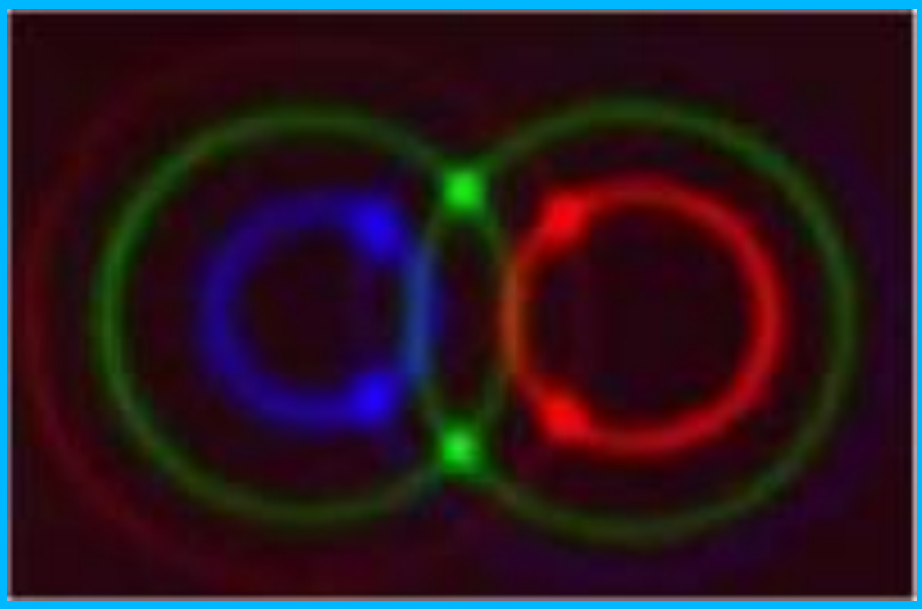
QUANTUM TELEPORTATION

Paper on a Topic in Nanoscience



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Abstract

The formal definition of quantum teleportation is the transfer of information of an object without sending the object itself. This might look like an image from science fiction series, however several research groups spread out all over the world are currently working on this subject. To explain the concept of quantum teleportation, a closer look should be taken on the scientific debates of the early 20th century. By rejecting quantum non-locality, on which teleportation is essentially based, the famous Einstein-Podolsky-Rosen paradox was brought about. It is stated that quantum mechanics is an incomplete and non-local theory. At first the change to accepting long range correlations, the main effect of non-locality will be considered. Therefore great minds (Einstein, Bohr, etc.) of the past century should be addressed. In the sixties experimental tests on the conclusions of the EPR paradox were proposed, soon after which the experimental realisation followed. The tests performed were all in favour of quantum mechanics. A conclusion thus was that quantum mechanics was a complete and therefore a non-local theory. Now it is kwown why quantum teleportation works, the attention can be shifted to the initial proposal in the early nineties. The greatest drawback to put this idea into practice was the need for a source of entangled particles. Two sections are dedicated to the main parts of the set-up: the Bell state analyser and one particular source of entangled states. Finally the experimental achievements and what quantum teleportation can bring us in the future will be dealt with. It would be nice to known whether or not this technique could be used to beam us over large distances as proposed in science fiction.

1. Introduction

Quantum teleportation can be considered as one of the most imaginative ways of transportation. The idea that quantum teleportation, sending information of an object, without sending the object itself (with a speed larger than light), might be applicable to human beings can clearly be considered as transport on the edge. Although the act of teleportation is widely known due to famous science fiction series (e.g. Startrek) where human beings are "beamed" to and from their starships, the current state of affairs would amaze most of the people. Quantum teleportation, is not some freaky future image, but a technique studied and brought into practice by several research groups.

To explain the concept of quantum teleportation, a closer look shall be taken on the scientific debates of the early 20th century. In the 1920's Einstein was finding faults with quantum mechanics. At first Bohr could answer all of Einstein's questions, but in 1935 this changed. In this year Einstein, together with his co-workers Podolsky and Rosen, published a paper (*12*) on a paradox concerning quantum mechanics. They came to the conclusion that quantum mechanics was an incomplete and local theory. Actually the non-locality of quantum mechanics forms the basis of quantum teleportation. While the original idea of quantum teleportation was brought about in the early 1990's (*13*), in the underlying 60 years there was a gradual change towards accepting quantum non-locality. In the 1960's experimental tests on the conclusions of the EPR paradox were proposed (*14*). It took some time before the experimental realisation followed due to the technical problems scientists had to overcome. However when the first test (after which numerous others followed) was performed in 1982 (*4*)it fully agreed with the quantum mechanical predictions and it was shown that quantum mechanics was a non-local theory. With the acceptance of this idea, it is known why quantum teleportation works.

To explain how quantum teleportation works the original proposal of quantum teleportation shall be addressed. The greatest drawback to put quantum teleportation into practice (*7*) was the need for a source of entangled particles. These entangled particles will form a pathway for the instantaneous data transfer of the information of the particle. However for the verification of quantum teleportation it classical information line between the sending and the receiving station is necessary, which excludes instantaneous teleportation. Two separate sections are dedicated to the main parts of the set-up of quantum teleportation: the sources of entangled states and the Bell state analyser. The Bell state analyser entangles the particle to be teleported with the entangled particles providing the pathway for this teleportation. Finally the experimental achievements and what quantum teleportation can bring us in the future will be dealt with. It would of course be nice to known whether or not this novel technique could beam us over large distances as proposed in science fiction.

In this essay several articles will be reviewed starting with the Einstein-Podolsky-Rosen paradox and slowly proceeding to the experimental realisation of quantum teleportation. In the 60 years this process lasted, numerous papers have been written on the paradox and on its consequences, both from a theoretical and an experimental point of view.

2. Non-Locality in Quantum Mechanics

The beginning of the 20th century brought a new theory for science: quantum mechanics. This theory remarkably fitted all experimental results that couldn't be explained before. However, in spite of filling up all the gaps in physics, there was a group of scientists starting to doubt the completeness of the theory and its interpretation. By interpreting quantum mechanics as a non-local and complete theory all problems were solved. This non-locality meant that measurements on distant systems could predetermine the measurement outcomes of other systems. With this the basis of quantum teleportation was set.

2.1 Rising Doubts

Scientist at the end of the 19th century thought that physics was complete. However, there were three problems that were not understood and could not be accounted for with the present classical physics. These three "minor problems" were black body radiation, the photoelectric effect and spectral lines. Respectively Planck, Einstein and Bohr resolved these matters and with this a new theory, referred to as quantum mechanics, came into use.
Bohr proposed an interpretation of quantum mechanics in 1927, which was later called the Copenhagen Interpretation. Bohr struggled for some time with the wave-particle dualism of electrons, after which he

proposed the principle of complementarity (*15*). The two separate appearances of the electron were only observable using different experimental set-ups. This made Bohr realise that neither of the two descriptions could be regarded as complete, but that they should be seen as complementary and partial. But still then the question remained how an electron knew in what way to behave when measured for example in a "double-slit" experiment. The more or less philosophical writing style of Bohr made it hard, even for his genius contemporaries like Einstein, to understand the full meaning of this complementary theory. No experiment could be thought of to simultaneously reveal the wave and particle behaviour of an electron as stated in Heisenberg's uncertainty principle. Thus, experiments unable to be held at the same time could lead to a complementary view of physical phenomena. At the Fifth Solvay Conference in Brussels, among others Bohr and Schrödinger gave a lecture. At the end Einstein objected to both the idea of Schrödinger that quantum mechanics described complete physical reality and the idea of *"ghostly action at a distance*[1]*"* In the end, Einstein had to accept the soundness of quantum mechanics after Bohr managed to counter all the Gedanken experiments Einstein had brought up to invalidate this new theory. In 1932 Heisenberg received Nobel Prize and in 1933 Schrödinger together with Dirac followed.

2.2 EPR Paradox

However, the debate between Einstein and Bohr was not over. In 1935 Einstein together with two co-workers, Podolsky and Rosen (further addressed as EPR) published an article(*12*) in which they brought forward a new argument that quantum mechanics was a correct but incomplete theory. At the basis of this theory stood the condition for reality of a physical quantity raised by the authors. This condition is as follows: *"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity".* For a theory to be complete, all the elements of physical reality must have a counterpart in theory. In quantum mechanics, the wave function was assumed to give a complete description of the state of a system. This seemed reasonable, because the information from the wave function agrees well with the information acquired experimentally (without influencing the state). However EPR showed that this assumption and their criterion of reality lead to a paradox. For the proof of this EPR brought out two systems of which the states are known at t=0, subsequently they let the systems interact between t=0 and t=T. The Schrödinger equation provided a description of the combined state at all times, however the individual states after interaction could not be calculated. The latter could only be done with further measurements by reduction (or collapse) of the wave packet[2]. Measurement of the quantity A resulted in the combined wave function represented by Eq.(1), with u_n ($v_r(x_1)$) describing the eigenstates of the first system (x_1) and $\psi_n(x_2)$ ($\varphi_r(x_2)$) describing the expansion coefficients of Ψ into the orthogonal functions $u_n(x_1)$.

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2) u_n(x_1) \qquad (1)$$

[1] "Ghostly action at a distance" was also referred to as non-locality. The formal definition of non-locality was: instantenous action between distant systems, not consistent with the general theory of relativity.

[2] Measurement of a quantity A could result in a value a_k leaving the first system in a state described by the wavefunction $u_k(x_1)$ and the second system in a state described by the wavefunction $\psi_k(x_2)$. Now the wavepacket is reduced to a single term $\psi_k(x_2)u_k(x_1)$.

However if a quantity B were measured, the wave packet would reduce to $\varphi_r(x_2)v_r(x_1)$. Thus, by performing two different measurements on the first system, the second system was left in two states with different wave functions. This was striking, because the measurements took place at t>T and therefore measurement at the first system should not influence the state of the second system. Because EPR denied non-locality, in this example *"it is possible to assign two different wave functions, ψ_k and φ_r, to the same reality"*. These wave functions could be eigenfunctions of two non-commuting operators (say, position and momentum), corresponding to the quantities P and Q. By measuring either A or B, without disturbing the second system, the value of quantity P (p_k) or the value of quantity Q (q_r) could be obtained, considering respectively P or Q as an element of reality. In the same article it was proved that two non-commuting quantities could not simultaneously have reality[3], starting from the assumption that the wave function was a complete description of the state. It is therefore necessary to conclude that *"the quantum-mechanical description of physical reality given by wave functions is not complete".* This conclusion made EPR consider a description of reality by a complete theory, which they thought possible (and so the hidden variable theories came up, see below).
Of course Bohr responded. In his article (*16*) Bohr criticises the condition of reality put forward by EPR. The passage singled out by Bohr and later also Bell, to oppose the article of EPR is as follows: *"Of course*

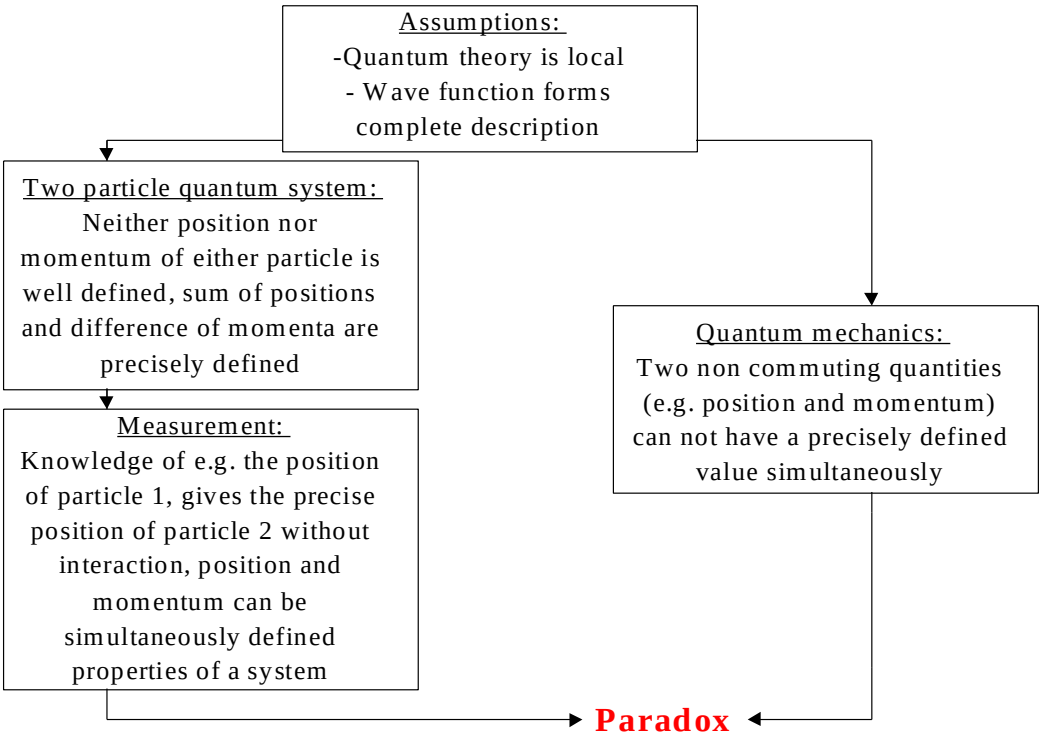

Figure 1: Simplified scheme of the EPR paradox. The basic assumptions lead in two different paths of reasoning to the paradox.

there is in a case like that just considered no question of a mechanical disturbance from the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an influence of the very conditions that define the possible types of predictions regarding the future behaviour of the system. Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusions that quantum-mechanical description is essentially incomplete." This prove of Bohr bewildered the great minds.

In the 1950's a young scientist, Bohm, took over in the debate and introduced a new version of the EPR paradox. Bohm had the opinion that only experiments (in the order of 10^{-13} cm, the region where the usual

[3] *"A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system."* (*12*)

interpretation was thought to break down) could prove that a new interpretation of the quantum theory was necessary (*17*). Bohm showed in his paper (*18*) that measurements violating the Heisenberg's uncertainty principle are conceivable. He had introduced the (classical) particle trajectory as a complicated hidden variable without violating the experimental agreement of quantum mechanics. Next to the conventional forces he let an extra force work on a particle, caused by the precisely definable wave function field, which could not be described by the valid mathematical formulation of quantum theory.
To illustrate this point he proposed an experiment with a molecule of total spin zero consisting of a pair of spin-½ atoms (*19*). This system could be described by Eq.(2) where ψ_+ denotes a particle (say A) with spin $+\hbar/2$.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+(1)\rangle|\psi_-(2)\rangle - |\psi_-(1)\rangle|\psi_+(2)\rangle\right] \tag{2}$$

The atoms were removed from the molecule and sent to two opposite Stern-Gerlach magnets. Depending on the spin (up or down) the particles were deflected and detected. The stunning outcome was that the results appearing in the left detector were complementary to the results of the right detector. According to quantum mechanics, both signals could not have simultaneous reality. However, the outcome at the right detector instantaneously resulted in knowledge of the signal at the other end. There was definitely a correlation between the spincomponents of the two atoms. This view was rejected by Einstein (too simple) and at first also by de Broglie.

2.3 Hidden Variable Theories

When it was accepted that quantum mechanics was incomplete, it was only a small step to consider supplementary variables influencing the exact state of a system (*20*). There were a lot of different theories describing different kinds of variables, but they all worked on the same basis: ψ and the hidden variable λ together determined the state of a system. This hidden variable λ had, similar to ψ, the form of a probability distribution. These hidden variables could be considered to govern the uncertainties in quantum mechanics.

2.4 Experimental Prove of the EPR Paradox

The first proposed practicable test of the EPR paradox concerned discrete variables: the polarisation of two photons (*19*). At the process of annihilation of a positron-electron pair (see section 3.2.1) two photons were released simultaneously with opposite momentum. Two photons in a polarisation entangled state (Appendix 1) could be represented by a total state Eq.(3).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2\right) \tag{3}$$

The photons were separated to large distance and the polarisation was measured with calcite crystals oriented at θ and φ for photon 1 and 2 respectively. The two possible outcomes at each end made a total of four possible results Fig.(2)

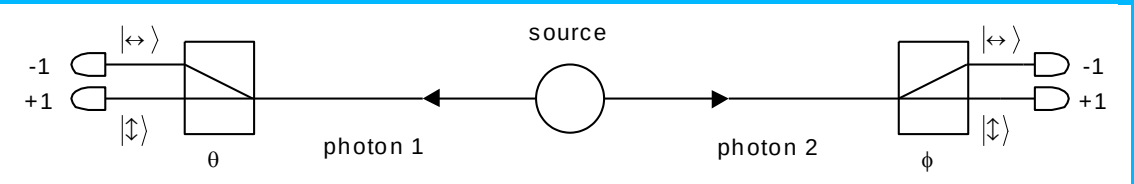

Figure 2: two possible results from the calcite crystal: the photon is transmitted in the ordinary directions θ and φ (resulting in +1), and is scattered in the extraordinary directions θ^{perp} and φ^{perp} (resulting in –1) (*3*).

To calculate the probabilities of the different outcomes, the total state $|\Psi\rangle$ must be rewritten in terms of $|\theta\rangle_1, \left|\theta^{perp}\right\rangle_1$ and $|\varphi\rangle_2, \left|\varphi^{perp}\right\rangle_2$, the superscript *perp* refers to perpendicular polarisation. Two orthogonal photons with linear polarisation could be represented by Eq.(4).

$$\begin{aligned} |\theta\rangle &= \cos\theta|\updownarrow\rangle + \sin\theta|\leftrightarrow\rangle \\ |\theta\rangle &= \sin\theta|\updownarrow\rangle - \cos\theta|\leftrightarrow\rangle \end{aligned} \tag{4}$$

By orienting the polarisers in the right direction it was possible two distinguish between photons in states represented by Eq.(4). The two equations above needed to be inverted to be of use in rewriting Eq.(3).

$$\begin{aligned} |\updownarrow\rangle &= \cos\theta|\theta\rangle + \sin\theta\left|\theta^{perp}\right\rangle \\ |\leftrightarrow\rangle &= \sin\theta|\theta\rangle - \cos\theta\left|\theta^{perp}\right\rangle \end{aligned} \tag{5}$$

Rewriting the total state $|\Psi\rangle$ in terms of $|\theta\rangle_{1,2}$ and $\left|\theta^{perp}\right\rangle_{1,2}$ results in the product Eq.(6), since the terms $|\theta\rangle_1|\theta\rangle_2$ and $\left|\theta^{perp}\right\rangle_1\left|\theta^{perp}\right\rangle_2$ have zero result.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\theta\rangle_1\left|\theta^{perp}\right\rangle_2 - \left|\theta^{perp}\right\rangle_1|\theta\rangle_2\right) \tag{6}$$

Setting both polarisers to the same angle θ, one of the photons was detected in the ordinary path, then the other photon had to be detected in the extraordinary path Fig.(2).

The second proposal to test the EPR paradox concerned nondegenerate parametric amplification (*21*). In this technique the pump photon was detached into two a signal and an idler photon, for further explanation see section 3.2.2.

The correlation between both photon numbers was extremely high, leading to reduced fluctuations in the intensity difference of the signal and idler photon. The paradox can be translated to spatially separate phase quadrature field amplitudes that can be measured by heterodyne detection techniques (appendix 2). The quadrature phase amplitudes of the signal and idler amplitudes, $\mathbf{X_\theta}$ and $\mathbf{P_\varphi}$ (analogue to the position and momentum operator) led to the following quantum mechanical Cauchy-Schwarz inequality Eq.(7).

$$\left|\left\langle X_\theta(L)P_\varphi(L)\right\rangle\right|^2 \le \left\langle\left[X_\theta(L)\right]^2\right\rangle\left\langle\left[P_\varphi(L)\right]^2\right\rangle \tag{7}$$

The accompanying quantum correlation coefficient was defined according to Eq.(8).

$$C_{\theta\varphi} = \frac{\left\langle X_\theta(L)\,P_\varphi(L)\right\rangle}{\left[\left\langle X_\theta(L)\right\rangle^2\left\langle P_\varphi(L)\right\rangle^2\right]^{1/2}} \tag{8}$$

A long time after the interaction of both photons, this coefficient $C_{\theta\varphi}$, could be approximated, leading to Eq.(9).

(9)

In case of perfect correlation, $|C_{\theta\varphi}| = 1$, which occurred for $\theta+\varphi = \pi/2$. Fulfilling this condition, there was perfect correlation between $\mathbf{X_1}$ and $\mathbf{P_2}$, after which the situation became comparable to the one in the EPR paradox. However, this maximum correlation could only be achieved for infinitely high amplification (of the pump beam intensity) factor r. While this amplification factor could never be obtained in experiment it was considered whether the demonstration of the EPR paradox would work without realising the maximum correlation. When deducing an observable for system (particle) 2 from the result of a measurement on system (particle) 1 without perfect correlation, there will be an *"inference error"*. In his article Reid calculated how small this error could be to remain in the order of the EPR paradox. A measurement on the idler amplitude $\mathbf{P_\varphi}(L)$ would result in an estimate of the value of $\mathbf{X_1}(L)$ by scaling it with parameter g Eq.(10).

(10)

This resulted in an average error between the estimated and the real signal $\Delta_{inf}\mathbf{X_1}(L)$ Eq.(11).

$$\Delta_{inf}^2 X_1(L) = \left\langle \left[X_1(L) - X_1^0(L)\right]^2 \right\rangle = \left\langle \left[X_1(L) - g\, P_\varphi(L)\right]^2 \right\rangle \tag{11}$$

This led to a minimum error, with g = tanh(2r), by taking the derivative and setting it to zero Eq.(12).

$$\Delta_{inf}^2 \left[X_1(L)\right]_{min} = \frac{1}{\cosh(r)} \tag{12}$$

Measurement of $\mathbf{P_2}(L)$ of the idler beam immediately led to information on the signal beam amplitude $\mathbf{X_1}(L)$ with an error of $\Delta_{inf}\mathbf{X_1}(L)$. Similarly, measurement of the idler beam $\mathbf{P_1}(L)$ led directly to $\mathbf{X_2}(L)$ with an error of $\Delta_{inf}\mathbf{X_2}(L)$. According to quantum mechanics $\mathbf{X_1}(L)$ and $\mathbf{X_2}(L)$ were noncommuting operators and could therefore not have a definite value at the same time with certainty larger than the value dictated by the Heisenberg principle Eq.(13).

(13)

Thus if the values of the signal beam $\mathbf{X_1}(L)$ and $\mathbf{X_2}(L)$ were defined with an accuracy satisfying Eq.(14), there is a contradiction with quantum mechanics.

(14)

In this experiment the value on the right hand side of Eq.(14) was replaced by 1/cosh(2r) and therefore the paradox was obtained. Because is it experimentally possible to measure the averaged error $\Delta_{inf}^2\mathbf{X_1}(L)$ the demonstration of the paradox was possible.

Three years later the experimental demonstration followed (*11*). The quantum noise in the output of the nondegenerate optical amplifier was measured. The quantum correlations in the quadrature phase amplitudes between the signal and idler beam tested the conclusions of the EPR paradox Fig.(3). The product of the two inference variances was smaller than 1, thus lower than predicted by the Heisenberg uncertainty principle.

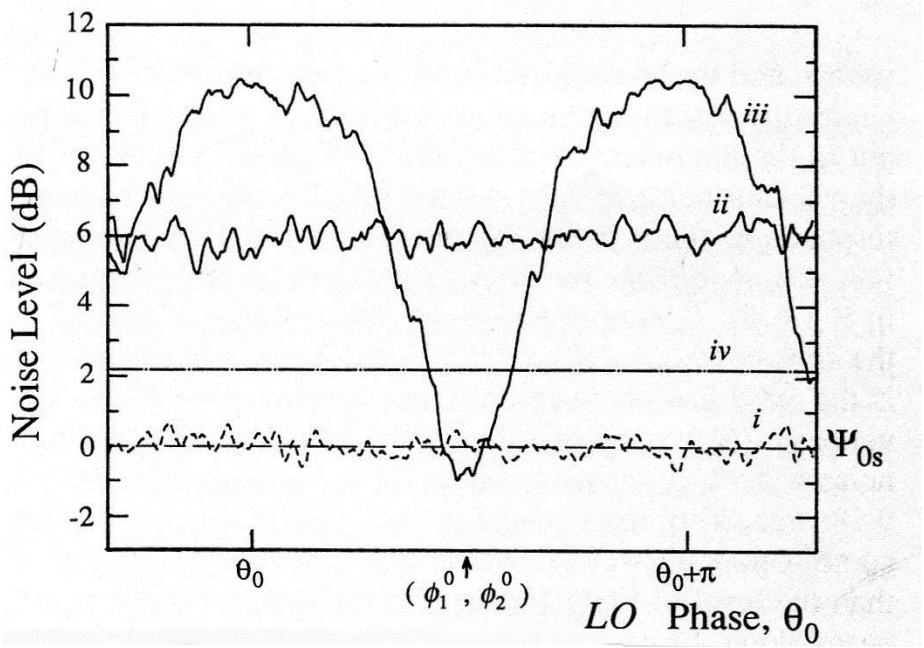

Figure 3: measurement of the phase-dependent correlation between the signal and idler beams from the non-linear optical parametric amplifier (NOPA). Where (i) represented the noise level of the signal beam alone (Ψ_{0s}); (ii) is the phase-insensitive noise from the NOPA; (iii) is the phase sensitive noise with attenuation; (iv) is the noise level of the photocurrent. The phase-sensitive noise (iii) for some value of phases (φ_1^0 ,φ_2^0) was lower than the noise level of the signal beam alone (i) and the noise level of the photocurrent (iv). This meant that the Cauchy-Schwartz inequality Eq.(7) was violated (*11*).

The authors claim that there is no paradox at all, because *"the quantum description for the system of the nondegenerate optical parametric amplifier as well as for the system originally discussed by EPR is consistent with deterministic local realism*[4]*. Therefore no paradox exists either in our system or in theirs: the local realism demanded by EPR is not violated in these cases."*

The implications of the demonstration of the EPR paradox made it unavoidable to make a choice (*22*). Either macroscopic realism must be rejected or accepting the latter, quantum mechanics was incomplete. By rejecting macroscopic local realism the Schrödinger cat could not be considered dead or alive, but this had to be confirmed by measurement on a second correlated spatially separated system. Even if macroscopic local realism is rejected, it was not sufficient to find the solution to the EPR paradox (*11*)

2.5 Bell's Inequalities

In 1964 Bell provided a solution to the EPR paradox (*14*). He published a paper in which he started with the assumptions that quantum mechanics was incomplete and that there existed hidden variables. In this paper he found mathematical prove that the idea of EPR about hidden variables restoring causality and locality

[4] Macroscopic realism is usually explained with the example of the Schrödinger cat. Consider a measurement on a cat to determine whether it is alive or dead (two macroscopically distinct states). It would be illogical to consider the cat as being both alive and dead before the measurement took place. In a system with two or more macroscopically distinct states, macroscopic realism claims that the system must always be in one of the two states.

was incompatible with quantum mechanics. "*It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty*". Using the example of Bohm and Aharonov, measurement of the spin of the first particle predicted the value of the spin op particle two. Thus the results of this measurement was determined beforehand. However the wave function did not lead to a complete result for an individual measurements, which means that there must be hidden parameters λ to complete the state. The results of measuring the spin of the particles 1 and 2 ($\boldsymbol{\sigma}_1$,$\boldsymbol{\sigma}_2$) were respectively A and B. Measurement of $\boldsymbol{\sigma}_1.\mathbf{a}$, with **a** some unit vector, yielded the value +1. Then, according to quantum mechanics, measurement of $\boldsymbol{\sigma}_2.\mathbf{b}$ must yield –1. The result of the last measurement was therefore predetermined. This predetermination implied a more complete description of the state than given by quantum mechanics. The parameter λ was introduced to complete the state. The outcome of the measurements was determined by unitvectors **a** and **b** and the variable λ Eq.(15). The unitvectors **a** and **b** corresponded to the setting of the magnets to detect the selected components of the spin.

$$A(a, \lambda) = \pm 1$$
$$B(b, \lambda) = \pm 1 \qquad (15)$$

The important assumption was that the result B for particle 2, should not be influenced by the setting of the magnet, **a**, for particle 1. The expectation value Eq.(16) of the product of the two components introduced in Eq.(3) with the distribution function of λ as $\rho(\lambda)$, should be equal to the quantum mechanical expectation value Eq.(17) of the spins (σ_1 and σ_2).

$$[illegible] \qquad (16)$$

(17)

Bell proved, by setting the difference between Eq.(16) and Eq.(17) to ε and showing that this value cannot be made extremely small, that the results of quantum mechanics and hidden variable theories did not match. Thus the form chosen in Eq.(16) could not be a true representation. This led to the conclusion that however remote, measuring devices influence the output elsewhere. The information from the measuring device had to be transferred instantaneously (faster than the speed of light), so non-locality was a fact.

2.6 Violation of Bell's Inequalities

The theorem of Bell was generalised (*23*) to test this theory experimentally. The proposed experiment made use of the polarisation correlation of photon pairs. During the experiment the polarisers had appropriate relative orientation and tests were also performed with subsequently one of the two polarisers removed Fig.(2). By defining the correlation function Eq.(16) a mathematical derivation led to the inequalities in a new form Eq.(18) which all local hidden variable theories must satisfy (*3*). In this case the variables a,a' and b,b' did not represent settings of magnets, but polarisation angles in polarisers.

$$[illegible] \qquad (18)$$

There were relative orientations (settings of the polarisers: a,b and a',b') that violated these inequalities, showing that quantum mechanical results could not be reproduced by local hidden variable theories.

Ten years later this proposed test was put into practice by using time-varying analysers (*4*). All previous attempts to test Bell's inequalities experimentally were based on a static set-up. This did however not exclude exchange of information. The time-varying analysers consisted of polarisers, jumping between orientations

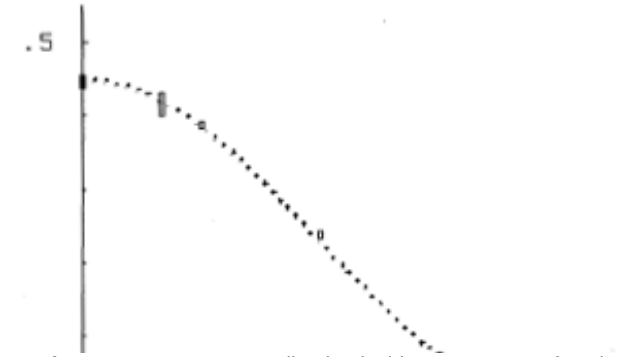

Figure 4: Average normalised coincidence rate as a function of the orientations of the polarisers. The dashed curve was not to fit the data, but are the predictions from quantum mechanical calculations (*4*).

during the time of flight of the photons. During the flight of a photon it was in theory possible to investigate its path and send information of it to the other photon. Therefore the settings of the polarisers should be changed during the flight of the photons. In this experiment Bell's inequalities were violated by 5 orders of magnitude, however the results were compatible with a quantum mechanical prediction Fig.(4). Thus disagreement with both the inequality and the predictions of quantum mechanics showed that hidden variables were not a right option to supplement quantum theory. In 1990 a paper was published (*24*) bringing to light, the (stronger) incompatibility of the EPR ideas with quantum systems of at least three particles. Bell's theorem could be tested even without inequalities. Only perfect correlations (instead of statistical) were treated. Perfect correlations implied a certain prediction of the outcome of a measurement on a particle when the result of a measurement of another particle in the system was known. Although it was not the first paper concerning proof of Bell's theorem without inequalities, it had the most understandable and optimal demonstration. Taken into account a system of 4 particles, particles 1 and 2 were spatially separated as were the oppositely moving particles 3 and 4. The spin-½ particles were created from a single spin-1 particle. Equation(19) represented the total state of this system.

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left|\psi_+(1)\right\rangle\left|\psi_+(2)\right\rangle\left|\psi_-(3)\right\rangle\left|\psi_-(4)\right\rangle - \left|\psi_-(1)\right\rangle\left|\psi_-(2)\right\rangle\left|\psi_+(3)\right\rangle\left|\psi_+(4)\right\rangle \tag{19}$$

The particles were measured at several angles φi Fig.(2). Setting $A_\lambda(\varphi_1) = +1$ when the particle was detected in the ordinary direction and setting $A_\lambda(\varphi_1) = -1$ when detected in the extraordinary direction (equal for $B_\lambda(\varphi_2)$, $C_\lambda(\varphi_3)$, $D_\lambda(\varphi_4)$).
The expectation value E^Ψ of the product of the joined outcomes in the arbitrary directions ($\mathbf{n_1}$, $\mathbf{n_2}$, $\mathbf{n_3}$, $\mathbf{n_4}$) could be simplified by only looking at the orientations of $\mathbf{n_i}$ in the x-y plane Eq.(20)

$$E^\Psi\left(n_1, n_2, n_3, n_4\right) = -\cos\left(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4\right) \tag{20}$$

Perfect correlation occurs in two cases Eq.(21) leading respectively to a value of E^Ψ of –1 and +1.

$$\begin{aligned} \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 &= 0 \\ \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 &= \pi \end{aligned} \tag{21}$$

The assumptions in the EPR paradox, connecting perfect correlations to Eq.(21) could be shown wrong in the case of 4 particles. Rewriting Eq.(21) in terms of the introduced functions A, B, C, D resulted in respectively $A_\lambda(\varphi_1)B_\lambda(\varphi_2)C_\lambda(\varphi_3)D_\lambda(\varphi_4) = -1$ and $A_\lambda(\varphi_1)B_\lambda(\varphi_2)C_\lambda(\varphi_3)D_\lambda(\varphi_4) = +1$.

$$\begin{aligned} A_\lambda(0)B_\lambda(0)C_\lambda(0)D_\lambda(0) &= -1 \\ A_\lambda(\varphi)B_\lambda(0)C_\lambda(\varphi)D_\lambda(0) &= -1 \\ A_\lambda(\varphi)B_\lambda(0)C_\lambda(0)D_\lambda(\varphi) &= -1 \\ A_\lambda(2\varphi)B_\lambda(0)C_\lambda(\varphi)D_\lambda(\varphi) &= -1 \end{aligned} \tag{22}$$

A combination of Eq.(22a) and Eq.(22b) resulted in $A_\lambda(\varphi)C_\lambda(\varphi) = A_\lambda(0)C_\lambda(0)$ and combining Eq.(22a) and Eq.(22c) gave $A_\lambda(\varphi)D_\lambda(\varphi) = A_\lambda(0)D_\lambda(0)$. Putting these results together had as a consequence $C_\lambda(\varphi)/D_\lambda(\varphi) = C_\lambda(0)/D_\lambda(0)$, because the function D is ± 1, it equalled its inverse and the division can be changed into a product. The last result together with Eq.(22d) yielded $A_\lambda(2\varphi)B_\lambda(0)C_\lambda(0)D_\lambda(0) = -1$, changing the second term by part of Eq.(22a) led to the end result: $A_\lambda(2\varphi) = A_\lambda(0)$ = constant for all φ.
By taking $A_\lambda(\varphi_1)B_\lambda(\varphi_2)C_\lambda(\varphi_3)D_\lambda(\varphi_4) = 1$, which led to $A_\lambda(\theta+\varphi) = -A_\lambda(\theta)$, a contradiction was raised if $\varphi=\pi/2$ and $\theta=0$. Thus there appeared an inconsistency in the original assumption of EPR concerning perfect correlations.

2.7 Conclusion

The considerations and experimental proof of the past sections make it inevitable to consider quantum mechanics as a complete and therefore non-local theory. In a non-local system distant measurements influence each other instantaneously. These so-called long-range correlations have as a result that instantaneous information transfer is possible. This can be used to teleport the information of systems. Scientist came up with the idea that this idea could actually be put into practice. The concept could provide whole new ideas on communication and even on transportation. What would it be like if this could be implemented on human beings who would for instance use it to circumvent traffic jams and to beam to work in the morning In the next section quantum teleportation will be illustrated further and in the end it will be considered if humanity can expect to see this technique working in their direct benefit in the near future.

3. Quantum Teleportation

The idea that long-range quantum correlation could be used to transport data in 1993 was used in a protocol for transporting the state of a quantum system to a distant location. This process is known as quantum teleportation. At first sight it seems laborious to use quantum teleportation for transporting the information of the object instead of sending the object itself. However, by sending the object by a regular classical channel, the information of the object could be destroyed (partly) or the position of the receiver might not be known. These problems are overcome by using quantum teleportation. With this technique it is neither necessary to know the state of the object that is going to be teleported nor the position of the receiver (broadband message).

3.1 Original Idea

The long range correlations (or non-locality) brought up by the EPR paradox could be used for information transfer. It was known that instantaneous information transfer was impossible, however entangled states could be used in teleporting a quantum state to a receiver, where both the exact state and the location of the receiver could be unknown. The first scheme for quantum teleportation Fig.(5) was brought up in the early nineties and included the famous Alice and Bob (*13*).

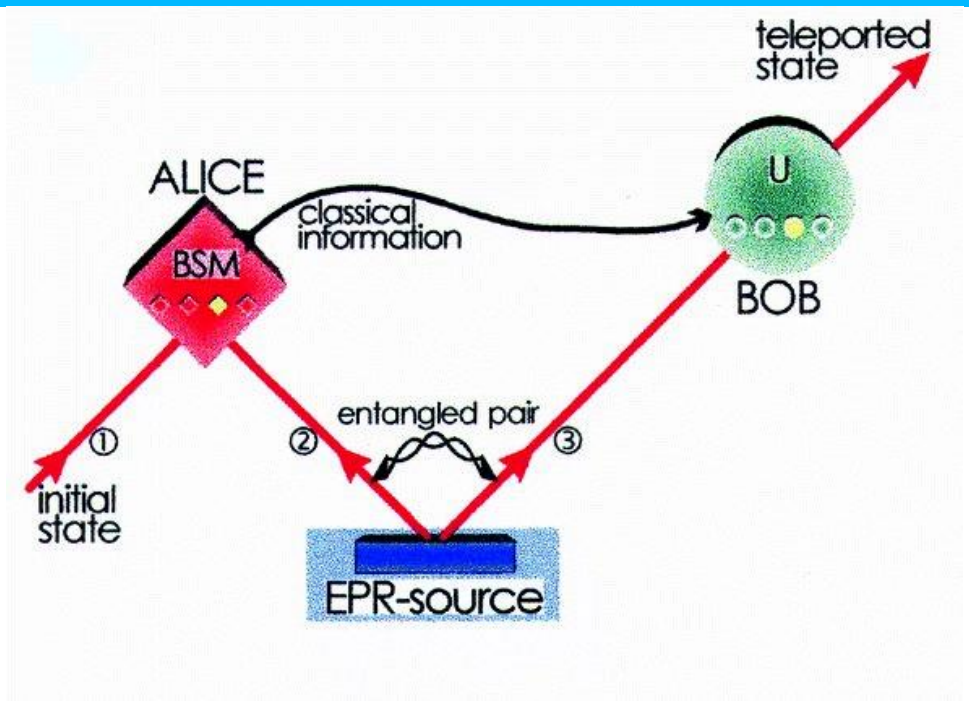

Figure 5: Schematic set-up showing the principles of quantum teleportation. Alice wanted to teleport particle 1. Alice and Bob shared an EPR entangled pair of photons. When Alice performed a Bell state measurement (BSM) on the initial particle and one of the EPR pair projecting them onto an entangled states. After this she sent the information to Bob who transformed his EPR particle by a unitary transformation into the original state (*7*).

The problem to be solved was that Alice would like to provide Bob, who was at a distant location, with a full description of a particle in state $|\psi\rangle$. It was of course trivial that Alice could send the particle to Bob, but assume that this took too long or that this classical channel was not good enough for the state to fully survive. There existed no measurement that Alice could use to obtain sufficient information for Bob to reconstruct the state of the particle. This kind of measurement will project the total state on only one of the superposed states of which a quantum state usually consists. However, using this same projection postulate it was possible for Alice to let the particle interact with a particle with known state $|a_0\rangle$ leaving it in state $|a\rangle$ containing all the information about $|\psi\rangle$. By sending the known particle to Bob, he could reverse Alice's actions to obtain a replica of the original state $|\psi\rangle$. During the interaction between the two particles Alice destroyed the quantum state to be teleported, obeying the no-cloning theorem of quantum mechanics. Alice could divide the information of $|\psi\rangle$ into a classical and a non-classical part after which she could transmit these to Bob through different channels. It is this classical channel holding back instantaneous teleportation. To teleport the state of a spin-½ particle, the use of an entangled pair of particles, shared between Alice and Bob, is unavoidable. Suppose that the initial state of an initial particle 1 and the entangled state of particle 2 and 3 could be represented by Eq.(23).

$$\begin{aligned} |\Psi\rangle_1 &= \alpha|\leftrightarrow\rangle_1 + \beta|\updownarrow\rangle_1 \\ |\Psi^-\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_2|\updownarrow\rangle_3 - |\updownarrow\rangle_2|\leftrightarrow\rangle_3\right) \end{aligned} \tag{23}$$

It is not necessary that the three particles are of the same kind. One of the entangled particles is given to Alice and the other to Bob. To be able to teleport the information an entanglement between particles 1 and 2 should be brought about. A convenient way to describe the total state of the three particles makes use of the maximally entangled Bell states. These states Eq.(24) form a basis that represent entangled states (*25*).

$$\begin{aligned} |\Psi^+\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_2|\updownarrow\rangle_3 + |\updownarrow\rangle_2|\leftrightarrow\rangle_3\right) \\ |\Psi^-\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_2|\updownarrow\rangle_3 - |\updownarrow\rangle_2|\leftrightarrow\rangle_3\right) \\ |\Phi^+\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_2|\leftrightarrow\rangle_3 + |\updownarrow\rangle_2|\updownarrow\rangle_3\right) \\ |\Phi^-\rangle_{23} &= \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_2|\leftrightarrow\rangle_3 - |\updownarrow\rangle_2|\updownarrow\rangle_3\right) \end{aligned} \tag{24}$$

The joined polarisation state of particles 1 and 2 could be written as a superposition of the states in Eq.(24) to form the total state Eq.(25) of the three particles (*1*).

$$\begin{aligned} |\Psi\rangle_{123} &= |\Psi\rangle_1 \otimes |\Psi\rangle_{23} \\ |\Psi\rangle_{123} &= \frac{1}{2}\left[\begin{array}{l} |\Psi^-\rangle_{12}\left(-\alpha|\leftrightarrow\rangle_3 - \beta|\updownarrow\rangle_3\right) + |\Psi^+\rangle\left(-\alpha|\leftrightarrow\rangle_3 + \beta|\updownarrow\rangle_3\right) \\ + |\Phi^-\rangle_{12}\left(\alpha|\updownarrow\rangle_3 + \beta|\leftrightarrow\rangle_3\right) + |\Phi^+\rangle\left(\alpha|\updownarrow\rangle_3 + \beta|\leftrightarrow\rangle_3\right) \end{array}\right] \end{aligned} \tag{25}$$

While Alice performed a measurement (a Bell state measurement) on the particles 1 and 2 in her possession she projected the particles on one of the four Bell states of Eq.(25) and ensured the entanglement. As a result particle 3 at Bob's position was projected onto on of the four "initial" states in Eq.(25). The outcomes of this measurement each occurred with the probability ¼. The "initial" states were simply related (by a

unitary transformation, corresponding to rotations around the x,y,z axes) to the original state of particle 1. This could easier be verified by looking at the antisymmetric wave function Eq.(23). Whatever state particle 1 was in after the Bell state measurement, it was known that particle 2 must be in a state orthogonal to this. But because particle 2 and 3 were initially in an antisymmetrized entangled state, particle 3 must be in a state orthogonal to the state of particle 2. The polarisation is a discrete variable, therefore this only could occur when particle 3 is in the original state of particle 1 (*7*). The teleportation could take place when Bob classically received the outcome of Alice's measurement. Because teleportation is a linear process it does not only work for pure states, but also for entangled states (section 3.4.4).
The experimental realisation of teleportation could not take long, because this process only required a source emitting entangled states (section 3.2) and an analyser capable of measuring Bell states.

3.2 Preparation of Entangled States

Entangled states are an essential key to quantum teleportation. The particles in these states need to be shared between a sending station (Alice) and a receiving station (Bob), to perform the act of teleportation. The entangled particles provide a path for instantaneous data transfer. This section only concerns entangled states solely involving photons. However similarly entanglement between other objects are realised: nuclei (*26*),(*27*), trapped atoms and a single photon (*28*), pairs of atoms (*29*), spins of macroscopic objects (*30*).

3.2.1 Entanglement between Single Photons

In their proposed experiment for testing the conclusions EPR paradox Bohm and Aharonov came up with correlated photons emitted during annihilation of an electron-positron pair. During this process two photons with opposite momentum were emitted, forming the distinct qubit[5] states of the system. To come to the entangled state, creation operators were defined: C_1^x, C_1^y for the photon moving in the +**k** direction and C_2^x, C_2^y for the photon moving in the –**k** direction. This gave four possible wave functions for the radiation field Eq.(26).

$$\left|\psi_1\right\rangle = C_1^x C_2^y \left|\psi_0\right\rangle \qquad \left|\psi_2\right\rangle = C_1^y C_2^x \left|\psi_0\right\rangle$$
$$\left|\psi_3\right\rangle = C_1^x C_2^x \left|\psi_0\right\rangle \qquad \left|\psi_4\right\rangle = C_1^y C_2^y \left|\psi_0\right\rangle \tag{26}$$

The first two wave functions represent photons excited in orthogonal directions, while the latter two represent photons excited in the same direction. This is only valid for the x and y axes of the eigenstates of the single photon excitation energy (*19*). When the polarisations were measured along arbitrary axes x' and y' the states in Eq.(27) must be transferred to rotating axes to obtain general correlations (similar for the other three wave functions).

$$\begin{aligned}\left|\psi_1\right\rangle &= \left(C_1^{x'}\cos(\alpha) + C_1^{y'}\sin(\alpha)\right)\left(-C_2^{x'}\sin(\alpha) + C_2^{y'}\cos(\alpha)\right)\left|\psi_0\right\rangle \\ \left|\psi_1\right\rangle &= -\sin(\alpha)\cos(\alpha)\left|\psi_3'\right\rangle + \sin(\alpha)\cos(\alpha)\left|\psi_4'\right\rangle + \cos^2(\alpha)\left|\psi_1'\right\rangle - \sin^2(\alpha)\left|\psi_2'\right\rangle\end{aligned} \tag{27}$$

However Eq.(27) no longer represented an orthogonal function which is needed to form entangled states. Correct combination of the wave functions in the rotating basis again gave an antisymmetrized, entangled function Eq.(29).

$$\left|\varphi_1\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\psi_1\right\rangle - \left|\psi\right\rangle_2\right) = \frac{1}{\sqrt{2}}\left(C_1^x C_2^y - C_1^y C_2^x\right)\left|\psi_0\right\rangle \tag{28}$$

[5] A qubit is the quantum analog of a classical bit. A qubit is a two quantum state system, where the two states need to have the properties for coherence and superposition leading to .

3.2.2 Parametric Down-Conversion

The second technique producing single entangled photons is parametric down-conversion. This technique was based on a non-linear optical crystal.

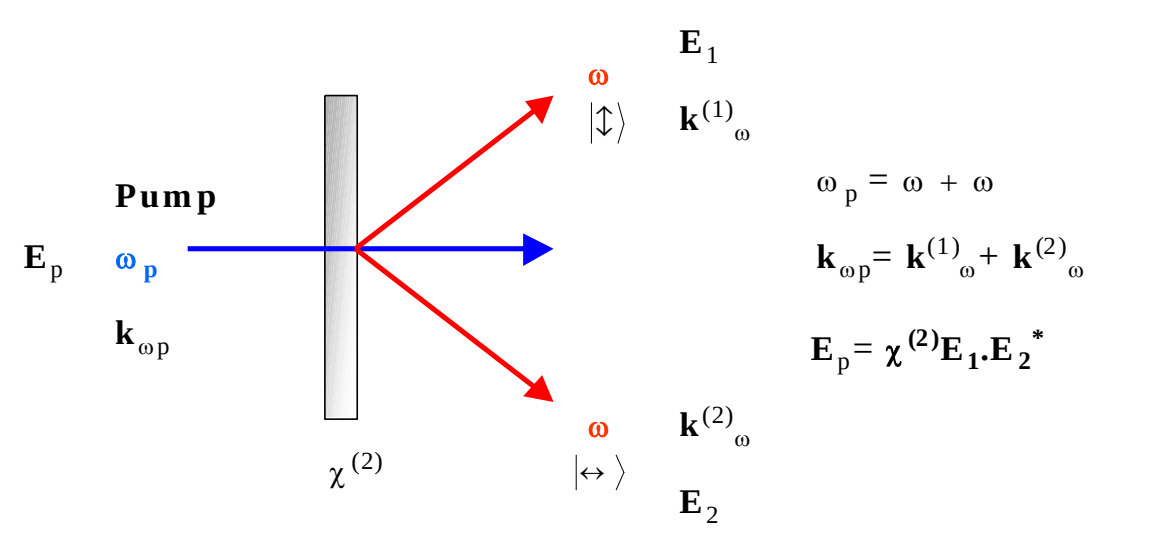

Figure 6: degenerate parametric down-conversion. A pump photon (2ω) is transformed by a χ(2) non-linear optical medium to a signal and an idler photon, in the case of degenerate down conversion both of frequency ω

When an intense beam of frequency ω_p is directed onto the medium Fig.(6), the pump photon is readily fragmented into two new photons ω_1 and ω_2. This $\chi^{(2)}$ non-linear process must obey the conservation of energy $\omega_1 + \omega_2 = \omega_p$ together with the phase-matching condition $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_p$. There exist two different phase-matching schemes: type-I phase-matching where the created photons have identical polarisations and type-II phase-matching where the created photons have opposite polarisations. There are actually three types of correlations that can be observed from optical parametric down-conversion: time entanglement, momentum entanglement and polarisation entanglement.

During parametric down-conversion one of the correlations that could occur was time entanglement (*31*). The source producing the basic qubit states essentially existed of a Mach-Zehnder interferometer. In this device a one-photon beam, with a pulse length short compared to the armlength of the interferometer, [illegible] coefficients α and β were determined by the settings of the interferometer. The two pulses were then sent through a non-linear crystal, splitting the pump photon into a signal and an idler photon. The down-conversion by the non-linear crystal yielded a final state Eq.(29).

$$|\psi\rangle = |\alpha\rangle_{p_1}|\beta\rangle_{p_2} \rightarrow |\alpha\rangle_{s_1}|\beta\rangle_{i_2} \tag{29}$$

A great advantage over other sources that produce time entangled photons was the fact that here the coherence of the pump laser was not important. The Mach-Zehnder interferometer accounted for the necessary coherence. This implies that any standard laser diode could be used to pump the process. The technique will also be easy to implement, because the analyser will not need an optical element, the photons can be distinguished upon their arrival time. This technique offered a leap forward in the length scale of quantum communication, under which quantum teleportation.

A different kind of entanglement that arose from down-conversion was momentum entanglement (*32*). After splitting of the pump photon into two photons of approximately half of the original frequency, photon pairs (consisting of a and b) were selected making use of double apertures. The photon pairs were emitted in different modes; a_1, b_2 or a_2, b_1 leading to the following state Eq.(30).

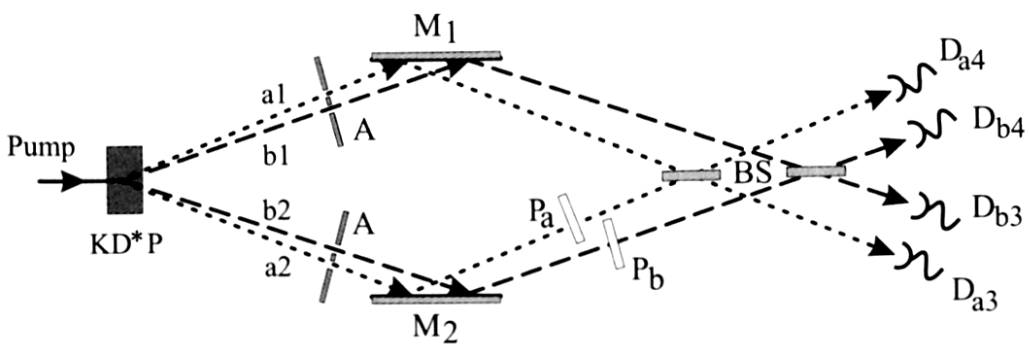

Figure 7: schematic set-up of the experiment on momentum-entanglement. Using the two double apertures A, photon pairs were selected. Recombination occurred at the beamsplitter (BS) after which the detectors D_{a4}, D_{b4}, D_{b3}, and D_{a3} measured the outcomes (*1*).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[e^{i\varphi_b}|a\rangle_1|b\rangle_2 + e^{i\varphi_a}|a\rangle_2|b\rangle_1\right] \tag{30}$$

This state was not yet fully entangled, because the states were distinguishable upon their positions of impact on the two mirrors Fig.(7). Redirecting the beams to two beamsplitters yielded four outputs, of which the lower and upper parts were indistinguishable. The total entangled state after the beamsplitters then was Eq.(31).

$$|\Psi\rangle = \frac{1}{2}\begin{bmatrix} \left(e^{i\varphi_a} - e^{i\varphi_b}\right)|a\rangle_4|b\rangle_3 + \left(e^{i\varphi_b} - e^{i\varphi_a}\right)|a\rangle_3|b\rangle_4 \\ + i\left(e^{i\varphi_a} + e^{i\varphi_b}\right)|a\rangle_4|b\rangle_3 + i\left(e^{i\varphi_a} + e^{i\varphi_b}\right)|a\rangle_3|b\rangle_4 \end{bmatrix} \tag{31}$$

The amplitudes of the states led to the probability of coincidence on the four detectors. The experimental coincidence rate between detectors a_3 and b_4, as a function of the path-length difference in the two arms of the interferometer, showed cosinusoidical behaviour. This occurred when the phase shifts, φ_a and φ_b, regulated by two glass plates in the paths of the beam were close to zero. This method suffered from difficult alignment and thermal instability.

The third and last (as known so far) correlation arising from optical parametric down-conversion was polarisation entanglement (*9*). In this experiment the entangled state was obtained directly from the non-linear crystal (beta-barium borate, BaB_3O_6 (BBO)) without need for beamsplitters and mirrors. Using type-II down-conversion, the photons were emitted in two cones Fig.(8) that intersected when the angle between the crystal optical axis and the pump beam had a certain value.

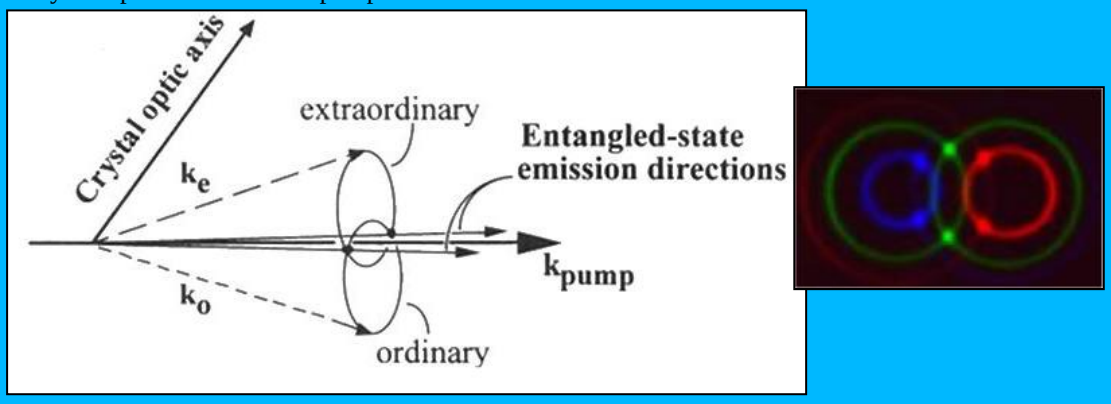

Figure 8: emission cones of type-II parametric down-conversion and a photo taken perpendicular to the propagation direction. The correlated photons lie on opposite sides of the two circles. At the intersect of the cones the polarisation of the photons is unknown, but opposite, resulting in entanglement (*9*).

The entangled state, obtained when the two cones intersected, could be written as Eq.(32) with vertical (ordinary) and horizontal (extraordinary) polarisation.

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\updownarrow\right\rangle_1\left|\leftrightarrow\right\rangle_2 + e^{i\varphi}\left|\leftrightarrow\right\rangle_1\left|\updownarrow\right\rangle_2\right] \tag{32}$$

This state was not fully entangled, while the horizontally and vertically polarised photons moved with different velocities through the crystal. The states could be distinguished by their incidence time. A second non-linear crystal (of half the length of the first) placed in the 90° rotated beams, erased this effect. This same crystal could regulate the phase φ between the two components. A half wave plate (λ/2) made it possible to produce the other Bell states as well. Relatively easy alignment and remarkable stability (phase shifts are not disadvantageous, unless they are polarisation dependent) make this technique interesting for future use in quantum communication.

3.3 Bell State Analyser

To perform a Bell state analysis it is necessary to project the state under investigation onto the Bell state basis Eq.(24). By repeating this measurement the probability of finding the teleported state in each of the four Bell states can be determined. However, for polarisation entangled photons (most frequently used to test quantum teleportation) projection on each of the four Bell states is impossible as explained below (*1*). The basis of the Bell state analyser is the fact that only one of the four Bell states is antisymmetric, namely $\left|\Psi^-\right\rangle$. The total state of a system consists of a spatial part and a part described by the Bell states. Assuming that the teleported particle behaves boson like (implying that the total state must be symmetric), the spatial part of the wave function must be antisymmetric to result in a symmetric total state (naturally for fermions this spatial part must be symmetric). An entangled photon pair incident on a beamsplitter can be described by either a symmetric or an antisymmetric spatial state Eq.(33).

$$\begin{aligned}\left|\Psi_A\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|\leftrightarrow\right\rangle_1\left|\updownarrow\right\rangle_2 - \left|\updownarrow\right\rangle_1\left|\leftrightarrow\right\rangle_2\right)\\ \left|\Psi_S\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|\leftrightarrow\right\rangle_1\left|\updownarrow\right\rangle_2 + \left|\updownarrow\right\rangle_1\left|\leftrightarrow\right\rangle_2\right)\end{aligned} \tag{33}$$

Because the total state is symmetric, the combination of the spatial part with the Bell states yielded Eq.(34).

$$\left|\Psi^{+}\right\rangle\left|\Psi_{S}\right\rangle \qquad \left|\Phi^{+}\right\rangle\left|\Psi_{S}\right\rangle$$
$$\left|\Psi^{-}\right\rangle\left|\Psi_{A}\right\rangle \qquad \left|\Phi^{-}\right\rangle\left|\Psi_{S}\right\rangle \tag{34}$$

The beam splitting thus results in only one total state with an antisymmetric spatial component (the beamsplitter leaves the Bell states and the spatial states intact as can be proven by making use of the Hadamard transformations). The result of this is that in 25% of the cases 1 photon appears on each side of the beamsplitter and give rise to coincidence in two oppositely placed detectors. Therefore there will be full distinction between $\left|\Psi^{-}\right\rangle$ and the other three states. Further measurement of the polarisation leads to discrimination between $\left|\Psi^{+}\right\rangle$, consisting of two photons with opposite polarisation and the resulting two Bell states with equally polarised photons $\left|\Phi^{+}\right\rangle, \left|\Phi^{-}\right\rangle$ that are degenerate in detection.

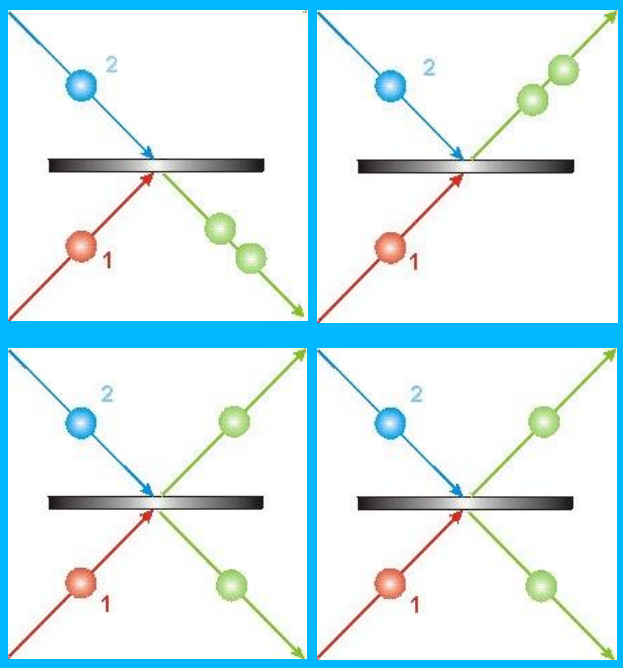

Figure 9: schematic representation of the four possible outcomes of the Bell state analyser. The upper two situations correspond to states in which the two green photons can be distinguished. The situation of the lower to represent the creation of entangled states. Here it is no longer possible to tell if photon 2 is reflected or transmitted, equal for photon 1.

3.4 Experimental Realisation and Recent Achievements

In this section a closer look is taken on the experimental realisation of quantum teleportation. The original theoretical scheme was soon followed by its practical application. Although this first scheme makes use of discrete variables (photon polarisation) it soon proved to be possible to teleport with continuous variables (heavy particle mass or quadrature phase amplitudes). Based on parametric down-conversion entangled photons (section 3.2.2) it appeared possible to swap the entanglement between two photon pairs and to entangle more than two photons. The current distance record in quantum teleportation is held by a research group from Geneva (*33*) that have transported the quantum state of photons successfully between two laboratories 55 m apart, using 2 km of standard telecommunication fibre.

3.4.1 Original Scheme

The first experimental realisation of quantum teleportation was based on the scheme originally proposed by Bennet *et al*. The source of the entangled photons was parametric down-conversion as described below. For a projection of Alice's particles 1 and 2 onto a Bell state basis it was necessary that both particles could not be distinguished. To achieve the projection both particles were led to a beamsplitter after which coincidence in two oppositely placed detectors was sufficient (projection onto the antisymmetric Bell state $|\Psi^-\rangle$ (*7*). Photons 1 and 2 were generated by using a pulsed laser beam (pulse duration 200 fs) that was sent through a narrow-bandwidth filter (4 nm) to result in a coherence time of 520 fs. Thus, it was prevented that particles 1 and 2 would be distinguishable by means of their arrival time. It was demonstrated for the qubit consisting of two states linearly polarised at –45° and +45° (already a superposition of horizontal and vertical polarisation). When the detectors f_1 and f_2 Fig.(10) fired simultaneously, projection on the $|\Psi^-\rangle$

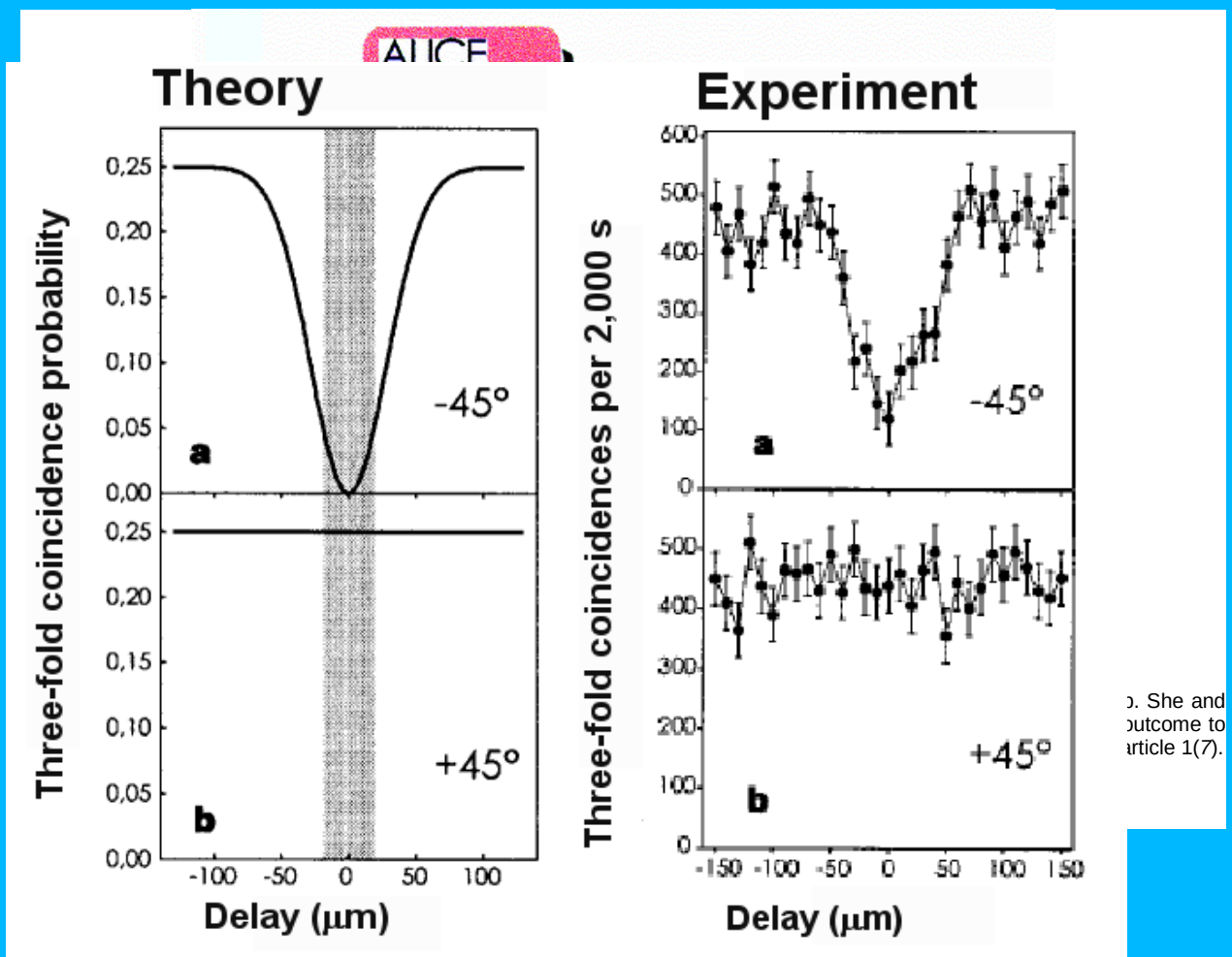

Figure 11: theoretical and experimental results for three-fold coincidence rates, plotted as a function of the delay time between photon 1 and 2. For polarisation at +45°the threefold coincidence dips to zero with the detector analysing-45° ($d_1f_1f_2$), for detector at +45° ($d_2f_1f_2$) the value is constant (*7*).

state had succeeded (25% of the cases). If photon 1 was polarised at +45° the coincidence at Alice's detectors identified Bob's particle as having the same initial state as particle 1 (which meant +45°). Full proof of the teleportation of particle 1 was obtained when the three-fold coincidence $d_2f_1f_2$ (+45°) was measured while $d_1f_1f_2$ (-45°) was absent. By changing the position of the retroflection mirror (casting the UV-beam the second time onto the non-linear crystal), a second pair of entangled photons (1 and 4) was created and the temporal overlap between photon 1 and 2 could be regulated. Photon 4 acts as a trigger that

photon 1 is on its way. Only in the region of temporal overlap quantum teleportation is possible, while outside this region photon 1 and 2 travel independently towards f_1 and f_2. This means that in this classical case the coincidence between f_1 and f_2 is 50%. Because photon 3 is part of an entangled pair, observation at d_1 and d_2 is equally probable. The chance of classical teleportation outside the region of temporal overlap of photon 1 and 2 results in 25% for $d_2f_1f_2$ (+45º) coincidence Fig.(11).

When photon 3 is completely uncorrelated, the coincidence rate in Fig.(11) was predicted to go down by half. The fact that it actually went down to zero showed that there were full correlations and that teleportation was complete. The fidelity of teleportation is defined by the overlap of the initial and the teleported qubit and was in this experiment 70% ± 3%, that corresponded to the number of teleported photons in the right polarisation state.

3.4.2 Two-Photon Approach

In an experimental scheme proposed only one year later only two particles were needed for the complete process of teleportation (*10*). The EPR pair in this scheme Fig.(12) was not based on polarisation, but **k**-vector entanglement. This two-photon approach was limited with respect to the unknown state, this state must be prepared on one of the photons of the EPR pair and therefore must be a pure state. The projection of states on the Bell state basis distinguished between all four Bell states, resulting ideally in 100% success rate of teleportation. The authors stated that the chance of teleporting a particle classically must be smaller than ¾. To prove the effect of quantum teleportation this inequality must be violated. The experimental set-up starts with polarisation entangled photons (by parametric down-conversion) with state Eq.(3).

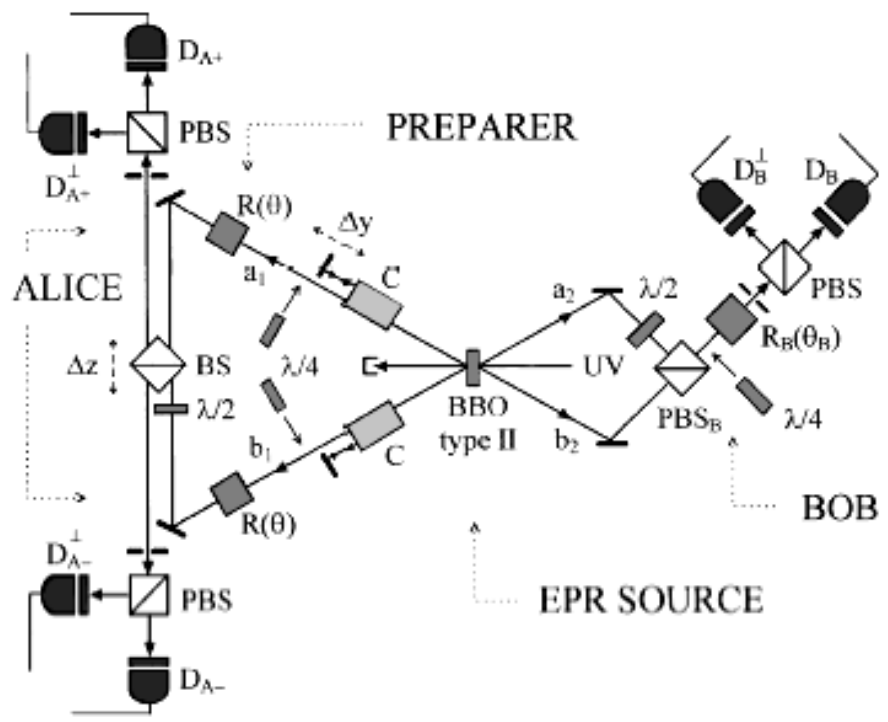

Fig 12: set-up for the 2-photon approach of quantum teleportation showing the role of Alice and Bob. An UV beam is sent through a BBO crystal after which they are sent to Alice. The backreflection of photons at the calcite crystals C, creates the beams travelling to Bob by paths a_2 and b_2 (*10*).

These photons were sent through calcite crystals to turn to a **k**-vector entanglement Eq.(35)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle\right)|\updownarrow\rangle_1|\leftrightarrow\rangle_2 \tag{35}$$

Here a and b denote the paths of the photons leading to Alice and Bob respectively. On its way to Alice, photon 1 is intercepted by the preparer who turns the polarisation $|\leftrightarrow\rangle$ into an arbitrary superposed state that will be teleported. The total state of the system could be represented by Eq.(36).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle\right)\left(\alpha|\updownarrow\rangle_1 + \beta|\leftrightarrow\rangle_1\right)|\leftrightarrow\rangle_2 \tag{36}$$

Consider four orthonormal states analogue to the maximally entangled Bell states Eq.(37).

$$\begin{aligned} |c_\pm\rangle &= \frac{1}{\sqrt{2}}\left(|a_1\rangle|\updownarrow\rangle_1 \pm |b_1\rangle|\leftrightarrow\rangle_1\right) \\ |d_\pm\rangle &= \frac{1}{\sqrt{2}}\left(|a_1\rangle|\leftrightarrow\rangle_1 \pm |b_1\rangle|\updownarrow\rangle_1\right) \end{aligned} \tag{37}$$

These four states were used in rewriting the total state of the system to Eq.(38).

$$|\Psi\rangle = \frac{1}{2}\left[\begin{aligned} &|c_+\rangle\left(\alpha|a_2\rangle + \beta|b_2\rangle\right)|\leftrightarrow\rangle_2 + |c_-\rangle\left(\alpha|a_2\rangle - \beta|b_2\rangle\right)|\leftrightarrow\rangle_2 \\ &+|d_+\rangle\left(\beta|a_2\rangle + \alpha|b_2\rangle\right)|\leftrightarrow\rangle_2 + |d_-\rangle\left(\beta|a_2\rangle - \alpha|b_2\rangle\right)|\leftrightarrow\rangle_2 \end{aligned}\right] \tag{38}$$

For measurement on the Bell state basis at Alice's, the polarisation and the **k**-vector of the photons need to be entangled. This was achieved by a beamsplitter and combining the vertical component of a_1 with the horizontal component of b_1. Effectively this is put into practice by rotating the path of b_1 by 90º, changing the state $|b_1\rangle|\updownarrow\rangle_1$ into $-|b_1\rangle|\leftrightarrow\rangle_1$ and the state $|b_1\rangle|\leftrightarrow\rangle_1$ into $|b_1\rangle|\updownarrow\rangle_1$. This changed the Bell state basis to Eq.(39).

$$\begin{aligned} |c_\pm\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|a_1\rangle \pm |b_1\rangle\right)|\updownarrow\rangle_1 \\ |d_\pm\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|a_1\rangle \mp |b_1\rangle\right)|\leftrightarrow\rangle_1 \end{aligned} \tag{39}$$

The paths a_1 and b_1 were led to a beam splitter which was followed by a polarising beamsplitter coupled to detectors $D_{A\pm}$ and $D_{A\pm}^{\perp}$. Firing of both detectors respectively corresponded to measurement of $|d_\pm\rangle$ and $|c_\pm\rangle$. The position of the beamsplitter was chosen to make sure that the detectors in the left upper and lower corner Fig.(12) were excited by photons with respectively states $\frac{1}{\sqrt{2}}\left(|a_1\rangle + |b_1\rangle\right)$ and $\frac{1}{\sqrt{2}}\left(|a_1\rangle - |b_1\rangle\right)$. This offered the possibility of detecting all four Bell states simultaneously. The photons sent to Bob made use of paths a_2 and b_2 and originated from backreflection of the calcite crystals. By rotating the path a_2 by 90º and combining it with b_2 at a beamsplitter Eq.(38) turns into Eq.(40).

$$|\Psi\rangle = \frac{1}{2}\begin{bmatrix} |c_+\rangle\left(\beta|\leftrightarrow\rangle_2 + \alpha|\updownarrow\rangle_2\right) + |c_-\rangle\left(-\beta|\leftrightarrow\rangle_2 + \alpha|\updownarrow\rangle_2\right) \\ +|d_+\rangle\left(\alpha|\leftrightarrow\rangle_2 + \beta|\updownarrow\rangle_2\right) + |d_-\rangle\left(-\alpha|\leftrightarrow\rangle_2 + \beta|\updownarrow\rangle_2\right) \end{bmatrix} \tag{40}$$

After Alice has sent her measurement outcome, Bob can transform his photon back to the original state by a simple unitary transformation. The measurement outcome was a teleportation success rate of 0.853, which clearly violated the classical limit. Although the classical line from Alice to Bob is not directly vissible in Fig.(12) it is an essential part of this set-up. This channel is used to inform Bob on the outcome of the Bell state measurement at Alice's after which he can perform the necessary unitary transformation.

3.4.3 Continuous Variables

Teleportation of discrete variables (e.g. photon polarisation) can also be extended to continuous variables (optical fields, motion of massive particles). Shortly after the introduction of quantum teleportation by Bennet *et al*. the scheme for this was proposed (*34*) and four years later this scheme was realised (*35*). Consider the case when the position (x) and momentum (p) entanglement of two particles (2 and 3) led to the following initial conditions: $x_2 + x_3 = 0$, and $p_2 - p_3 = 0$ (*34*) with only the joined properties specified. The particle and the momentum operator do not commute $[\hat{x},\hat{p}] = i\hbar$, on the other hand, the operators of the joined position and momentum do commute. This means that Alice can measure the exact state of the joined properties. Similarly to the experiments above, Alice had to perform a Bell state measurement to yield information on the joined properties of the initial particle and the EPR particle: $x_1 + x_2 = a$, and $p_1 - p_2 = b$ (with a and b real numbers). Bob could retrieve the following information from his part of the entangled state: $x_3 = a - q_1$, and $p_3 = p_1 - b$. When Alice had send him the measured values of a and b, Bob had to displace his particle linearly by a and b to regain the state of particle 1. Although the continuous variables x and p had been introduced, they were not used in this experiment. The reason for their introduction is the fact that the operators describing electromagnetic fields obey the same commutation relations.
A single mode of the electronic field can be described by the Hamiltonian of a harmonic oscillator Eq.(41), with m the mass of the particle and ω the oscillation frequency.

$$H = \frac{p^2}{2m} + \frac{m}{2}\omega^2 x^2 \tag{41}$$

To change this classical Hamiltonian into its quantum mechanical analogue, the position and momentum should be interpreted as operators. Both operators can be defined on a basis of creation and annihilation operators $\hat{a}^+$ and $\hat{a}$.

$$\begin{aligned} \hat{x} &= \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}^+ + \hat{a}\right) \\ \hat{p} &= i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a}^+ - \hat{a}\right) \end{aligned} \tag{42}$$

This led to the quantum mechanical analogue of the Hamiltonian for the classical harmonic oscillator Eq. (43).

$$\hat{H} = \hbar\omega\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) \tag{43}$$

A single mode of the opto-electric field can also be rewritten to a basic form in terms of the creation and annihilation operators Eq.(44)

(44)

The entangled state shared between Bob and Alice is a highly squeezed mode of the electromagnetic field, where the quadrature phase amplitudes ($\hat{X}, \hat{P}$) were the analogues of position and momentum (*36*). Both amplitudes could not be completely characterised simultaneously. By similar arguments to the position and momentum operators, the quadrature field amplitudes could be described by Eq.(45).

$$\begin{aligned} \hat{X} &= \left(\hat{a}^{+} + \hat{a}\right) \\ \hat{P} &= i\left(\hat{a}^{+} - \hat{a}\right) \end{aligned} \tag{45}$$

The non-linear crystal was usually placed in a resonant cavity to generate longer interaction times and therefore increase the amount of squeezing (appendix 3). The evolution of the quadrature phase amplitudes can be described by Eq.(46).

$$\begin{aligned} \frac{d\hat{X}}{dt} &= S\hat{X} \rightarrow \hat{X}(t) = \hat{X}(0)e^{St} \\ \frac{d\hat{P}}{dt} &= -S\hat{P} \rightarrow \hat{P}(t) = \hat{P}(0)e^{-St} \end{aligned} \tag{46}$$

With the coupling constant S depending on the nonlinearity of the crystal and the intensity of the pump beam. The effect of degenerate parametric amplification was amplification of the in-phase amplitude $\hat{X}$ and damping (squeezing) of the out-of-phase amplitude $\hat{P}$. The Bell-state measurement for position and momentum entangled states in this scheme is simple. A combination of the initial beam (X_1, P_1) with one of the beams of the entangled pair (say X_2, P_2) onto a beamsplitter resulted in two output beams.

$$\begin{aligned} \left(X_c, P_c\right) &= \left(X_1 - X_2, P_1 - P_2\right) \\ \left(X_d, P_d\right) &= \left(X_1 + X_2, P_1 + P_2\right) \end{aligned} \tag{47}$$

Using her two heterodyne detectors (appendix 2) Alice could now experimentally determine the X component of beam D and the P component of beam C. This gave the results: a = $X_1 + X_2$, and b = $P_1 - P_2$, which she sends to Bob. The second beam of the EPR entangled pair (X_3, P_3) at Bob's was reflected into a mirror (R=99%) through which a field is added which has been shifted with the values a and b. This results in a shift of Bob's particles of a and b, leaving him with the teleported state of particle1. Due to the highly sophisticated and sensitive set-up the fidelity of transportation was only 0.58 ± 0.02, which is still higher than the classical limit of 0.5.

3.4.4 Swapping

In general this was another possibility to create entangled states. It was strongly related to the original experimental realisation of quantum teleportation described above and it made use of Type-II parametric down-conversion. This method projected the state of two arbitrary particles onto an existing entangled state. The particles need not interact for this projection, it was sufficient to perform a Bell state measurement that collapsed the two particles into an entangled state. This projection scheme was referred to as swapping. The newly formed entangled pair consisted of particles that had no common past (*37*). The experimental set-up for entanglement swapping contained two sources that produced entangled states. These sources were set to simultaneously emit entangled pairs, without mutual entanglement between the two sources. Assuming that the photons were polarisation entangled, the total state of the system was Eq.(48).

$$\left|\Psi\right\rangle_{1234} = \frac{1}{2}\left(\left|\leftrightarrow\right\rangle_1\left|\updownarrow\right\rangle_2 - \left|\updownarrow\right\rangle_1\left|\leftrightarrow\right\rangle_2\right)\left(\left|\leftrightarrow\right\rangle_3\left|\updownarrow\right\rangle_4 - \left|\updownarrow\right\rangle_3\left|\leftrightarrow\right\rangle_4\right) \tag{48}$$

With the use of the maximally entangles Bell states in Eq.(24), this total state could be rewritten to the form of Eq.(49).

$$|\Psi\rangle_{1234} = \frac{1}{2}\begin{pmatrix} |\Psi^+\rangle_{14}|\Psi^+\rangle_{23} + |\Psi^-\rangle_{14}|\Psi^-\rangle_{23} \\ +|\Phi^+\rangle_{14}|\Phi^+\rangle_{23} + |\Phi^-\rangle_{14}|\Phi^-\rangle_{23} \end{pmatrix} \tag{49}$$

When a Bell state measurement was performed on particle 2 and 3, as a consequence of Eq.(49) particle 1 and 4 collapsed to the same state as measured for 2 and 3. The set-up for this experiment was practically the same as for the quantum teleportation scheme in section 3.4.1 (*8*).

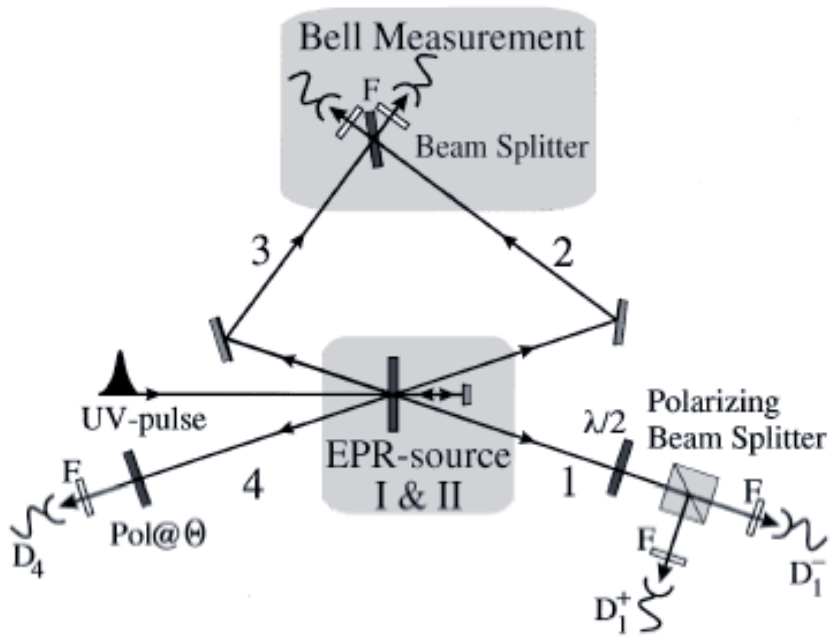

Figure 13: experimental set-up for entanglement swapping. An UV pulse creates a pair 1-2 of entangled photons when passing through a non-linear crystal. Photon 2 meets a beamsplitter and on its second round through the crystal creates a second entangled pair 3-4 (*8*). Although not shown in this figure a classical line is present between the BSM and the detectors at the receiving station (D).

Only now, photon 4 was not used as a trigger but had its role in the entanglements. As a consequence of this scheme, the state that was teleported was not well defined. In this case not the state of the system, but the entanglements to other particles were teleported. The choice was made to look only at the Bell state for which photon 2 and 3 were projected onto the state $|\Psi^-\rangle_{23}$ (and similarly photon 1 and 4 onto $|\Psi^-\rangle_{14}$). To check whether entanglement swapping had occurred, a fourfold coincidence must be detected. This fourfold coincidence was obtained by the detection of $|\Psi^-\rangle_{23}$ together with either $D_1^+D_4$ or $D_1^-D_4$ coincidence. The detector D_4 in Fig.(13) was preceded by a polariser that could be set at arbitrary polarisation angles θ. The use

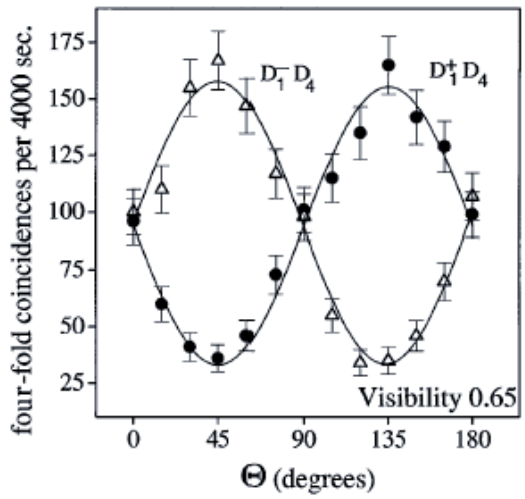

Figure 14: verification of entanglement swapping. Rate of the fourfold coincidences when varying the polariser angle θ.

of the λ/2 retardation plate provided the possibility to measure photon 1 in any linear polarisation basis (chosen is for 45º). The two-fold coincidences (assuming $|\Psi^-\rangle_{23}$ was present) $D_1^+D_4$ and $D_1^-D_4$ showed sinusoidal behaviour. When measuring at the setting of θ = 45º, the coincidence of $D_1^+D_4$ should go to zero. D_1^+ only detects polarisations at +45º, because the polarisation of photon 1 and 4 was orthogonal, the combination with θ = 45º could not lead to coincidence. However when detecting the coincidence in $D_1^-D_4$, the two detectors measured orthogonal polarisation and the rate went to a maximum Fig.(14). The visibility in Fig.(14) is 0.65 ± 0.02 which is clearly larger than the classical limit of 0.5. This procedure can be extended to a theoretical survey of multiple particle swapping (*2*). Multiple particle entangled states originate from states with entanglement between only 2 or 3 particles. The basic set-up could consist of a source of GHZ particles and Bell state measurement devices. To obtain an N+1 particle entangled state the following general scheme could be followed.

The arrow denotes a Bell state measurement between two particles of the different systems.
As an example the conversion of two EPR states and a GHZ (3-particle state) to a 3-particle and a 4-particle GHZ state Fig.(15). The bold lines denote entanglement relations while the dashed lines represent Bell state measurements.

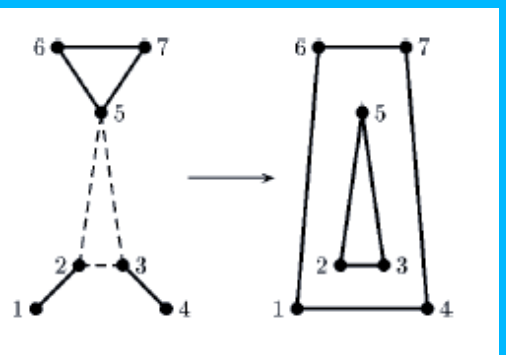

Figure 15: conversion of two Bell states and a 3-particle GHZ state to a 3- and a 4-particle GHZ state (*2*).

3.4.5 Three-Photon Entangled States

Multi-particle entangled states are important in quantum teleportation as well as in other forms of quantum communication. An entangled state of more than two particles is known as a Greenberger-Horne-Zeilinger (GHZ) state. These states were originally brought up to test whether or not quantum mechanics was complete (*38*). It can be shown experimentally that there exist entanglements between more than two spatially distant systems (*5*). The set-up of this experiment was based on the techniques that were used in above (quantum teleportation and swapping). The basics of the GHZ particle creation were two entangled pairs that can be turned into a three-particle entangled state and a fourth individual photon. This GHZ entanglement was only complete if both the independent photon and the three-particle entangled state were simultaneously observed. The polarisation entangled pairs were produced by sending short UV pulses through a non-linear crystal Fig.(16), resulting in a state of Eq.(3). The beamsplitter in arm a reflected vertically polarised photons and transmitted horizontally polarised photons. Firing of detector T must therefore be due to horizontally polarised photons. Beam b hit a 50/50 polarisation independent beamsplitter. Both from path a and path b a beam was directed to the final polarising beamsplitter. In between the two beamsplitters, the polarisation in path a was changed from vertical to 45º with a λ/2 plate.

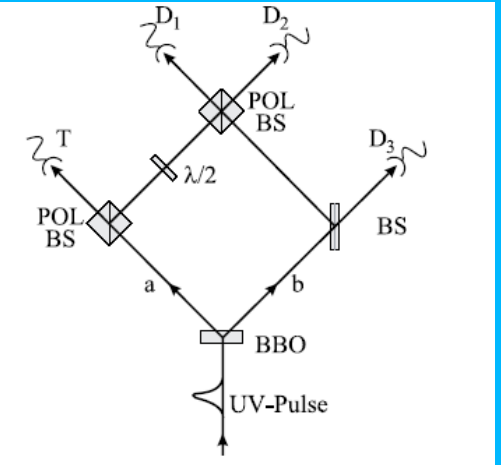

Figure 16: experimental set-up for the realisation of GHZ entanglement for spatially separate photons (37).

The three output beams towards D_1, D_2, and D_3 were led through interference filters and single mode fibres.

At the end the detection probability of a photon was about 10%. In the case when two EPR pairs were created by a single pulse (in contrary to the previously mentioned set-ups desirable here), fourfold coincidence detection of T, D_1, D_2, and D_3 implied detection of a three-particle GHZ state. The arguments below led to this conclusion. If a photon was detected in T, it must have horizontal polarisation, the other photon in the entangled pair in path b must therefore have vertical polarisation. This photon in path b had 50% chance of being reflected towards the final beamsplitter and 50% chance of being transmitted to D_3. When considering this second probability, detectors D_1 and D_2 could only fire due to a second entangled pair. The photon of this second pair must be vertically polarised, to be reflected towards the final beamsplitter. Therefore the accompanying photon in path a was horizontally polarised and arrived after reflection in the first beamsplitter at detector D_1. To be transmitted by the beamsplitter, the photon that was detected by D_2 must have equal polarisation as the photon detected in D_1. However the photon in path a originally had vertical polarisation, this was changed to 45º by the λ/2 plate (superposition of horizontal and vertical polarisation). So, it had 50% chance to hit at detector D_2 with horizontal polarisation. Thus, if the photon at T was entangled with the photon at D_3, coincidence by D_1, D_2, and D_3 led to the state (50).

(50)

Similarly, if the photon detected at D_2 was entangled with the photon detected at T, coincidence by D_1, D_2, and D_3 led to the state (51).

(51)

However these two states did not form a GHZ state yet, they could be distinguished by noting that different pairs of detectors fire simultaneously (e.g. for state (50), T and D_3 or D_1 and D_2). To prevent this, the photon detectors were placed behind narrow bandwidth filters, the coherence time of the photons were now much larger than the pulse duration. The superposition of the states (50) and (51), led to Eq.(52) which is a GHZ state.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle_T\left(|\leftrightarrow\rangle_1|\leftrightarrow\rangle_2|\updownarrow\rangle_3 + |\updownarrow\rangle_1|\updownarrow\rangle_2|\leftrightarrow\rangle_3\right) \tag{52}$$

The fourfold coincidence detection performed a projection onto this equation and terms for two photons hitting at the same detector were filtered out. The way to prove GHZ entanglement was to show that a combination of T with either $H_1H_3V_3$ or $V_1V_2H_3$, but with no other states was observable. The ratio between the desired and the undesired states (e.g. $H_1H_2H_3$ or $V_1V_2V_3$) was 12:1. But proven existence of these states alone was not sufficient, they could as well be present in a statistical mixture. This superposition was shown by polarising photon 1 along +45º leaving it in the state (53).

$$|+45^\circ\rangle_1 = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_1 + |\updownarrow\rangle_1\right) \tag{53}$$

Now Eq.(52) could be transformed to the form of Eq.(54).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\leftrightarrow\rangle_T|+45^\circ\rangle\left(|\leftrightarrow\rangle_2|\updownarrow\rangle_3 + |\updownarrow\rangle_2|\leftrightarrow\rangle_3\right) \tag{54}$$

Rewriting the entangled states of photon 2 and 3 on the basis of the +45º polarisation resulted in Eq.(55).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+45^\circ\rangle_2|+45^\circ\rangle_3 - |-45^\circ\rangle_2|-45^\circ\rangle_3\right) \tag{55}$$

This meant that photon 2 and 3 were polarised along the same direction. The mixed polarisation terms +45°. -45° were not present due to destructive interference, thus indicating the form of Eq.(55). To test this experimentally, the detectors D_1, D_2, and D_3 were placed behind polarisers (respectively +45º, -45º, and either +45º or -45º). The difference in arrival time between the photons at detector D_1 and D_2 was varied. Fourfold coincidence was measured when the polariser in front of D_3 is set at -45º at zero delay, this was what could be expected from the GHZ state and was shown in Fig.(17). At longer delay times the photons were no longer indistinguishable and there no longer was a superposed state. The observed visibility was quite high, 75% and was mainly limited by the finite width of the interference filters and the finite pulse duration.

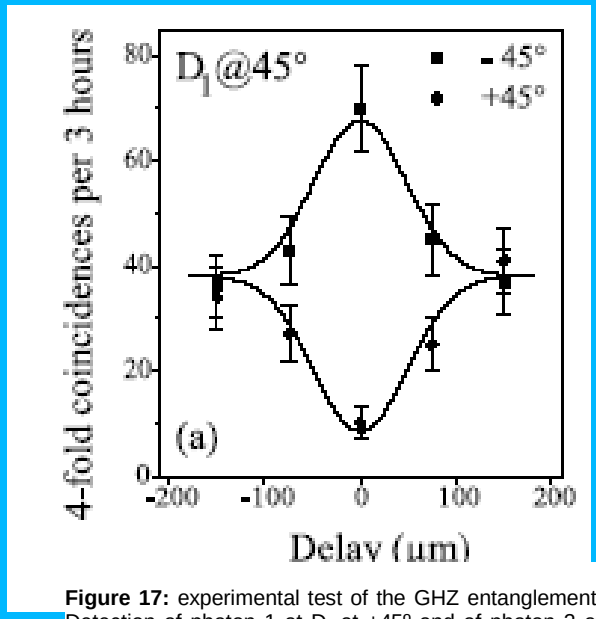

Figure 17: experimental test of the GHZ entanglement. Detection of photon 1 at D_1 at +45º and of photon 2 at 45º, detection of photon 3 at variable polarisations (*5*).

3.4.6 Four-Photon and Multi-Photon Entangled States

Four-photon GHZ entanglement was based on the creation of two photon pairs (*6*). Suppose that the total state could be represented by a tensor product of the individual states of the photon pairs Eq.(56).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_1|\updownarrow\rangle_2 - |\updownarrow\rangle_1|\leftrightarrow\rangle_2\right) \otimes \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_3|\updownarrow\rangle_4 - |\updownarrow\rangle_4|\leftrightarrow\rangle_3\right) \tag{56}$$

One photon of each pair was led to a polarising beamsplitter that reflected vertical and transmitted horizontal polarisation. Figure (18) showed that photon 2 and 3 were either both vertically polarised or both horizontally. This implied that the state in Eq.(56) was transformed [redacted] the polarising beamsplitter, the GHZ state corresponding to a fourfold coincidence could be described by Eq. (57), where the primes denote that the photons have passed a beamsplitter. The success probability of fourfold coincidence was 50%, 4 times more efficient than for three-photon GHZ entanglement (section 3.4.5). The verification of the fourfold GHZ correlations was performed in a way similar as in section 3.4.5 resulting in a ratio between desired and undesired fourfold events of 200:1. The coherent superposition was demonstrated with a visibility of 0.79± 0.06 that could be raised

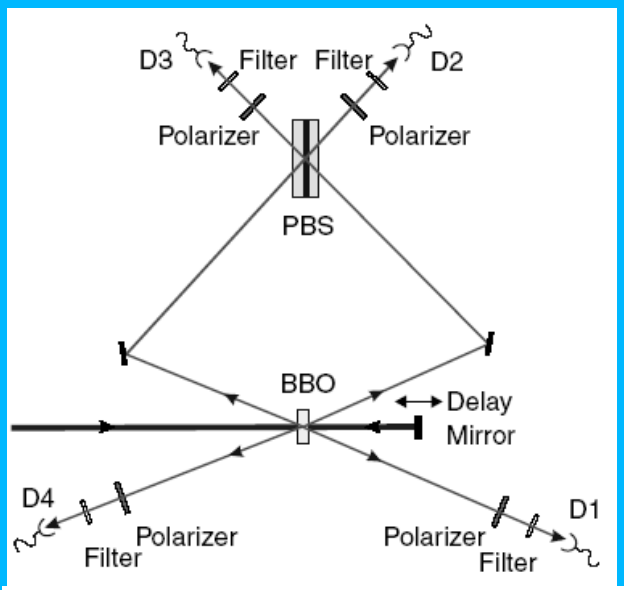

Figure 18: schematic set-up for the measurement of fourfold GHZ coincidence. A pulse of UV light passes a BBO crystal twice to produce two entangled pairs of photons (*6*).

with high quality beamsplitters and bandwidth filters. The fidelity of teleportation was 89%.

$$|\Psi\rangle_{12'3'4} = \frac{1}{\sqrt{2}}\left(|\leftrightarrow\rangle_1|\updownarrow\rangle_{2'}|\updownarrow\rangle_{3'}|\leftrightarrow\rangle_4 + |\updownarrow\rangle_1|\leftrightarrow\rangle_{2'}|\leftrightarrow\rangle_{3'}|\updownarrow\rangle_4\right) \tag{57}$$

This method could in principle be extended to multiple photons as long as one has optimal pair sources, however, the correlation could be expected to decrease (*39*).

3.3.7 Teleportation of the State of Massive Particles

Only very recently scientists have succeeded in teleporting the state of massive particles unconditionally. Two different approaches to teleport the state of trapped ions were published in the same issue of Nature (*40-42*). The main idea which the authors used to teleport the state of the ions is exactly the same as that explained for photons. However, the preparation of the state to be teleported is more complex. This development is a large step forward in realising the teleportation of macroscopic objects (and once maybe human beings) deterministically.

4. Conclusion

This paper began with describing the situation in the 1920's when the discussion that eventually led to the idea of quantum teleportation was started. Einstein and his co-workers Podolsky and Rosen found a paradox while reasoning about quantum mechanics (*12*). This paradox was based on the fact that quantum theory was considered to be a local theory (no influence by distant systems and no long range correlations) and resulted in the interpretation of quantum theory as an incomplete theory. This made scientists consider complementing theories, the so-called hidden variable theories. They were based on the fact that there were hidden variables that determined together with the wave function the complete state of the system. Soon after the publication of Einstein's ideas experiments were proposed using down-conversed polarisation entangled photons to show that there was a paradox indeed.

Bell, who published the famous Bell inequalities (*14*) to which all local hidden variables should obey, however overthrew the interpretation of quantum mechanics as an incomplete theory, complemented by local hidden variables. Again this was followed by experiments that all showed that Bell's inequalities were violated (*4*)and the results were in good agreement with quantum mechanics. It was even shown without the use of Bell's inequalities that there was a problem with the EPR paradox. The result of the discussions was that the assumption of locality on which the EPR paradox was based was considered to be unjustified. It was accepted that quantum mechanics is a complete but non-local theory, which meant that long range correlations were present. These long-range correlations formed the basis of quantum teleportation, which concerned the second part of the paper.

The idea rose that long-range correlations could be used in data transfer (*13*). The non-locality made it possible to teleport the massles information of objects with a speed larger than that of light. With the use of polarisation entangled particles (of which the state could not be described without making use of a superposition of product states) the state of a particle could be teleported without knowing the exact position of the receiver and the exact state of the initial particle. To explain the set-up of quantum teleportation (*7*) Alice (the sending station) and Bob (the receiving station) were introduced. Alice and Bob shared an ancillary pair of entangled particles created by an EPR source. For teleportation Alice needed to create an entanglement between the initial particle 1 and her particle of the EPR pair (say particle 2). This she did by performing a Bell state measurement. This measurement projected the combined state of particle 1 and 2 on a Bell state basis leaving particle 1and 2 entangled and particle 3 in a pure state. Alice should send her measurement outcome to Bob after which he could turn his particle 3 by a unitary transformation into the initial state of particle 1 and quantum teleportation is complete. While Alice sent her measurement outcome to Bob by a classical channel, it was impossible to send information with a speed larger than that of light.

The experimental set-up essentially consisted of two main parts; the Bell state analyser that performed the projection of particle 1 and 2 on an entangled state and a source of entangled particles 2 and 3. The preparation of entangled states was limited to photons, because they are most widely used and well developed. The first source of entangled photons proposed was the annihilation of an electron-positron pair. During this process two photons with orthogonal polarisation were emitted and a combination of the wave functions on a rotating basis led to an antisymmetrized entangled state. A second proposal concerned parametric down-conversion and is now widely used. In this technique a pulsed UV beam was directed onto a non-linear $\chi^{(2)}$ optical medium. The pulse beam (2ω) was split into a signal (ω) and an idler (ω) photon and the created photons could be time, momentum or polarisation entangled.

In general the different experimental achievements were all based on a single set-up. In this set-up a mirror reflected the UV beam a second time onto the non-linear crystal creating a second EPR pair (particle 1 and 4). Particle 4 was in the original scheme used as a trigger for the arrival of photon 1, however in the set-up for entanglement swapping photon 4 was incorporated in the entanglement. At Alice's photons 1 and 2 were entangled by the BSM and Alice could send the information from this measurement to Bob classically. Validation of teleportation was obtained by recording a three-fold coincidence between the two detectors at Alice's and one of the detectors at Bob. By showing that the coincidence probability of the undesired polarisation setting goes to zero when teleporting, the validation was obtained.

Very recently scientists have succeeded in teleporting the state of massive particles (ions) unconditionally. The analogy to the original idea of quantum teleportation is striking. However, the preparation of the state to be teleported is extremely complex (*40-42*).

Having taken a scientific look on quantum teleportation and knowing its current state of development it would be nice to consider what this concept could bring us in the (near) future. Will we be beamed to work instead of making use of regular transport? Can it be a solution to traffic jams and can we travel to the moon by this mechanism? In the near future and with the foreseeing techniques quantum teleportation will not be used in transport, but in quantum communication and computing. The use in quantum communication over distances of a few kilometres is realised for two-photon states. Already for quite some time scientists are trying to develop a quantum computer to increase the speed and decrease the size of data stream processes. The fidelity of teleportation is in most methods treated just above the classical limit, however in all the articles is states that by optimising especially the optical elements in the set-up, the fidelity can be increased to almost ideal values. It is not surprising that this technique is not yet completely optimised, while it is still in its early days. However the vast amount of research groups working on quantum teleportation or closely related subjects promises fast progress. Lately there appeared a numerous amount of articles on high-fidelity teleportation. Next to this improvement that is not treated just because of the limitations on this paper, papers appeared on entanglement purification, communication over noisy networks and quantum error correction. All interesting and promising subjects on which more information can be found for instance in ref.18. To teleport human beings a lot of problems have to be overcome. Although currently 2 ions can be entangled and teleported successfully and unconditionally it is an enormous step to a human being which consists of 10^{29} different atoms, joined in complex molecules (e.g. proteins and DNA). All these atoms would have to be represented by qubits, which assuming that Moore's law will hold will not be possible the next 200 years. Even if this would be possible it would still the question what to do with the soul.

The scientists that started the discussion finally leading to quantum teleportation would never have imagined that this could be possible and probably would have great difficulties in believing what they were seeing. It would be interesting to see what a few great minds and hard work will bring us in the next century.

Appendix 1: Entangled States

Looking back to the proposed experiment of Bohm (section 2.2), it was learned that instead of magnetic atoms, polarised photons could be used. To perform the experiment with photons, these should be of a special kind: entangled (*3*). The state of the combined system of two photons, separately prepared and not interacted, could be written as a product of the two individual states. However if the phonons were prepared in the same process or if they had interacted, this representation could not be used. Instead it must be written as a sum of product terms. The combined system could be represented as follows.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2\right) \tag{58}$$

An entangled state was a state, which could not be represented simply by a product of individual states. These entangled states could lead to communication faster than light. The two entangled photons seemed to be in contact even if the interaction is finished.

Appendix 2: Heterodyne Detection

Heterodyne detection is based on interferometry between the signal beam and a local oscillator. This type of detection is preferred for squeezed states (*43*). Quadrature squeezed states have a component that fluctuates less than in the vacuum state and a component that fluctuates more. For heterodyne detection, the signal squeezed beam is mixed by a beamsplitter with a strong oscillator beam (from a local oscillator) with the same frequency and a variable phase θ. The reflectivity of the beamsplitter is extremely small. Thus, the signal beam is not greatly affected. The combined beam falls onto a photodetector that counts the number of photons, n, in a time interval shorter than the decoherence time. The phase θ of the light of the local oscillator is varied to identify the squeezed field quadrature. This is visible as reduced fluctuations of the combined beam (*44*).

Appendix 3: Squeezing:

Amplitude squared squeezing exists only in quantum mechanical states and is an effect of parametric down-conversion (*44*). By describing the quadrature phase amplitudes in terms of the creation and annihilation operators as in Eq.(45), the result was a new expression for the electric field mode Eq. (59).

$$\hat{E}(t) = E_0\left(\hat{a}^+(t) + \hat{a}(t)\right) \tag{59}$$

The dimensionless quadrature phase amplitude operators obey the commutation relation Eq.(60).

$$\left[\hat{X}, \hat{P}\right] = 2\,i\left(\hat{n} + \frac{1}{2}\right) \tag{60}$$

The state of a system is quadrature squeezed when the following inequality applies (61).

$$\left\langle\left(\Delta\hat{X}\right)^2\right\rangle < \hat{n} + \frac{1}{2} \tag{61}$$

When two maximally squeezed beams fall onto a 50/50 beamsplitter, this resulted in two fields (2 and 3) described by: $X_2 + X_3 = 0$, and $P_2 - P_3 = 0$, which corresponded perfectly to the entangled state of particle 2 and 3 in section 3.4.3. The evolution of the fields inside the non-linear crystal could be described by the interaction Hamiltonian Eq.(62) (*1*).

$$\hat{H} = \hbar\omega\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + S\cos\left(2\omega t\right)\left(\hat{a}^+ - \hat{a}\right)^2 \tag{62}$$

The second term described the interaction, where 2ωt denotes the pump field and ($\hat{a}^+ - \hat{a}$) denoted the created fields, with S the coupling constant. This interaction term reduced due to energy conservation to Eq.(63).

(63)

The time evolution of the electromagnetic field was reduced to the time evolution of the creation and annihilation operators Eq.(64).

$$\frac{d\hat{a}}{dt} = -\frac{i}{\hbar}\left[\hat{a}, \hat{H}\right] = -i\omega\hat{a} - iS\hat{a}^{+}e^{-i2\omega t}$$
$$\frac{d\hat{a}^{+}}{dt} = -\frac{i}{\hbar}\left[\hat{a}^{+}, \hat{H}\right] = i\omega\hat{a}^{+} + iS\hat{a}e^{+i2\omega t} \tag{64}$$

The evolution of the quadrature phase amplitude then reduced to Eq.(46). Where the in-phase amplitude $\hat{X}$ is amplified and the out-of-phase amplitude $\hat{P}$ was damped (squeezed).

References

1. Bouwmeester, D., Ekert, A., and Zeilinger, A. (2001) *The Physics of Quantum Information* Springer, Berlin.

2. Bose, S., Vedral, V., and Knight, P. L. (1998) *Physical Review A 57*, 822-829.

3. Hardy, L. (1998) *Contemporary Physics 39*, 419-429.

4. Aspect, A., Dalibard, J., and Roger, G. (1982) *Physical Review Letters 49*, 1804-1807.

5. Bouwmeester, D., Pan, J.-W., Matthew, D., Weinfurter, H., and Zeilinger, A. (1999) *Physical Review Letters 82*, 1345-1349.

6. Pan, J.-W., Daniell, M., Gasparoni, S., Weihs, G., and Zeilinger, A. (2001) *Physical Review Letters 86*, 4435-4438.

7. Bouwmeester, D., Pan, J.-W., Mattle, K., Eibl, M., Weinfurter, H., and Zeilinger, A. (1997) *Nature 390*, 575-579.

8. Pan, J.-W., Bouwmeester, D., Weinfurter, H., and Zeilinger, A. (1998) *Physical Review Letters 80*, 3891-3894.

9. Kwiat, P. G., Mattle, K., Weinfurter, H., and Zeilinger, A. (1995) *Physical Review Letters 75*, 4337-4341.

10. Boschi, D., Branca, S., De Martini, F., Hardy, L., and Popescu, S. (1998) *Physical Review Letters 80*, 1121-1125.

11. Ou, Z. Y., Pereira, S. F., and Kimble, H. J. (1992) *Applied Physics B 55*, 265-278.

12. Einstein, A., Podolsky, B., and Rosen, N. (1935) *Physical Review 47*, 777-780.

13. Bennet, C. H., Brassard, G., Crepau, C., Jozsa, R., Peres, A., and Wootters, W. K. (1993) *Physical Review Letters 70*, 1895-1899.

14. Bell, J. S. (1964) *Physics 1*, 195-200.

15. Wick, D. (1995) *The Infamous Boundary, seven decades of controversary in quantum physics* Birkhäuser, Boston.

16. Bohr, N. (1935) *Physical Review 48*, 696-702.

17. Bohm, D. (1951) *Physical Review 85*, 166-179.

18. Bohm, D. (1951) *Physical Review 85*, 180-193.

19. Bohm, D. and Aharonov, Y. (1957) *Physical Review 108*, 1070-1076.

20. Belinfante, F. J. (1974) *A survey of Hidden Variables Theories* Pergamon Press, Oxford.

21. Reid, M. D. (1989) *Physical Review A 40*, 913-923.

22. Reid, M. D. (1996) *Europhysics Letters 36*, 1-6.

23. Clauser, J. F., Horne, M. A., Shimony, A., and Holt, R. A. (1969) *Physical Review Letters 23*, 880-884.

24. Greenberger, D. M., Horne, M. A., Shimony, A., and Zeilinger, A. (1990) *American Journal of Physics 58*, 1131-1143.

25. Braunstein, S. L., Mann, A., and Revzen, M. (1992) *Physical Review Letters 68*, 3259-3263.

26. Nielsen, M. A., Knill, E., and Laflamme, R. (1998) *Nature 396*, 52-55.

27. Gershenfeld, N. A. and Chuang, I. L. (1997) *Science 275*, 350-356.

28. Blinov, B. B., Moehring, D. L., Duan, L.-M., and Monroe, C. (2004) *Nature 428*, 153-157.

29. Hagley, E., Maitre, X., Nogues, G., Wunderlich, C., Brune, M., Raimond, J. M., and Haroche, S. (1997) *Physical Review Letters 79*, 1-5.

30. Julsgaard, B., Kozhekin, A., and Polzik, E. S. (2001) *Nature 413*, 400-403.

31. Brendel, J., Gisin, W., Tittel, W., and Zbinden, H. (1999) *Physical Review Letters 82*, 2594-2597.

32. Rarity, J. G. and Tapster, P. R. (1990) *Physical Review Letters 64*, 2495-2498.

33. Marcikic, I., de Riedmatten, H., Tittel, W., Zbinden, H., and Gisin, W. (2003) *Nature 421*, 509-513.

34. Vaidman, L. (1994) *Physical Review A 49*, 1473-1476.

35. Furusawa, A., Sorensen, J. L., Braunstein, S. L., Fuchs, C. A., Kimble, H. J., and Polzik.E.S. (1998) *Science 282*, 706-709.

36. Braunstein, S. L. and Kimble, H. J. (1998) *Physical Review Letters 80*, 869-872.

37. Zukowski, M., Zeilinger, A., Horne, M. A., and Ekert, A. (1993) *Physical Review Letters 71*, 4287-4290.

38. Zeilinger, A., Horne, M. A., Weinfurter, H., and Zukowski, M. (1997) *Physical Review Letters 78*, 3031-3034.

39. Weinfurter, H. and Zukowski, M. (2001) *Physical Review A 64*, 010102-010104.

40. Kimble, H. J. and Van Enk, S. J. (2004) *Nature 429*, 712-713.

41. Riebe, M., Häffner, H., Roos, C. F., Hänsel, W., Benhelm, J., Lancaster, G. P. T., Körber, T. W., Becher, C., Schmidt-Kaler, F., James, D. F. V., and Blatt, R. (2004) *Nature 429*, 734-436.

42. Barrett, M. D., Chiaverini, J., Schaetz, T., Britton, J., Itano, W. M., Jost, J. D., Knill, E., Langer, C., Leibfried, D., Ozeri, R., and Wineland, D. J. (2004) *Nature 429*, 737-739.

43. Yariv, A. (1989) *Quantum Electronics* John Wiley& Sons, New York.

44. Mandel, L. and Wolf, E. (1995) *Optical coherence and quantum optics* Cambridge University Press, Cambridge.