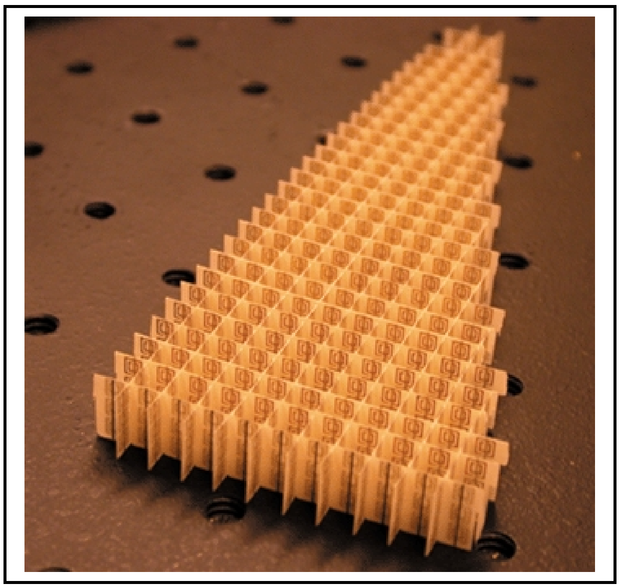
RijksUniversiteit Groningen

Nanoscience TopMaster 2006 Symposium

NEGATIVE REFRACTION



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Abstract

Negative refraction: Physics in the mirror.

One of the most fundamental phenomena in the interaction of matter and electromagnetic radiation is refraction. The refractive index of all naturally occurring materials is positive (and except for some cases, always larger than 1).

In 1967 physicist Victor Veselago hypothesized that a material with a negative refractive index (showing simultaneously negative permeability and permittivity) would not violate any known physical laws. He also predicted that these hypothetical materials would manifest a cascade of exotic phenomena (reversed geometrical optics, Doppler shifts and Cerenkov radiation).

In the last six or so years, practical and theoretical understanding of this phenomenon has grown owing to the advent of actual experimental proof that negative refraction may be achieved, as well as thanks to new ways of treating theoretical studies of the phenomenon. These breakthroughs have shown that negative refraction may be demonstrated in systems more easily accessible than what was thought possible (i.e. photonic crystals) and have brought the promise of amazing new applications such as perfect optical lenses.

Chapter 1 – Background

1.a Introduction

When electromagnetic radiation (EMR) interacts with matter, multiple phenomena such as reflection, absorption and phase shift for the radiation may occur on the interface of the material. In the case where EMR may propagate through the medium (e.g. light propagating through an optical medium) new phenomena manifest: scattering, absorption and refraction.

Classical physics has provided us with coefficients that can be used to quantify such phenomena in a straightforward way. Absorption and scattering have the same effect, namely the attenuation of the propagating beam. The propagation of the beam through a transparent medium is described by the refractive index. Refraction forms the basis of lenses and imaging. Any finite section of a material with a refractive index differing from that of its environment will alter the direction of incident rays that are not normal to the interface. Lenses can be designed to focus or steer radiation over a wide range of wavelengths, from radio frequencies to optical.

The refractive index (or index of refraction) of a material is the factor by which the phase velocity of electromagnetic radiation is slowed in that material, relative to its velocity in a vacuum. It is usually given the symbol n, and defined for a material by:

$$n = \sqrt{\varepsilon_r \mu_r}$$

where, ε_r is the material's relative permittivity and μ_r is its relative permeability.

1.b Permittivity and Permeability

The permittivity ε is a physical quantity that describes how an electric field affects and is affected by a dielectric medium and is determined by the ability of a material to polarize in response to an applied electric field, and thereby to cancel, partially, the field inside the material. Permittivity relates therefore to a material's ability to transmit (or "permit") an electric field. Permittivity is defined through this equation:

$$\vec{D} = \varepsilon \times \vec{E}$$

where E is an electric field (V/m) and D is the electric displacement (Cb/m^2), representing the materials' response (the organization of electrical charges including charge migration and electric dipole reorientation. If the medium is isotropic ε is a scalar coefficient, otherwise it is a 3x3 matrix. In S.I. units permittivity is measured in Farad per meter (F/m = Cb·V^{-1}·m^{-1})

Permittivity is not a constant as it can vary with the position in the medium, the frequency of the field applied, humidity, temperature, and other parameters. In a nonlinear medium, the permittivity can depend upon the strength of the electric field. The relative permittivity ε_r of a material is given by the equation:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi_\varepsilon$$

where ε_0 is the permittivity of vacuum ($8.8541 \cdot 10^{-12}$ F/m) and χ_ε is the material's electric susceptibility.

The permeability μ is defined in a completely analogous manner: Permeability is the degree of magnetization (the amount of magnetic moment per unit volume) of a material that responds linearly to an applied magnetic field. A material's permeability μ can be defined by way of the equation:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$$

where B is the magnetic flux density in the material (in T), M is the material's magnetization (A/m), H is the auxiliary magnetic field (A/m), μ_0 is the permeability of vacuum ($4\pi \cdot 10^{-7}$ N/A^2) and χ_m is the magnetic susceptibility (dimensionless).

The relative permeability μ_r of a material is defined as the ratio:

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$$

Permeability is also measured in henrys per meter.

1.c Refraction – Snell's Law

Refraction occurs when EMR traversing a medium of a given refractive index enters a medium of a different index. At the boundary, the radiation's phase velocity changes, and the EM waves will propagate through the new medium with a velocity calculated according to the well known formula:

$$u_g = \frac{c}{n}$$

where u_g is the radiation's group velocity and c is the speed of light. The wavelength may be increased or decreased but the frequency is not altered. In geometrical optics, Snell's law is used to calculate the entry angle of a ray in the new medium:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

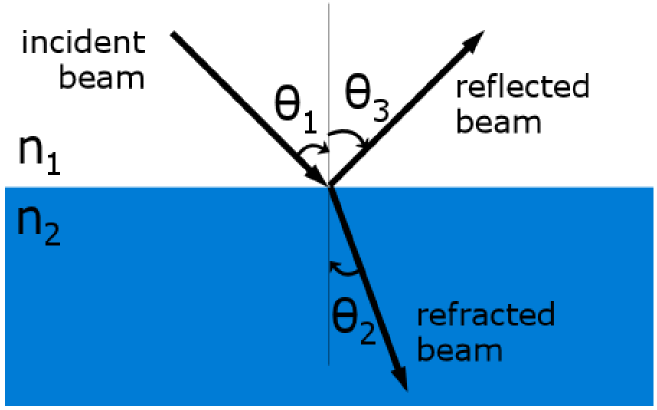

Figure 1: Snell's law (drawn for $n_1<n_2$)

Where θ_1 is the angle of incidence, θ_2 is the angle of refraction and θ_3 is the angle of reflection ($=\theta_1$). Naturally, this law is the basis for a direct measurement of a material's refractive index.

Although Snell's law was apparently known before the first millennium [1] it had to be re-discovered by scientists in earlier times. The sinθ form that is currently used was arrived at in 1621 by the Dutch mathematician Willebrord Snell (this result remained unpublished until 1703 when Huygens included it in his tome *Dioptrica*) [2]. René Descartes also derived the law independently in 1637, however the law has been accredited to Snell (in France it is actually called Descartes' Law or Snell-Descartes Law).

Many methods have been used to explain the experimental result of refraction. Huygens' suggestion of the wave nature of light involved the mediation of ether ("luminiferous aether") and also took for granted that light has a finite speed. According to this interpretation a small portion of each incident angled wavefront (angled relative to the interface) should impact the second medium before the rest of the front reaches the interface (Fig. 2). This portion will start to move through the second medium while the rest of the wave is still traveling in the first medium, but will move more slowly due to the higher refractive index of the second medium. Because the wavefront is now traveling at two different speeds, it will bend into the second medium, thus changing the angle of propagation. He was also able to describe how each point on the wave could produce its own wavelets, which then add to form a wavefront.

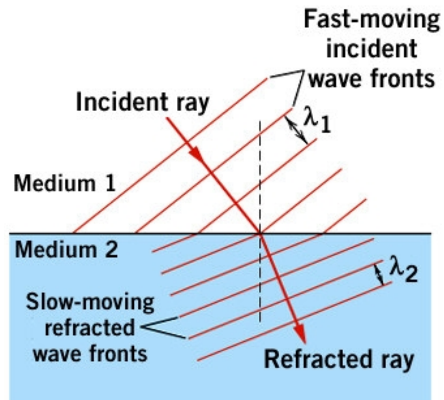

Figure 2. (from [3])

Snell's law may also be derived from Fermat's principle, which states that light will travel through the shortest path (in an analogy by R. Feynman, if the interface is the beachhead and the denser medium (n_2) is the sea, then the fastest route for a rescuer, who was on the beach, to save a drowning person is to follow the path calculated by Snell's law).

1.d The Drude-Lorentz Atomic Model

After Maxwell's electromagnetic theory, it was shown that an oscillating electric dipole would emit electromagnetic waves. We can now understand how EMR of frequency ω interacts with a material. Using the Drude-Lorentz model, we may model the atoms as oscillators, the electrons being bound to the nucleus via a spring. Each oscillator will have a natural resonant frequency ω_0. The AC electric field exerts forces on the electron and the nucleus and drives the oscillations of the system at ω. If ω coincides with one of the natural frequencies of the atom (there are many oscillators = electrons in most atoms) then we have a resonance phenomenon, a fact that is of great importance to negative refraction as will be discussed later (resonance conditions are also crucial for the existence of photonic crystals, which have been used for some time now as negative index materials [4]).

If ω does not coincide with any of the resonant frequencies then the atoms will not absorb the radiation and the medium will (ideally) be transparent to it. The radiation will still drive the non-resonant oscillations of the atoms at its own frequency. The oscillations of the atoms follow those of the driving wave but with a phase lag which is a standard feature of forced oscillation and is caused by damping. The oscillating atoms all re-radiate instantaneously, but the phase lag acquired in the process accumulates throughout the medium and retards the propagation of the wavefront. The propagation velocity is therefore smaller than in free space. This reduction is what is characterized by the refractive index [5]

It has been found that the averaged response of artificially structured materials (such as the ones employed in negative refraction research and the photonic crystal field) follows well known forms of response that occur in conventional materials. If the polarization is linearly related to the applied fields, then Maxwell's equations combined with the oscillator model yield the following effective material parameters [4]:

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{0e}^2 + i\Gamma_e \omega} \qquad \mu_{eff}(\omega) = 1 - \frac{\omega_{pm}^2}{\omega^2 - \omega_{0m}^2 + i\Gamma_m \omega}$$

where ω_p is the plasma frequency, ω_0 is the resonant frequency and Γ is the damping factor. The parameters are indexed with an "e" for an electric and m for a magnetic response. The free electron contribution in metals corresponds to $\omega_{0e}=0$.
These are the standard Drude-Lorentz forms of the permittivity and permeability. Their form stems from the universal response of an harmonic oscillator to an external frequency dependent perturbation. It describes correctly the electromagnetic response of materials over the range from microwaves to UV [4].

The slowing-down of the wave due to non-resonant interactions can be considered as a repeated scattering process. The scattering is both coherent and elastic, and each atom behaves like a Huygens point source. The scattered radiation interferes constructively in the forward direction and destructively in all other directions, so the direction of the beam is unchanged by the repetitive scattering process. However, each scattering event introduces a phase lag which causes the propagation of the phase front to slow down throughout the medium. [5]

We have to note that Snell's law is only generally true for isotropic media (or, at least, media that appear to be isotropic to the radiation as will be shown below). In anisotropic media we may observe birefringence which is light splitting into to beams, the ordinary (o-beam) which follows Snell's law and the extraordinary (e-beam) which will not necessarily be coplanar with the incident beam. One final remark is that the above hold if the absorption of the medium is insignificant. In the opposite case, we have to define the complex index of refraction ñ.

Chapter 2 – Negative Refraction

2.a Introduction to Negative Refraction

The first analysis of a (hypothetical) system of a negative refraction index was done by the physicist Victor Veselago in a paper published in 1968 [7], in which he studied the interaction of EMR with a material for which both the electric permittivity ε and the magnetic permeability μ were simultaneously negative. He arrived at some incredible predictions: Firstly, such a material would not violate any known physical laws, so the fact that such a material had not been discovered until then was not an indication that it couldn't exist (exactly why it hadn't been found is explained in paragraph 2.b). It was also shown that a planar slab of this material would focus light (albeit, not parallel rays) [7] and that the energy flow in an electromagnetic wave would be in the opposite direction to the wave vector (Fig.3) [7]. More specifically, the wave propagating through the Negative Index Material (NIM) will have the triad $\vec{k}, \vec{E}, \vec{H}$ left-handed and hence exhibit phase and group velocities in opposite directions [4]. (for a very interesting visual demonstration of this see: http://sagar.physics.neu.edu/lhm-intro-1.html and also http://www.csupomona.edu/~ajm/materials/animations/packets.html for a collection of animations of group velocity Vs phase velocity). Because of this, Veselago termed these materials "left-handed materials" (LHM), a term used to the present day.

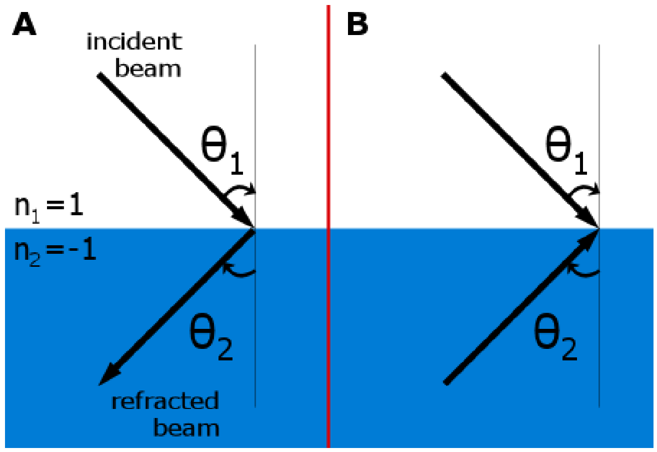

Figure 3: (A) A NIM bends light away from the interface normal. The direction of the ray also shows the direction of energy flow. (B) Meanwhile, the wave vector points in the opposite direction to the energy flow because the group velocity is negative. [7], [8]

Most importantly, Veselago was able to show that this material would exhibit a negative index of refraction. The latter leads to reversal of the Cerenkov radiation phenomenon (Cerenkov radiation will emerge in the opposite direction to the direction of the incident particle [7]), reversal of the Doppler shift (a light source moving toward an observer being down-shifted in frequency) and of Snell's law [7]. Through further theoretical study of this material, more counter-intuitive properties arised e.g. the prediction that this material will exhibit almost zero reflectivity at any angle of incidence [4].

2.b Resonant Response in Materials

The electrical permittivity ε and the magnetic permeability μ summarize the response of a homogeneous medium to magnetic and electric fields. These quantities are usually thought of as positive numbers but in principle negative values are allowed for both. The phenomena that interest us manifest when ε, n and μ become negative, either separately, or together [7].

Electromagnetic radiation will propagate through a medium that has a real index of refraction. If either the electrical permittivity ε or the magnetic permeability μ are negative, then n will be imaginary and there will be no propagation through a thick sample [9] (It is interesting to note that one of the theoretical predictions for negative index materials for which $\varepsilon = \mu = -1$ is that they will be completely transparent in the steady state [4]).

Most familiar materials such as glass or water have positive values for both ε and μ. A lot of materials (e.g. silver and gold) have negative ε at wavelengths in the visible. Materials with negative permeability include resonant ferromagnetic or antiferromagnetic systems [10]. A negative permittivity is explained thus: Going back to the Drude-Lorentz model, we conceptually replace the atoms and molecules of a real material by a set of harmonically bound electron oscillators, resonant at a frequency ω_0. At frequencies far below ω_0 an applied electric field displaces the electrons from the positive core inducing a polarization in the same direction as the field. At frequencies near the resonance, the induced polarization becomes very large, as is typically the case with resonance phenomena. The large response represents accumulation of energy over many cycles such that a considerable amount of energy is stored in the resonator (medium) relative to the driving field. So large is this stored energy that even changing the sign of the applied electric field has little effect on the polarization near resonance [10]. That is, as the frequency of the driving electric field is swept through resonance the polarization flips from in-phase to out-of-phase with the diving field and the material exhibits what is referred to as negative response [10]. The classical analogue is a mass on an ideal spring: Below the resonant frequency the mass is displaced in the same direction as the driving force. However, above the resonant frequency, the mass is displaced in a direction opposite to the applied force. Because the material can be modeled as a set of harmonically bound charges, the negative resonance response translates directly to a negative material response (with the applied electric or magnetic field acting on the bound charges corresponding to the force and the responding dipole moment corresponding to displacement) [8].

The negative material parameters will occur near a resonance. This has two important consequences: First, negative material parameters will exhibit frequency dispersion i.e. they will vary as a function of frequency. Secondly, the usable bandwidth of negative index materials will be relatively narrow. The resonances in existing materials that give rise to electric polarizations typically occur at very high frequencies (in the optical region for metals or at least in the THz to IR region for semiconductors and insulators and result from phonon modes, plasma like oscillations of the conduction electrons, or other fundamental processes). On the other hand, resonances in magnetic systems typically occur at much lower frequencies (usually tailing off toward the THz and infrared region) plus generally occur in inherently magnetic materials. In short, the fundamental electronic and magnetic processes that give rise to resonant phenomena, simply do not occur at the same frequencies, although no physical law would forbid this. This is then the reason for which material with both ε and μ negative are not readily found, and thus Veselago's analysis remained for a long time an academic curiosity.

2.c The Negative Refractive Index

Solving Maxwell's equations to determine how electromagnetic waves propagate within a medium we can arrive at a wave equation of the form:

$$\frac{\partial^2 E(x,t)}{\partial x^2} = \varepsilon\mu \frac{\partial^2 E(x,t)}{\partial t^2}$$

In this equation ε and μ enter as a product and it would not appear to matter whether the signs of ε and μ were both positive or both negative. Solutions of the wave equation have the form $\exp[i(nkd - \omega t)]$ where $n = \sqrt{\varepsilon\mu}$ is the refractive index. Propagating solutions exist in the material for ε and μ being both positive and negative. Actually though, ε and μ are analytic functions and are generally complex numbers.

Instead of writing $\varepsilon = -1$ and $\mu = -1$ we can write $\varepsilon = \exp(i\pi)$ and $\mu = \exp(i\pi)$, then $n = \sqrt{\varepsilon\mu} = \exp(\frac{i\pi}{2}) \times \exp(\frac{i\pi}{2}) = \exp(i\pi) = -1$. An important thing is that is that the square root of either ε or μ alone must have a positive imaginary part, which is necessary for a passive material [10].

2.d Metamaterials

In the 90's researchers started looking into the possibility of engineering artificial materials that have a tailored electromagnetic response.

Because the radiation used has a wavelength hundreds of times larger than the atoms of our material, the atomic details lose importance in describing how the material interacts with it. In practice we can average over the atomic scale by (conceptually) replacing the otherwise inhomogeneous medium by a homogeneous material characterized just by two macroscopic electromagnetic parameters, the permittivity and the permeability. From the electromagnetic point of view the wavelength λ determines whether a collection of atoms or other objects can be considered a material [8]. The electromagnetic parameters need not strictly arise from the response of atoms or molecules: Any collection of objects whose size and spacing are much smaller than λ can be described by an ε and μ. Here, the values of ε and μ are determined by the scattering properties of the structured objects. Although such an inhomogeneous collection may not satisfy our intuitive definition of a material, an EM wave passing through cannot tell the difference [8]. From this point of view we have created a metamaterial (the term is widely used in the negative index material literature and is usually defined as a material whose unique properties are not determined by the fundamental physical properties of its constituents but by the shape and the distribution of the specific patterns included in them [4]).

To form an artificial material we start with a collection of repeated elements designed to have a strong response to applied EM fields. At lower frequencies conductors are excellent candidates from which to form an artificial material as their response to EM fields is large. A metamaterial mimicking the Drude-Lorentz model can be straightforwardly achieved by an array of wire elements into which cuts have been periodically introduced (Fig. 4). The effective permittivity for the cut-wire medium has the form [10]:

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}$$

where the plasma frequency ω_p and the resonance frequency ω_0 are determined only by the geometry of the lattice rather than by the charge, effective mass and density of electrons, as is the case in natural materials. At frequencies above ω_0 and below ω_p the permittivity is negative. The resonant frequency can be set to virtually any value in this kind of materials, so the negative ε can be reproduced at low frequencies rather than just the optical region.

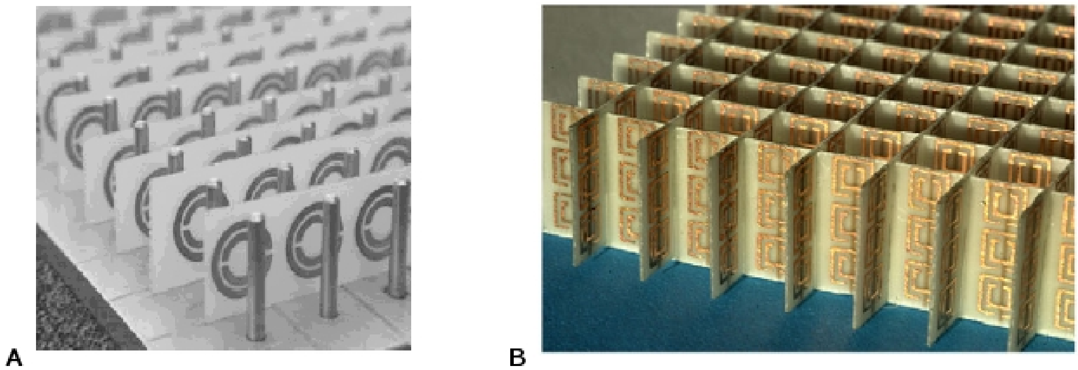

Figure 4: (A) A composite left-handed medium employed in [11]. The medium consists of metallic posts and Split Ring Resonators (SRR's) created lithographically on a circuit board. (B) A split ring structure etched into circuit boards with Cu wires. This material gives negative electric and magnetic response [12].

The path to achieving magnetic response from conductors is slightly different: From the basic definition of a magnetic dipole moment

$$\vec{m} = \frac{1}{2}\int_V \vec{r}\,' \times \vec{j}\,dV$$

it is calculated [10] that a magnetic response (a magnetic dipole) can be obtained if local currents can be induced to circulate in closed loops (solenoidal currents). By circulating currents in a closed SRR (CSRR) we get a magnetic moment m with magnitude given by the product of the current and the area of the CSRR and direction perpendicular to the plane of the ring. Therefore the CSRR behaves as an inductor, storing magnetic energy:

$$U = mB = \frac{LI^2}{2}$$

where L is the self-inductance of the loop. If a CSRR is combined with a capacitor C then one obtains an LC circuit with a resonant frequency $\omega_{LC} = 1/\sqrt{LC}$. Such a capacitor can be realized by making a cut in the ring leading to a normal SRR (Fig. 5). Thus SRR's act like an electromagnetic resonator, producing at ω_{LC} resonant circular currents leading to resonant magnetization i.e. resonant effective permeability [4].

Introducing a resonance into the element should enable a very strong magnetic response, one that can lead to a negative μ. In 1999 Pendry *et al* [13] proposed a variety of structures that, was predicted, would form magnetic metamaterials. These nonmagnetic structures consisted of arrays of wire loops or tubes with a gap inserted. A gap in the plane of the structure introduces capacitance into the planar unit as explained. This Split Ring Resonators (SRR's) in their various forms can be viewed as the metamaterial equivalent of a magnetic atom. In the same paper [13] it was shown that the SRR medium could be described by the resonant form

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}$$

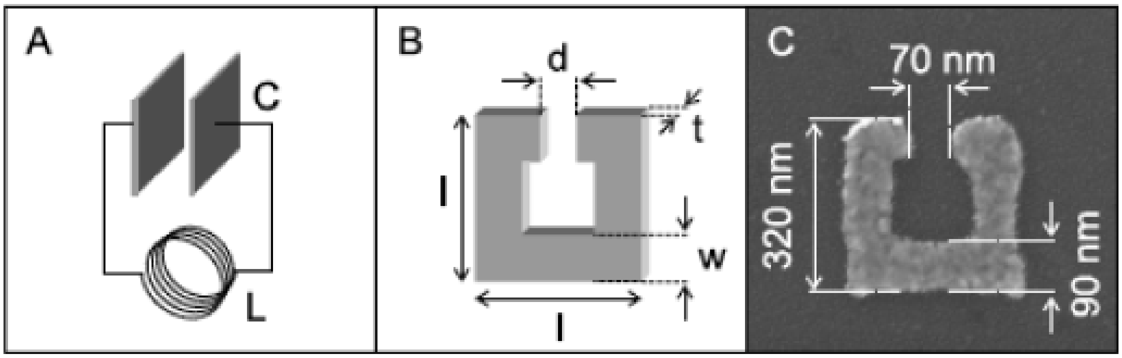

Figure 5: (A) An LC circuit, analogue to an SRR (B) Geometry of a SRR (C)Micrograph of an actual structure made of gold on a glass substrate (taken from [4], original: [14])

The wire medium and the SRR medium represent the two basic building blocks (one magnetic and the other electric respectively) for a large range of metamaterial response, including Veselago's hypothesized material.

Scaling these structures down in size we increase the usable frequency and this is the direction in which research is moving today.

It is worthwhile (and somewhat paradoxical after the previous discussion!) to note that a rigorously defined negative index-of-refraction may not necessarily be a prerequisite for negative refraction phenomena. An alternate approach to attaining negative refraction uses the properties of photonic crystals (PC's) [10]. Photonic crystals derive their properties from Bragg reflection in a periodic structure engineered in the body of a dielectric, typically by drilling or etching holes [10]. Since the PCs can be made from only dielectrics they can, in principle, have much less losses than the metallic LHMs, especially at high frequencies, and even at the optical range. In PCs, to achieve negative refraction, the size and the periodicity of the "atoms" (elementary units) should be of the order of the wavelength. In regular LHMs the size of the unit cell is much smaller than the wavelength and therefore the effective medium theory can be applied and one is able to define effective values of ε and μ as we have seen. In photonic crystals no effective ε or μ can be defined, although the phase and group velocities are reversed as in regular left-handed materials. It is important for these systems to distinguish between left-handed behavior and Bragg scattering effects [15]. Both negative refraction and left-handed behavior have been demonstrated in PC's [4]. Most of these experiments have been performed at microwave frequencies, two experiments [4] however have shown negative refraction in the near IR.

2.e Experimental Verification

The first experiment to verify a negative index of refraction was performed in 2001 by R. Shelby *et al* [12]. A Snell's law measurement was performed on a wedge-shaped metamaterial designed to have a negative index of refraction at microwave frequencies (Fig 6). A beam of microwaves was directed onto the flat portion of the wedge sample, passing through the sample undeflected and then refracting at the second interface. The angular dependence of the refracted power was measured around the circumference establishing the angle of refraction. The result of the experiment (Fig. 7) indicated quite clearly that the wedge sample refracted the microwave beam in a manner consistent with Snell's law. A similar slab of Teflon was used as a control.

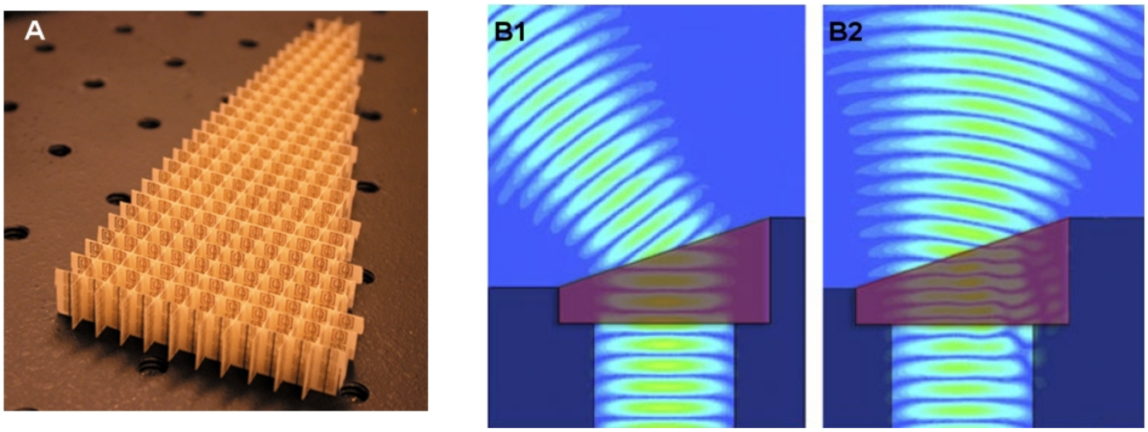

Figure 6: (A): The material used in [12], image from [16]
(B1): In this simulation [17] of a Snell's law experiment a negative index wedge ($\mu=\varepsilon=-1$) deflects an EM beam so that it emerges on the same side of the surface normal as the incident beam, confirming negative refraction. (B2): Positive refraction.

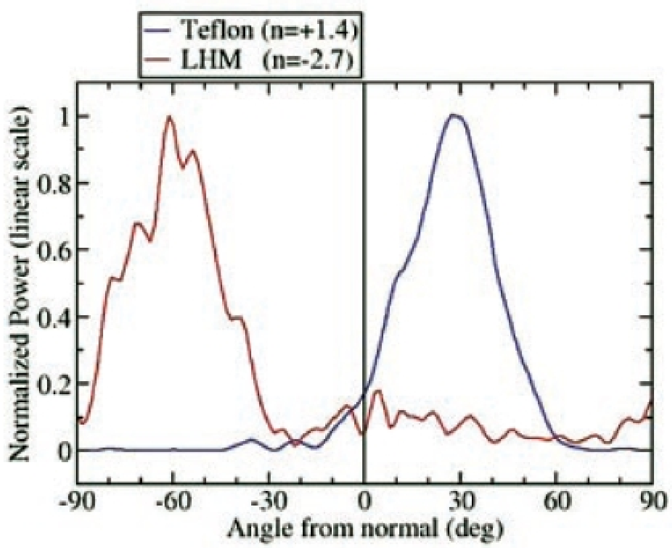

Figure 7 (from [12]), image from [8] 10.5 GHz

Subsequent experiments [18][19] have reaffirmed the property of negative refraction, giving strong support to the interpretation that these metamaterials can be correctly described by negative permeability due to the SRR's and negative permittivity due to the wires. Many other geometries of resonators are used with similar underlying physics. Figures. 8,9 illustrate some of them.

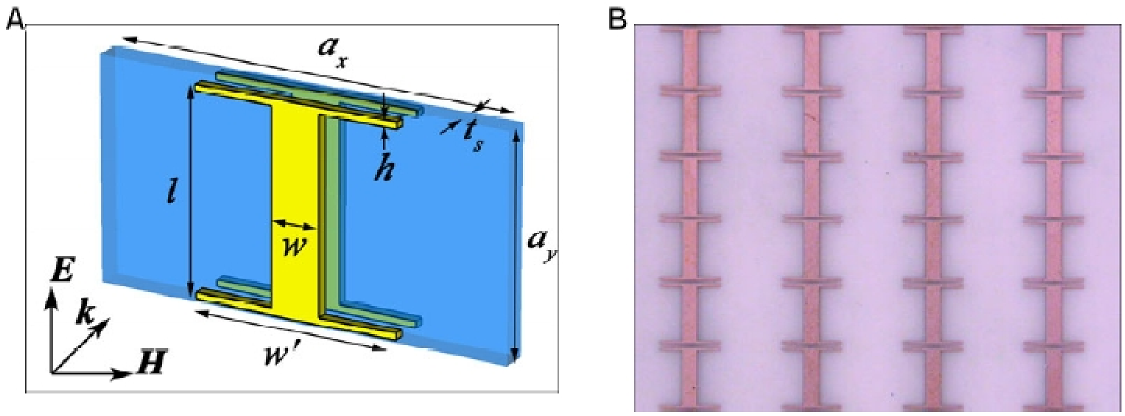

Figure 8 [from [4], refs 60,61] (A) Schematic representation of one unit cell of the wire-pair structure. (B) Photograph of fabricated microwave-scale wire-pair sample.

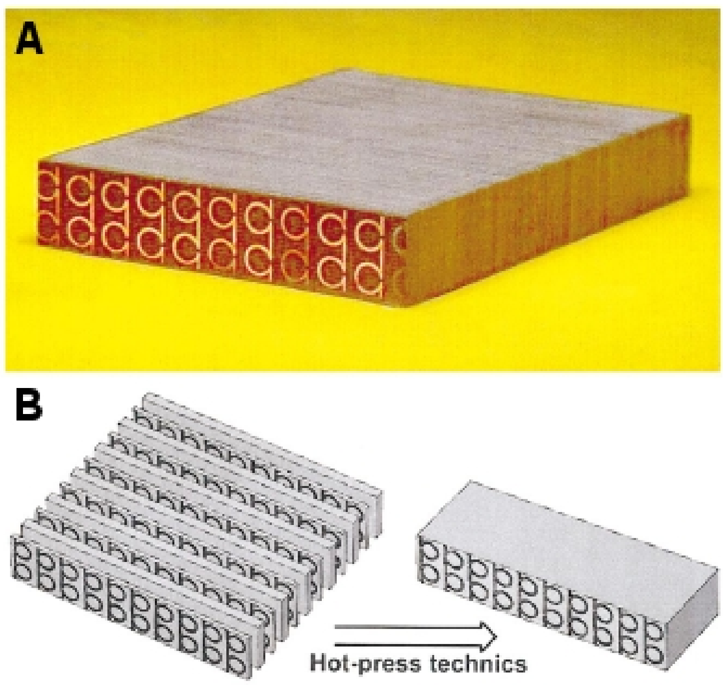

Figure 9: (A): photo of the solid state sample. Applying a standard hot-press technique for the manufacture of printed circuit boards, the researchers have created a solid-state metamaterial sample of the left-handed medium by compressing pieces of alternately stacked PC boards with and without the Ω patterns [20].

Chapter 3 – Applications of Negative Index Materials

3.a Lenses

The most prominent application of refraction is the manufacturing of lenses. What would we observe if we manufactured a lens using a left-handed material? A material with an index of n=1 has no refractive power, while a material of n= - 1 has considerable refractive power. In his early paper, Veselago noted that a negative index focusing lens would need to be concave rather than convex. First we can consider the formula for the focal length of a thin lens $f = R/|n-1|$ (where R is the radius of curvature of the surface). For a given R a lens with an index of n= - 1will have the same focal length as would one with index n=3. By the same reasoning if we compare two lenses of the same absolute index value but opposite signs, the negative index lens will be more compact. Researchers at Boeing [21] have designed a concave lens with an index very close to -1 (at microwave frequencies). This negative index “metalens” has a much shorter focal length as compared to a positive index lens (n=2.3) of the same radius of curvature. Moreover the metalens is considerably lighter than conventional ones (an advantage for aerospace applications).

Conventional positive-refractive index lenses require curved surfaces to bend the rays emanating from an object to form an image. Yet, Pendry and Veselago noted that negative refractive index lenses are not subject to the same constraint: they found that a planar slab of left-handed material could also produce an image (Fig. 10). For this lens, diverging rays from a nearby object are negatively refracted on the first interface (the left side of Fig. 10) reversing their trajectory so as to converge at a focus within the material. The rays diverge from this focus and are again negatively refracted at the second interface, finally converging to form a second image just outside the slab.

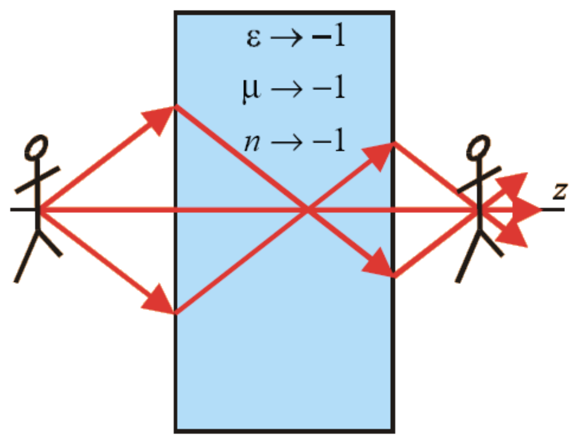

Figure 10: Focusing by a planar slab of LHM. Although it produces an image, the planar lens differs from conventional curved-surface lenses in that it does not focus parallel rays, and has a magnification that is always unity.

One of the most dramatic – and controversial- prediction of the LHMs was that by Pendry [22] which stated that a thin negative-index film should behave as a “superlens”, providing image detail with a resolution beyond the diffraction limit, to which all positive-index lenses are subject.

To make a conventional lens with the best possible resolution we seek to create a wide aperture. Each ray emanating from an object (Fig. 11) has wave vector components along the axis of the lens $k_z = k_0 \cos\theta$, and perpendicular to the axis $k_x = k_0 \sin\theta$. The former component is responsible for transporting the light from object to image and the latter represents a Fourier component of the image for resolution: the larger we can make k_x, the better. The best result that can be achieved is k_0 and hence the limit to resolution of

$$\Delta \simeq \frac{\pi}{k_0} = \frac{\lambda}{2}$$

which is the diffraction limit. This restriction is a significant problem in many areas of optics. The feature size that can be achieved lithographically is limited by the wavelength, as is the storage capacity of optical media.

In contrast to the image there is no limit to the electromagnetic details contained in the object (The electromagnetic field of an object includes not only propagating waves, but also near-field "evanescent" waves that decay exponentially as a function of the distance away from the object) but unfortunately not all of them make it across the lens to the image. The problem lies with the z component of the wave vector which we can write as $k_z = \sqrt{k_0^2 - k_x^2}$. For large values of k_x, corresponding to fine details in the object, k_z is imaginary and the (near field) waves acquire an evanescent character. By the time they reach the image plane they have negligible amplitude. Pendry found that in a planar negative-index lens an evanescent wave decaying away from an object grows exponentially in the lens; on exiting the lens, the wave decays again until it reaches the image plane, where it has the same amplitude with which it started (Fig. 11). Unlike any other lens, the resolution limit of the planar negative-index lens is determined by how many evanescent waves from the object can be recovered, rather by the diffraction limit (in practice several stringent requirements limit the perfect focusing).

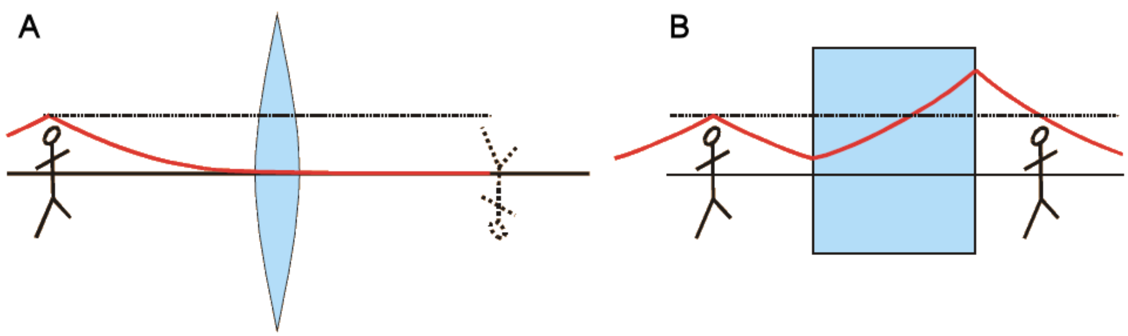

Figure 11: (A): The missing components of the image are contained in the near field which decays exponentially and makes negligible contribution to the image. (B) A new lens made from a slab of negative material has the capacity to amplify the near field so that it contributes to the image thus removing the wavelength limitation. However the resonant nature of the amplification places sever demands on materials: There must be very low loss.

In the case of a planar LH slab, the rays (shown in Fig. 10) only contribute details greater than about λ/2 just as for the conventional lenses. In contrast, the behavior of the near field is markedly different. It has the capacity to excite short wavelength resonances of the negative surface which are akin to the surface plasmons familiar on the surfaces of metals.

Through Maxwell's equations it can be shown [7] that the near fields can be focused by a LHM under only one condition, that $\varepsilon = \mu = -1$. Understanding exactly how the near field is amplified lies in recognizing that the condition $\varepsilon = \mu = -1$ is the condition for surface plasmon states to exist on the surface of the material. These states can be thought of as highly localized excitations which once excited do not propagate parallel to the surface. As a first step the weak incident near field excites surface plasmons on the first surface of the slab. Think of this as a sort of photographic negative - it is certainly not a focused image. Next the surface plasmons excited on the first surface begin a resonant interaction with the surface plasmons on the second surface and because the resonance is very strong the image is transferred to the second surface in a hugely amplified form. In fact it is still out of focus because the various components have been deliberately over-amplified so that the field escaping from the far side of the slab decays by just the right amount to bring all the components to exactly the right amplitude in the image plane [7].

3.b Other Applications

The possibility of magnetism without inherently magnetic materials turns out to be a natural match for magnetic resonance imaging (MRI), which we use as an example of a potential application area for metamaterials. In an MRI machine there are two distinct magnetic fields. Large quasi-static fields, between 0.2 and 3 tesla in commercial machines, cause the nuclear spins in a patient's body to align. The spins are resonant at the local Larmor frequency, typically between 8.5 and 128 MHz, so that a second magnetic field in the form of a radio frequency (RF) pulse will excite them, causing them to precess about the main field. Images are reconstructed by observing the time-dependent signal resulting from the precession of the spins. Although the resolution of an MRI machine is obtained through the quasistatic fields, precise control of the RF field is also vital to the efficient and accurate operation of the machine [8]. Any material destined for use in the MRI environment must not perturb the quasi-static magnetic field pattern, thus excluding the use of all conventional magnetic materials. However magnetic metamaterials that respond to time-varying fields but not to static fields can be used to alter and focus the RF fields without interfering with the quasi-static field pattern. All measurements are made on a length scale much smaller than a wavelength, which is 15 m at 20 MHz. On a subwavelength scale, the electric and magnetic components of electromagnetic radiation are essentially independent; so to manipulate a magnetic signal at RF, we need only control the permeability of the metamaterial: The dielectric properties are largely irrelevant. The metamaterial design best suited to MRI applications is the so-called "Swiss roll" (Fig. 12) manufactured by rolling an insulated metallic sheet around a cylinder. A design with about 11 turns on a 1-cm-diameter cylinder gives a resonant response at 21 MHz. Fig 13(A) shows one such cylinder. The metamaterial is formed by stacking together many of these cylinders. In an early demonstration, it was shown that Swiss roll metamaterials could be applied in the MRI environment [23]. A bundle of Swiss rolls was used to duct flux from an object to a remote detector [Fig. 13(B), 13(C)]. The metamaterial used in these experiments was lossy, and all the positional information in the image was provided by the spatial encoding system of the MRI machine. Nevertheless, it was clear from this work that such metamaterials could perform a potentially useful and unique function.

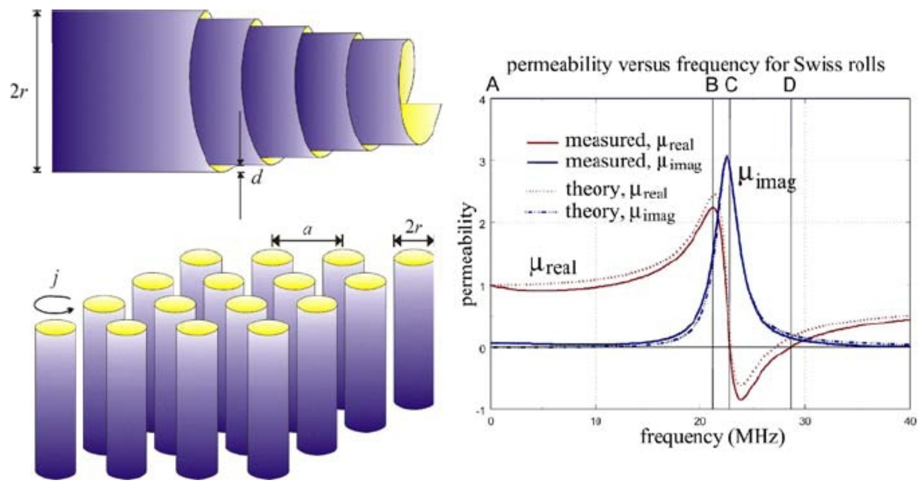

Figure 12: A sheet of metal coiled into a 'Swiss roll' responds to a magnetic field with a current. An array of Swiss rolls produces a material which has negative magnetic permeability over a range of frequencies. Improving the dielectrics used can considerably reduce the imaginary part of ε and μ and increase the resonant response.

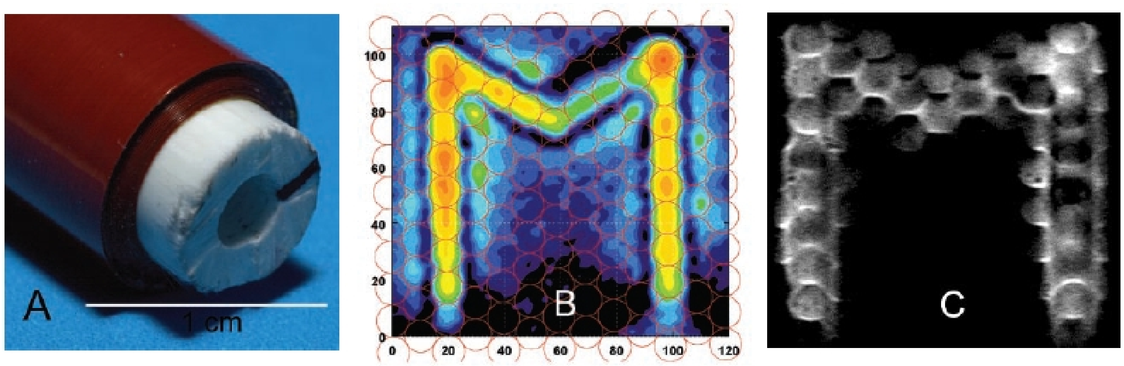

Figure 13: (A) A single element of Swiss roll metamaterial. (B) An array of such elements is assembled into a slab and the RF magnetic field from an M-shaped antenna, placed below the slab, is reproduced on the upper surface. The red circles show the location of the rolls, which were 1 cm in diameter. (C) The resulting image taken in an MRI machine, showing that the field pattern is transmitted back and forth through the slab (from [8], original reference: [23]).

Another impressive application is a potential "cloaking device" that has very recently been described [24] by researchers at the Imperial College London. The idea consists of steering light around an object rendering it invisible. The details of the report are very sketchy at the time as the paper has not been made available at the time of writing, but it has become clear that the new material (which is described as being wrapped around the target object) will be functional across a large part of the electromagnetic spectrum.

3.c Conclusion

The uniqueness and novelty of left-handed materials stem from:

(a) The ability to match the vacuum impedance (zero reflectance!) is a unique property of NIMs with many applications.

(b) Also, the possibility of creating patterns that allow for coupling with the magnetic component of an electromagnetic field without the presence of any magnetic material is a new capability of fundamental importance, especially in the THz region where no magnetic natural resonance exists.
(c) Finally, the capability of having a negative index of refraction opens up the possibility of new applications in optics and communications.

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