\tau)}[1 + \sum_{n=1}^{\infty} c_n (t/\tau)^n/n!] \tag{23}$$

Here the first term corresponds to the case of no collision at all and the term c_n represents the fraction of the velocity that remains after the nth collision.

An often used approximation of this generalization is the so-called single-scattering approximation, in which it is assumed that the velocity is retained for only one collision (i.e. $c_n = 0, \forall n > 1$). Applying this, equation 23 reduces to

$$\frac{j(t)}{j(0)} = (1 + c\tau/t)e^{(t/\tau)} \tag{24}$$

From this expression the real part of the dielectric function can be obtained

$$\epsilon_1(\omega) = 1 - \frac{\omega_p^2\tau^2}{(1+\omega^2\tau^2)}\Big[1 + \frac{2c}{(1+\omega^2\tau^2)}\Big] \tag{25}$$

which gives more insight is the implications of the Smith generalization. Equation 25 is the normal Drude form of the dielectric function added to an extra term which contains information about the persistence of the velocity of the electrons.

This model shows very versatile properties and interesting behaviour, especially for the cases where c is taken negative, which implies backscattering of electrons. For good metals ($c = 0$), one obtains the normal Drude behaviour: large and negative $\epsilon_1(\omega)$ at low frequencies. For values of c close to -1, however, a large and positive $\epsilon_1(\omega)$ is found at low frequencies. The inclusion of only this parameter c gives the model a great flexibility, something that lacked the traditional Drude model.

D. Frequency dependent damping

In 1970 M. L. Theye [13] conducted measurements of the dielectric properties of thin gold films and showed the obtained data deviated from the bulk Drude behaviour of gold. These deviates were explained by introducing a frequency-dependent damping term arising from electron-ion and electron-phonon interactions:

$$\gamma = \gamma_0 + A\omega^2 \tag{26}$$

A more sophisticated model to describe the damping term was given by Nagel [14], which accounts for a difference in preparation method

$$\gamma_{eff} = \gamma_a\Big[1+\frac{\omega_{pb}}{\omega_{pa}}\Big(\frac{\omega^2+\gamma_a^2}{\omega^2+\gamma_b^2}\Big)\Big]^{-1} + \gamma_b\Big[1+\frac{\omega_{pa}}{\omega_{pb}}\Big(\frac{\omega^2+\gamma_b^2}{\omega^2+\gamma_a^2}\Big)\Big]^{-1} \tag{27}$$

where the subscripts a and b refer to the regions a and b respectively. This equation reduces to that of equation 26 if $\omega\tau_a >> 1$ and $\omega\tau_b >> 1$. In these equations it is assumed that the effective masses of the electrons are the same in both regions, so that $N_a/N_b = \omega_{pa}/\omega_{pb}$, which implies that the thin film can be modeled by a Drude dielectric function with the above defined effective damping constant [15]. By fitting the plasma frequency and damping constants for annealed and nonannealed surfaces, a very good match between the experimental data of Theye [13] and theoretical model of Nagel [14]. As expected, annealing a film reduces the damping constant because the number of impurities will be smaller. Using this method R. Esquivel-Sirvent [15] managed to study theoretically the difference between the Casimir force for annealed and unannealed films, something that has been also experimentally performed by Svetovoy et al. [7].

E. Extended Drude Model

The Extended Drude model, often referred to as EDM, was first described by Shulga et al. in [16]. The key difference with the original Drude model is that the otherwise assumed to be constant plasma frequency (ω_p) and relaxation time (τ) are now made frequency dependent. Furthermore, to include the electron-electron interaction, a frequency dependent mass enhancement factor (λ) is introduced. The EDM is widely used and proved to be very successful in describing non-Fermi liquid behaviour due to interband transitions. As shown by Youn et al. [17] it is also an accurate model for describing noble metals like copper, silver and gold. In this section the important points of the derivation performed by Youn will presented and the implications thereof. In order to obtain the first two, the optical conductivity (see equation 18) needs to be split into its real and imaginary parts. This gives

$$\sigma_1(\omega) = \frac{\omega_p^2\tau}{(1+\omega^2\tau^2)} \tag{28}$$

$$\sigma_2(\omega) = \frac{\omega_p^2\omega\tau^2}{(1+\omega^2\tau^2)} \tag{29}$$

From these equations a time-dependent relaxation time can be obtained by dividing the two

$$\frac{1}{\tau(\omega)} = \frac{\omega\sigma_1(\omega)}{\sigma_2(\omega)} \tag{30}$$

The plasma frequency can be found by taking the imaginary part of the inverse optical conductivity, while the real part gives another equation for the relaxation time [17]

$$\frac{1}{\omega_p^2(\omega)} = \frac{1}{\omega} Im\Big[-\frac{1}{\sigma(\omega)}\Big] \tag{31}$$

$$\frac{1}{\tau(\omega)} = \omega_p^2(\omega) Re\Big[\frac{1}{\sigma(\omega)}\Big] \tag{32}$$

Using equations 30, 31 and 32 the Drude parameters can be obtained self consistently and more accurate, because of the frequency dependence. Whereas in the original Drude model a sum rule for the real part of the conductivity over all frequencies, which of course cannot be measured, is needed to obtain the plasma frequency, using this analyses it can be obtained for each frequency separately when the real and imaginary part of the optical conductivity are known for that specific frequency. Furthermore, it was shown by Allen and Mikkelsen [18] that the plasma frequency needs to be frequency dependent to satisfy the causality requirement of a dielectric function. That the relaxation time needs to be time-dependent in order to obtain a good fit to the experimental data, is something that was already well-known for a long time.

An important extension of the model was preformed by P. B. Allen [19]. The classical Drude model was derived without considering the electron-electron interaction, this means the metal was treated as a free electron gas. If this interactions are taken into account, the Fermi liquid theory should be used. This implies that for weak interactions, the electrons behave like free electrons with a renormalized effective mass [17]. This effect enters the model via the mass enhancement factor λ, which is defined as $\lambda = m^*/m - 1$. According to Allen, $\omega_p(\omega)$ and $\tau(\omega)$ should then be renormalized in the following manner

$$\omega_p^2(\omega) = \frac{\omega_{p0}^2}{1+\lambda(\omega)}, \ \tau(\omega) = [1+\lambda(\omega)]\tau_0(\omega) \tag{33}$$

Here, m^* is the effective mass of electrons with e-e interaction, m the effective mass without e-e interaction and ω_{p0} and τ_0 are the plasma frequency and relaxation time without e-e interaction respectively. Finally, this allows us to rewrite equations 31 and 32 as

$$1+\lambda(\omega) = \frac{\omega_{p0}^2}{\omega} Im\Big[-\frac{1}{\sigma(\omega)}\Big] \tag{34}$$

$$\frac{1}{\tau_0(\omega)} = \omega_{p0}^2(\omega) Re\Big[\frac{1}{\sigma(\omega)}\Big] \tag{35}$$

IV. Discussion

For Casimir force calculations both the plasma and the generalized-plasma model have been used. The plasma model in equation 9 is an oversimplified description of the dielectric properties of a metal, because it completely ignores relaxation of the electrons, which is responsible for the optical response of metals. Lambrecht and Reynaud [20], however, used this model to calculate the reduction factor of the Casimir force, which describes how much the obtained force deviates from the situation where perfect conductors are assumed. The obtained reduction factor was compared to the factor that was obtained when using the Drude model. The result of this calculation is shown in figure 3.

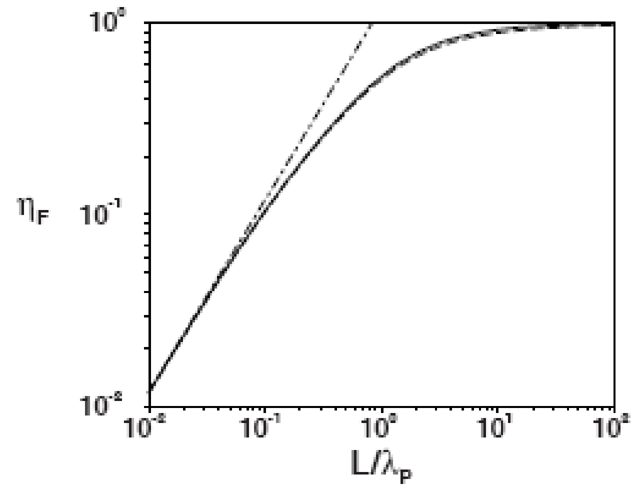

Fig. 3. *Reduction factor calculation using the plasma model (solid line) and the Drude model (dashed line). The dashed-dotted line is not considered in this paper. The graph is taken from [20]*

From figure 3 can be concluded that the difference between the plasma model and the Drude model for Casimir force calculations is very small. This implies that the effect of the relaxation parameter, that is included in the Drude model, is small. The influence of the parameter becomes larger at larger separations between the plates. However, according to Decca in [21] the plasma model is only valid at separations larger than the plasma wavelength, because the model completely ignores any dissipation effects, which will become more important at smaller separations. This implies that, assuming the calculations in [20] were performed correctly, the physically unrealistic plasma model and the more realistic Drude model are not accurate enough for describing the Casimir force. Indeed, this is the conclusion that can be drawn from [21]. In the graph shown in figure 4 it can be seen that the Drude model can be excluded at a 95% confidence level for separations from 162 nm to 746 nm and even at a 99.9% confidence level from 210 nm to 620 nm. Note that in this paper the Casimir pressure is shown, instead of the Casimir force, which is proportional to the gradient of the force

$$P_c = \frac{1}{2\pi r}\frac{\delta F_c}{\delta d} \tag{36}$$

Dielectric model	Formula	Features	Applied to Casimir
Plasma model	$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$	Very simple physics	Quite accurate for $a > \lambda_p$
Plasma model + extra oscillators	$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} + \sum_{j=1}^{K} \frac{f_j}{\omega_j^2 - \omega^2 - i g_j \omega}$	Simple, also describes interband transitions	Can also be applied for shorter separations
Drude model	$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$	Physically more realistic	Inaccurate for long wavelengths $\lambda > 15\mu$m
Drude-Smith model	$\epsilon_1(\omega) = 1 - \frac{\omega_p^2 \tau^2}{(1+\omega^2\tau^2)} \left[1 + \frac{2c}{(1+\omega^2\tau^2)}\right]$	Able to describe poor conductors	Describes Casimir force in very thin films accurately
Extended Drude model	$1 - \frac{\omega_{po}^2}{\omega^2[1+\lambda(\omega)] + i\omega\gamma_0(\omega)}$	Includes frequency dependence of ω_p, γ and e-e interaction	Not yet applied to Casimir force calculations
Frequency dependent damping	$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma_{eff}}$	Able to describe impurity influence	Increased accuracy over Drude model for films of thickness < 30 nm

TABLE I

An overview of the studied models, describing special features and the application for Casimir force calculations

where r is the radius of the parallel disks used in the experiment and d is the distance between the disks.

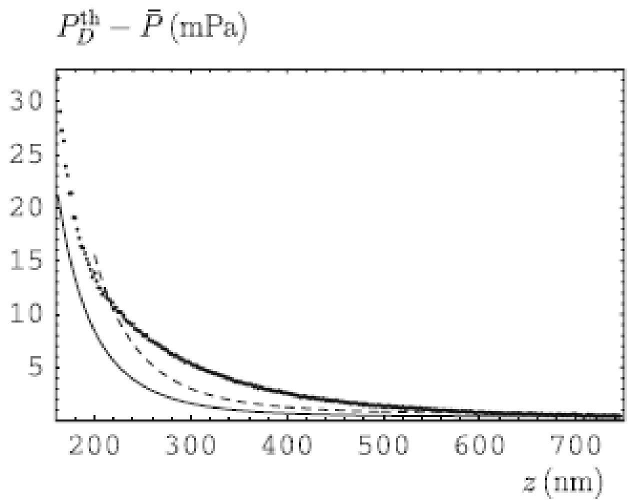

Fig. 4. *The difference between the theoretically obtained Casimir pressures, obtained using the Drude model, and mean experimental values as a function of the separation between the plates is shown in dots. The solid line and the dashed line show the limits of the 95% and 99.9% confidence intervals respectively [21].*

The same analyses was performed for the generalized plasma model. In order to apply this model, one needs to fit it to an experimentally obtained data set. An important feature of the model is that one can include as many oscillators as necessary to fit the possible transitions that may occur in the metal, each with a characteristic frequency, oscillator strength and relaxation time. Note that the presence of the latter term makes the model also more accurate for smaller separations. The graph in figure 5 shows that the generalized plasma model is confirmed to yield calculations that describe the experimental force data correctly, with a confidence level of at least 95%.

At this point it should be stressed that it is not only important to be able to match the calculated values with the experimentally obtained data, but that for understanding the origin of the force and inventing possible applications also

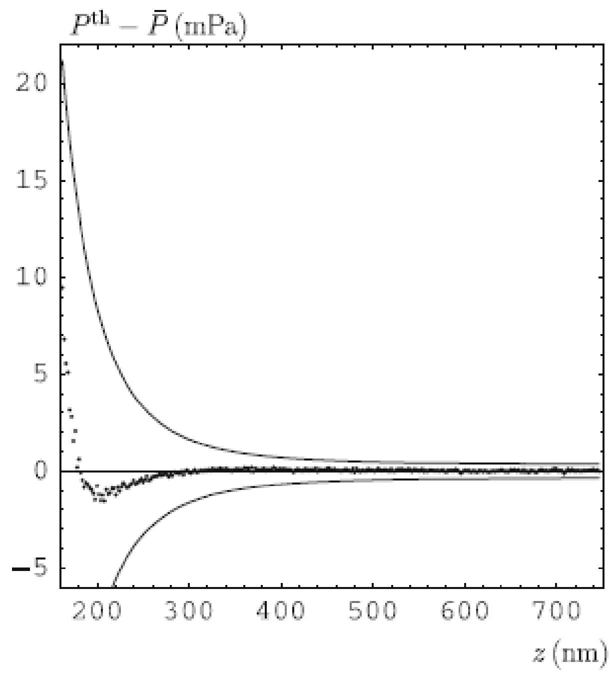

Fig. 5. *The dots show the difference between the experimental measured Casimir pressure and the theoretical calculated values using the generalized plasma model. The solid lines indicate the limits of the 95% confidence intervals [21]*

the physics behind a model needs to be described accurately and realisticly, especially when including external parameters like film thickness and impurity effects.

In [12], R. Esquivel-Sirvent calculates the Casimir force for thin gold films of different thicknesses, varying from 4 to 20 nm. There are many differences when comparing properties of such thin films to bulk properties. The thicker the film, the more the values will agree with the corresponding bulk values. When growing a thin gold film, at first islands (or clusters) will be formed. As long as these islands are not close to each other, there is no conductivity and therefore the film is an insulator. When depositing more and more gold, the island will become closer and closer to each other, eventually close enough for electrons to hop and therefore yield conductivity. This transition is called the insulator-conductor transition. After this, the more gold is deposed, the more the conductivity will be increased. An accurate dielectric model near this transition is the Drude-Smith model, discussed earlier in this paper.

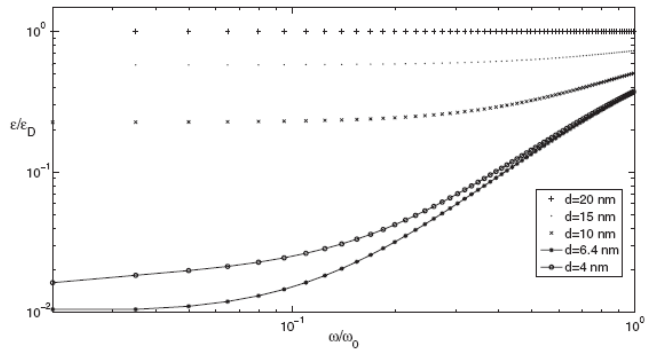

Fig. 6. *Dielectric function of thin Au films as a function of the frequency given by the Drude-Smith model relative to that obtained using the Drude model. The parameters were taken from experimental data given by [11]. Graph taken from [12]*

From figure 6 can be seen that the difference between the Drude and the Drude-Smith occurs mainly in the low frequency regime. Furthermore, this difference becomes larger for thinner films, as would be expected since the Smith generalization was derived for mimicking the dielectric properties of poor metals in the infrared regime. However, a special observation is the fact that there is a turning point: the dielectric constant decreases for thinner films, but then increases again for very thin films, the $4\ nm$-line lies higher then the $6.4\ nm$-line. Because the Drude-Smith model was the best model to describe the measurements on insulator-conductor transition performed by Walther et al. [11], it may be concluded that the persistence of the velocity is an important factor for determining the dielectric properties of thin metallic films. Indeed, the best fit to the experimental data for parameter c, corresponds to increasing negative values for thinner films, with even full backscattering ($c = -1$) for a thickness of $4\ nm$.

Knowing this, it seems logical to apply this model for calculations of the Casimir force for very thin films. In [12] indeed this model is used to calculate the Casimir force as a function of the film thickness. It should be noted that this experiment is more complicated since one also measures contributions from the underlying substrate. Figure 7 shows the result of the calculation. As can be clearly seen from this graph, the Casimir force decreases with decreasing film thickness, until it reaches a critical thickness (denoted by d_c, here $6.4\ nm$), after which the force increases again.

So far the discussed physical effects important for describing the dielectric properties of a metal in Casimir force calculations are the damping constant, adding extra oscillators to describe interband transitions in the plasma model and the persistence of the velocity of the electrons after colliding, but for an accurate physical description impurities or anomalies in the metal structure also need to be considered.

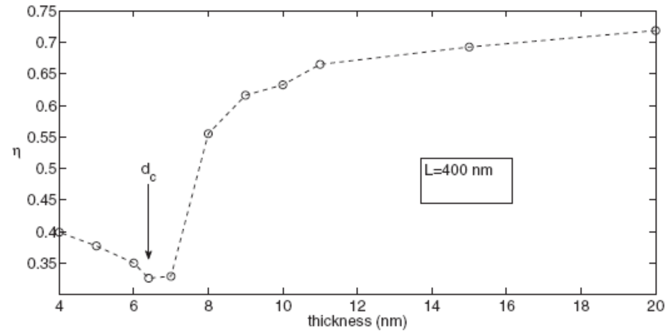

Fig. 7. *Reduction factor as a function of the Au film thickness for parallel plates with a separation of* $400\ nm$ *[12].*

A way to include these effects is to introduce an effective damping parameter, instead of a damping constant, like the frequency-dependent Drude damping model that was introduced in section III. Using this parameter gives the possibility to include electron-phonon and electron-ion interactions. Both experimental (e.g. Svetovoy et al. [7]) and theoretical (e.g. Esquivel et al. [15]) publications have commented on the influence of the sample preparations on the optical properties and therefore on the Casimir force. Of particular importance seems to be if the sample is annealed or not.

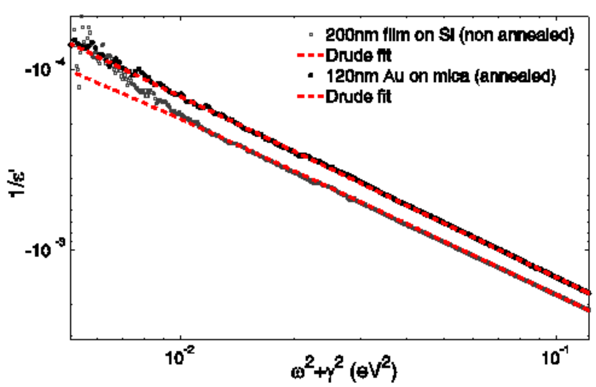

Fig. 8. *Drude fit to the experimental data for both annealed and non-annealed of Au of Si [7]*

As can be seen in figure 8 the non-annealed film shows non-Drude behaviour for low frequencies. Although the thickness of the films is also different, it was shown above that films that are thicker than $20\ nm$ can be described by the Drude model, so that the observed difference is really due to annealing. After annealing the film has lost a number of its impurities and the distribution of the atoms is more homogeneous, which implies that damping might also be homogeneous throughout the sample. This explains why the Drude fit is quite accurate for the annealed film. For a non-annealed film, however, several different distributions of atoms exit within the sample. A good approximation to describe this is the two carrier model, also described earlier in section III. This leads to an effective damping parameter that can replace the damping constant in the Drude model.

Experimental investigation of the influence of annealing on the Casimir force was also performed by Svetovoy et al. [7]. Annealing a film gives a larger value for the Casimir force, a decrease of only 5% relative to the perfect conductor case was obtained using the Drude model. This is shown in figure 9.

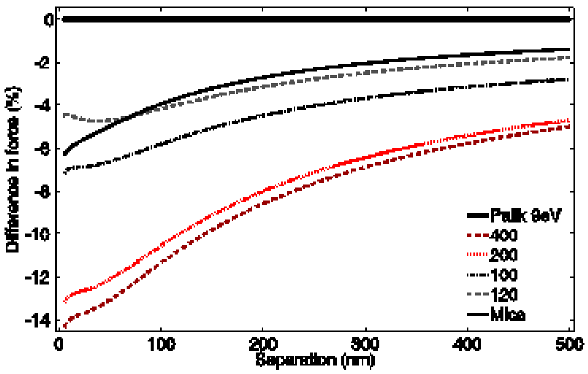

Fig. 9. *Casimir force difference for different samples. The thick line represents the perfect conductor value, the solid black line (Mica) represents the annealed Au film. Source: [7]*

In [15] R. Esquivel-Sirvent calculated the relative difference in the Casimir force when using the Drude model including the two carrier model compared to the normal Drude model. The result was that there is a small but significant difference: about 2.2% for nonannealed films and 1.6% for annealed films. Although these changes are small, using this more realistic model will give a significant difference for extrapolation to low frequencies. Futhermore, the work of Svetovoy et al. shows that for precision measurements it is important to characterize the used sample instead of using tablulated values.

So far it has been discussed that, although the Drude model is a physically realistic model, it is not very accurate for describing the dielectric function in Casimir calculations. Furthermore, there have been many discussions recently about whether the Drude model possibly violates the Nernst theorem ([21]) or not ([22]). Although it is not the aim of this paper to discuss this problem, it is something that should be kept in mind by finding the right dielectric function. Surprisingly, the very unrealistic plasma model is in quite good accordance with the experimental data. However, an explanation of why this model would be accurate is still to be found. In [23], Intravaia and Henkel give a possible explanation by studying the Casimir effect from magnetically coupled Eddy currents. The Drude-Smith model appears to be accurate enough to successfully describe the Casimir force for very thin films and by introducing a frequency dependent (effective) damping parameter in the Drude model, the influence of annealing a film can be studied.

V. Conclusions

Since each model is an approximation to the reality, it will not be possible to describe the dielectric properties under all different conditions. It has been shown that the experimental parameters of the film need to be determined in situ, so it is important to use a flexible model that is able to fit accurately to the experimental data, in order to get an accurate extrapolation to the lower frequency regime. In the opinion of the author of this paper, the Extended Drude Model (EDM) would be a suitable model which has the flexibility and accuracy that is needed. It allows determination of the frequency dependent plasma frequency and damping parameter without using the sum rule (equation 21) or any approximation. Furthermore, there is a possibility to include electron-electron interactions, something that has been ignored so far. Although it is only possible to study the importance of this effect for Casimir force calculations, when investigating this experimentally. It can however be reasoned that there is an effect, since noble metals, like Au, have free-electron-like s-electrons as well as d-electrons that are bound to the nuclei. The contribution of the s-electrons can be described without considering these interactions, since they are free. Electron-electron interactions will be important for the d-electrons however, so they need to be described by Fermi-liquid theory. Experimentally it was shown by Johnson and Christy [24] that indeed the relaxation times of noble metals follow the predictions of Fermi-liquid theory. Up to now, it has not yet been studied how big the influence of the electron-electron interactions is on the optical properties of metals. Once this is known, also predictions can be done about the influence of this interaction on the Casimir force. When comparing the EDM to the other models that have already been applied for Casimir force calculations, it can be concluded that, because of its great flexibility, it would be able to both describe thin and thick films, be more accurate than the Drude model and physically more realistic than the plasma model.

Suggestions for further research

Whether this quick analyses is correct or not needs to be investigated experimentally. Also theoretical investigation of this model is needed concerning the possible violation of the Nernst theorem. Another important point that has so far be ignored in all Casimir force calculations is the influence of the nonlocal dielectric response for p-polarized light. In 1976 P. J. Feibelman [25] derived the exact microscopic theory of surface contributions to the reflectivity and showed assuming a local dielectric response for s-polarized light is valid, but for p-polarized light, short-wavelength fields are induced in the surface region, and therefore the local response assumption is invalid. It needs to be investigated what the implications of this reasoning will be for extrapolations to the low frequency regime studied in current paper. Very recently G. Bimonte (in ref. [26]) suggested a way to avoid extrapolation to the lower and higher frequencies that are experimentally unreachable. This means that the dielectric constant at imaginary frequencies can be determined solely by using experimental optical data. The method however, assumes a perfect Kramers-Kronig consistency which is never the case in experimental data. It is a very interesting development, but the validity needs careful examination.

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