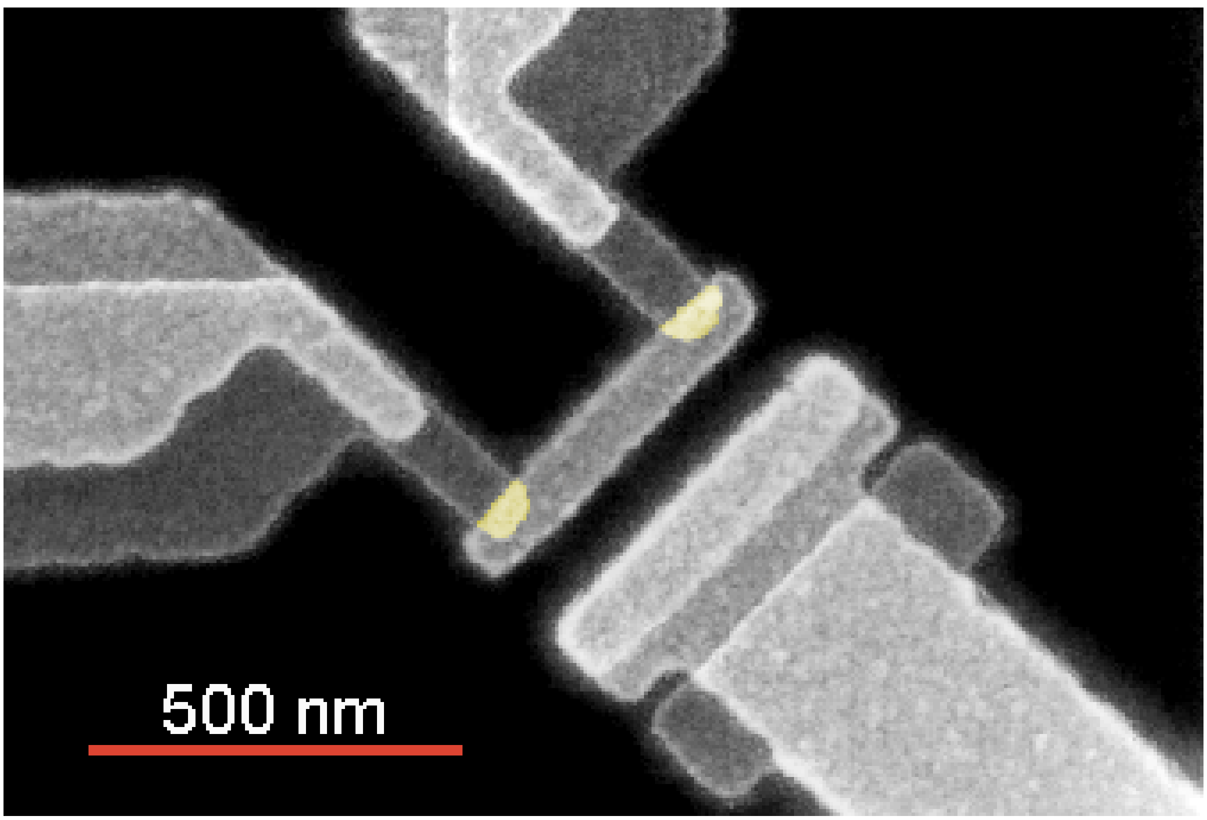
Single-electron Transistor

As Fast and Ultra-Sensitive Electrometer



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Abstract

The single-electron transistor (SET) is a nanodevice that can control the transport of single elementary charges on and off a metallic island. It can also function as transistor similarly to a nowadays FET. The principles of the operation of the SET is determined by the Coulomb blockade, an energy barrier that determines the current flow through the device and the charge placed on the metallic island. Regulating the gate charge of the device can modify the Coulomb blockade. The SET can also be used as an ultra-sensitive electrometer in DC ad RF mode. Theoretical calculations show a charge sensitivity h values lower than $1.7\times10^{-6}\,e/\sqrt{Hz}$ *for the SET and experimental research gives values of* $1.2\times10^{-5}\,e/\sqrt{Hz}$ *. The experimental value for the SET is 1000 times better than the field-effect transistor used as an electrometer. The SET can thus be used as ultra-sensitive electrometer and will be used in the future in the study of charged nanoscale systems.*

Contents

	Page
1. Introduction	**3**
2. Principles of the Single Electron Transistor	**4**
2.1 The Coulomb Blockade	**4**
2.2 Performance of the SET	**5**
2.3 Parameters that Influence the Performance of the SET	**8**
3. Ultra-sensitive Charge Measurements with the SET	**10**
3.1 The SET as electrometer	**10**
3.2 Noise that can affect the SET	**12**
3.3 The Sensitivity of the SET	**14**
4. Conclusions	**17**
5. References	**18**

1. Introduction

Miniaturization has brought today's electronic devices in scale very close to the size where quantum phenomena will dominate the operation of the device hence changing its whole properties. However quantum effects are not necessarily a downside to the electronic industry since they can be used to create new devices. Science has already entered new fields of technology that will be the future of modern electronics. One of these fields is nanoscience and the design of nano-scale devices called nanodevices. Nanodevices can often perform the same tasks as today's micro-scale devices such as FETs, yet the physical principles under which they work are very different. One of the most fascinating nanodevices is the single electron transistor (SET). This device exploits the quantum mechanical phenomenon of tunneling and it can perform as a switch or as an amplifier, similarly to common FETs, but it can also control the transport of single electrons. It is thus interesting to examine the working an properties of this device and to look into what its best applications could be.

The SET, similarly to the FET, can be used in its amplifier mode as an electrometer to measure the charge or charge variations of a specific system. Devoret and Schoelkopf (Nature 2000) claim that the SET is actually very suited to be an electrometer having a very high energy and charge sensitivity that comes close to the theoretical quantum limit. A very interesting question is if it is indeed true or feasible that the SET can reach such high values of charge sensitivity or if in the experimental setting the SET is not as a good candidate for charge measurements after all. The objective of this paper is to answer this question, if indeed the SET is an ultra-sensitive electrometer and if the optimal value set by Devoret and Schoelkopf is actually a feasible reality.

This paper will first treat the principles of the working of SETs. Then the application of the SET as ultra-sensitive electrometer will be discussed. In here the review by Devoret et. al. (Nature 2000) will be taken in consideration and viewed critically, focusing on the charge sensitivity of the device. Finally a conclusion will be given with some future considerations over the applications of the SET.

2. Principles of the Single Electron Transistor

2.1 The Coulomb Blockade

The single-electron transistor consists of a metallic island, placed between two tunneling junctions connected to a drain and a source and has a gate electrode as in a normal field-effect transistor. The tunneling junctions are simply a thin (*<10 nm*) oxide layer between the island and the electrodes. Quantum dots have also been used as islands for the SET. The schematics of the SET are given in figure 1. Each tunneling junction in the SET has intrinsic tunneling resistance and capacitance (parallel to each other). Yet, before we can fully understand the working of a SET we must first understand the concept of *Coulomb blockade*.

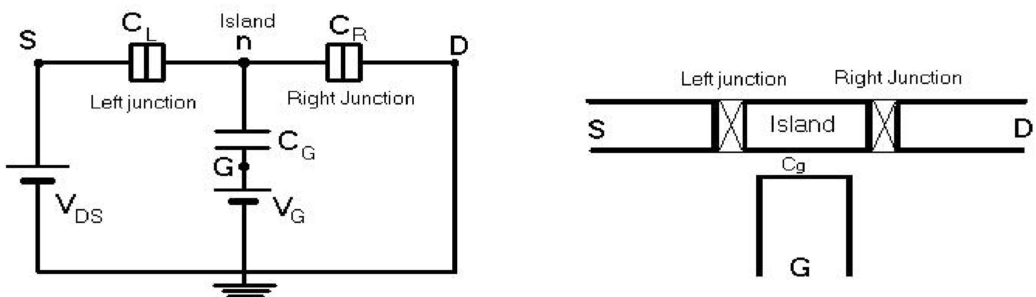

Fig. 1: Left: schematic circuit representation of the single-electron transistor. Right: a more realistic representation of what the 'core' of the single-electron transistor looks like.

The island of the single-electron transistor, even if very small (nanometric scale) still contains a very large number of electrons ($\cong 10^9$). Yet, through tunneling, one can add or subtract electrons from the island charging it either negatively or positively. The extra electrons that charge the island are called *excess electrons* and their number is designed by *n*. The number of excess electrons can also be negative, meaning that electrons have been removed leaving a positive charge on the island (one could talk of excess holes in this case). The presence of excess electrons affects the electrostatic energy of the system, which depends on the charging energy of the SET:

$$E_{ch} = \frac{1}{2}\frac{{Q_{isl}}^2}{C_\Sigma} = \frac{1}{2}\frac{n^2 e^2}{C_\Sigma} \qquad \textbf{(1)}$$

where Q_{isl} is the charge on the island, *n* the number of excess electrons, *e* the charge of one electron and C_Σ the total capacitance of the island which is equal to: $C_\Sigma = C_G + C_L + C_R$ (C_G, C_L and C_R are the gate capacitance and the intrinsic capacitances of the left and right tunneling junctions respectively). The energy scale applied when working with the SET is usually defined on the charging energy itself and the unit taken is usually: $E_C = \frac{e^2}{2C_\Sigma}$. The energy does not only depends on Q_{isl}, but also on the charge induced by the gate, the gate charge $Q_G=V_G C_G$ where V_G is the gate voltage. The electrostatic energy of the system is equal to $E_{el} = E_C(n - n_g)^2$, where *n* is the number of excess electrons of the island and n_g the number of elementary gate charges. The expression for the electrostatic energy of the system then becomes:

$$E_{el} = \frac{1}{2}\frac{Q^2}{C_\Sigma} = \frac{1}{2}\frac{(ne - V_G C_G)2}{C_\Sigma} = \frac{1}{2}\frac{(ne - Q_G)^2}{C_\Sigma} \quad \textbf{(2)}$$

This energy determines if tunneling through a junction is forbidden or allowed: if the adding of an extra excess electron causes the energy of the system to increase then tunneling will be energetically forbidden and the Coulomb charging energy will act as a blockade. This is known as the Coulomb blockade.

Two cases are thus possible. The first case is the one when we consider *n* excess electrons on the island and tunneling of one electron would cause the energy of the system increases (see also equation **(2)**). The system having *n+1* excess electrons on the island will be then energetically forbidden. No tunneling will occur through the junctions. This is the Coulomb blockade that, in this case, said to be active. In the second case tunneling of an extra electron on the island will *lower* the energy of the system, hence there will be no Coulomb blockade and tunneling will happen adding an excess electron to the island. The same principle applies if we wish to subtract electrons from the island, charging it positively. The drain-source voltage, V_{DS}, determines the energy of the electrons before the junction. When this energy is higher than the Coulomb blockade, the electrons will overcome the blockade and tunneling will occur. The height of the blockade is determined by the number of excess electrons on the island and the gate charge.

2.2 Performance of the SET

The SET controls the flow of electric current between the drain and the source through the gate electrode similarly to a FET. Unlike in a normal FET however the behavior of the SET (whether if it allows or not a current between the source and the drain) depends on the transport of single elementary charges and the gate voltage controls this via the Coulomb blockade. When the Coulomb blockade is overcome one electron will tunnel from the source to the island, adding one extra excess electron. Similarly a tunneling process will occur from the island to the drain. The energy of the source (E_{source}) and drain (E_{drain}) depend on their respective potentials, V_S and V_D. The change of the charging energy of the system if we go from *n* to *n+1* excess electrons will depend on the gate voltage V_G according to:

$$E_{n+1} - E_n = (n + \frac{1}{2} - \frac{V_G C_G}{q})\frac{e^2}{C_\Sigma} \quad \textbf{(3)}$$

So the adding of an excess electron on the island will be either favorable or unfavorable depending on the gate charge which on its turn depends on the gate voltage. We can se from equation **(3)** that if the gate charge is equal to integer values of elementary charge *e* the Coulomb blockade will be active and there will be no conduction. That is because the system has minimum energy when the island has a well defined number of electronic charges and the tunneling of an electron will only increase the global energy of the system (see figure 3a). The Coulomb blockade will be active and no tunneling of electrons in or out the island will occur (figure 2a). The transistor will be in the conducting state (meaning that tunneling will lower the energy of the system) if the gate charge is equal to values that are half integers of the electron charge, that is: $Q_G = \frac{(2N+1)}{2}e$ where *N* is an integer (recalling that *e* is the charge of one electron).

Then the system have an energy minimum which is in between two states with well-defined elementary charges (see figure 3b) on it ($Q \neq Nq$ where N is an integer number). This will cause a cascade of tunnel events, involving the two junctions sequentially, giving rise to a current between the drain and the source. This can be seen graphically in figure 2b. The another way to view the energy picture of the blockade and conduction modes of the SET is given in figure 3.

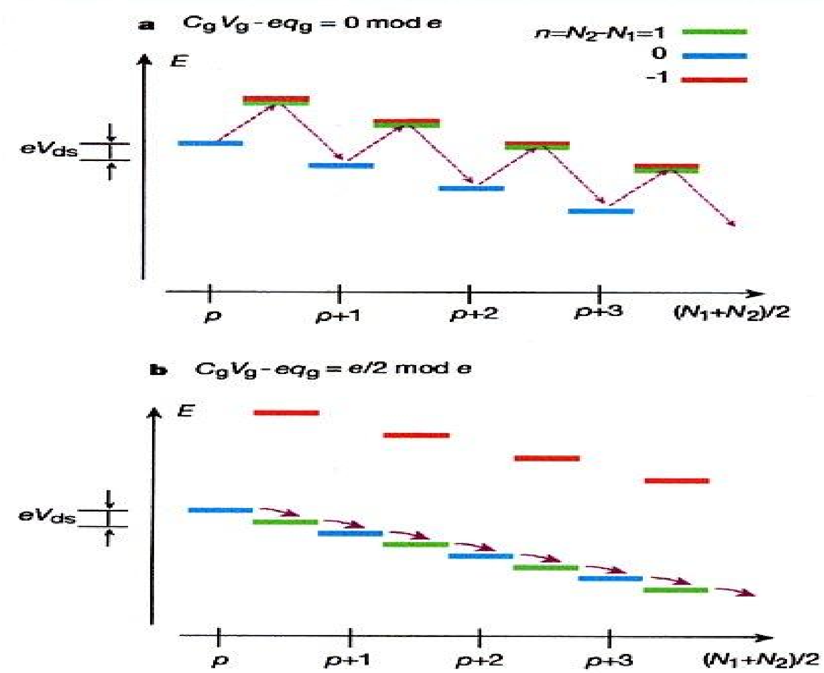

*Figure 2: Graphical representation of the Coulomb blockade in **a)** blocking state and **b)** in conducting state for the single electron transistor. The transistor will be in the conducting state if the gate charge is equal to e(2N+1)/2 where N is an integer. If the gate charge is equal to integer values of e the Coulomb blockade will be active and there will be no conduction. In the picture* N_1 *and* N_2 *denote the numbers of electrons having tunneled through the junctions and* p *is an integer. (Source: Devoret & SchoelKopf, Nature 2000)*

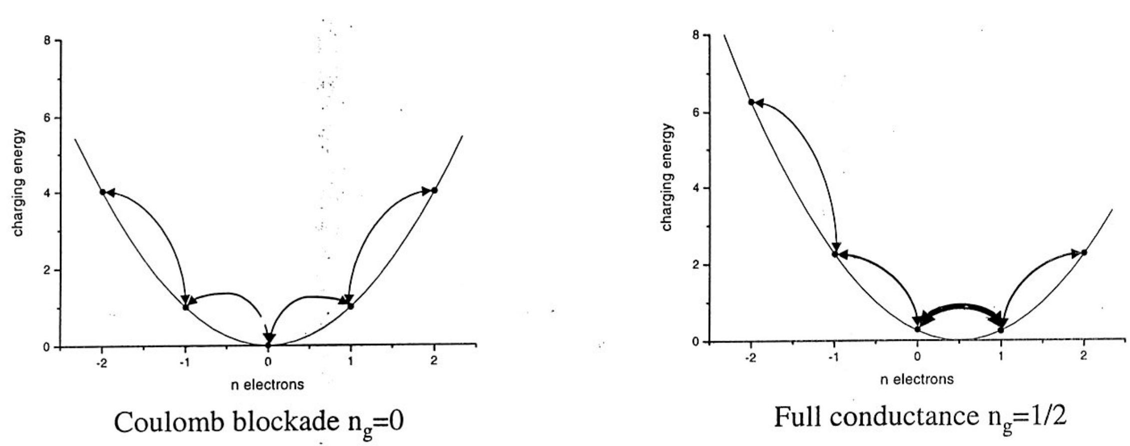

Figure 3: Charging energy against the charge present on the island n*. If the charge is integer the Coulomb blockade will be active (adding or subtracting an electron to the island will increase the charging energy). If the charge is ½ of an integer the SET will be in full conduction mode (adding an electron will not cause increase of energy).*

We can now write down the voltage conditions that have to be met to make tunneling possible. First we examine the case when the gate voltage V_G is zero. If there are no excess electrons on the island ($n = 0$) and no gate voltage then the threshold voltage, $V_{DS,T}$, must be equal to:

$$V_{DS,T} = \frac{1}{2}\frac{e}{(C_L + C_R + C_G)} = \frac{1}{2}\frac{e}{C_\Sigma} \quad \textbf{(4)}$$

where e is the charge of one electron, C_L and C_R are the (internal) capacitances of the left and right tunneling junction respectively, C_G is the gate capacitance and C_Σ is the total capacitance. The threshold voltage is an important quantity. It is the minimal drain-source voltage necessary if we want and electron to overcome the Coulomb blockade, hence have a current flow through our SET. That means that to achieve current flow the potential energy eV_{DS} must be higher than the charging energy $e^2/2C_\Sigma$. If we want to lower the threshold voltage down to zero we can achieve that by increasing the gate voltage and thereby lowering the Coulomb blockade. The voltage between drain and the source, V_{DS}, depends on the gate voltage V_G itself (assuming that V_{DS} is close to zero):

$$V_{DS} = \frac{C_G}{C_L + C_R + C_G} V_G = \frac{C_G}{C_\Sigma} V_G \quad \textbf{(5)}$$

The current will flow only if the voltage described in **(5)** is equal or higher than the threshold voltage given in **(4)**. Hence the gate voltage needed to lower the threshold voltage to zero is equal to:

$$V_G(V_{DS,T} = 0) = \frac{e}{2C_G} \quad \textbf{(7)}$$

Looking at equation **(7)** we see that it meets the condition mentioned above that the gate charge ($Q_G=V_G C_G$) must be one half the value of the charge of the electron in agreement with the conditions we set above for current conduction through the SET. From the above relations we can derive the *IV*-characteristics of the SET that are depicted in figure 4. The characteristic, as it can be seen from the picture, is symmetrical around the zero point. When $V_G = 0$ the current will start flowing only after the threshold voltage has been reached. Between $V_{DS}=e/2C_\Sigma$ and $V_{DS}=-e/2C_\Sigma$ there is a gap where no current will flow. This is the so-called *Coulomb gap* and it is the direct consequence of the Coulomb blockade. When $V_G = e/2C_G$ the threshold voltage is equal to zero and the *IV*-characteristic presents no gap since there is no coulomb blockade.

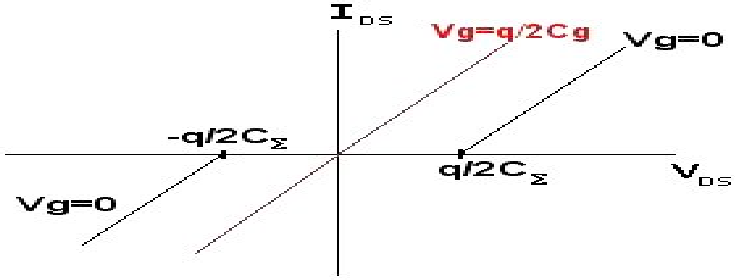

Fig. 4: IV-characteristics of the SET. The DS-current is plotted against the DS-voltage.

The current I_{DS} is given by the flow of electrons from the source to the drain (or vice versa for reversed bias). The electrons will tunnel from the source to the island to the drain, creating a current. One must keep in mind that even if the current is flowing, the number of excess electrons on the island can remain constant. Electrons will tunnel in the island from the source, also electrons will tunnel from the island to the drain, keeping the number of excess electrons unchanged, but creating a current. This process is not the same as co-tunneling, where electrons tunnel from the source to the island and from the island to the drain *simultaneously*. +

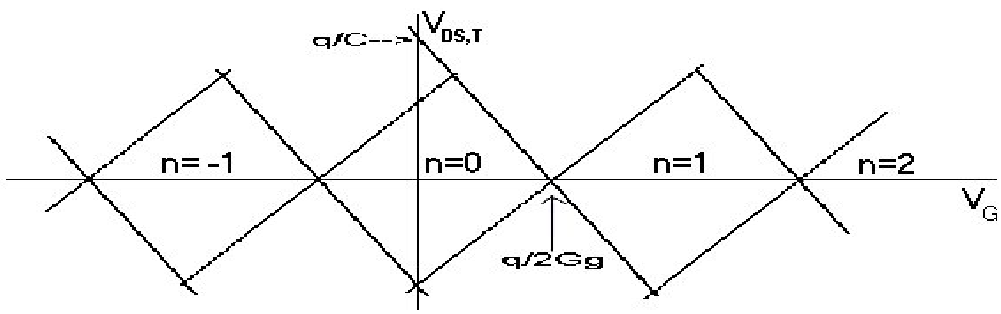

Fig. 5: The 'stability diagram' of the single electron transistor. No current will flow between the source and the drain when the DS- and gate voltages are inside the rhombic regions of the diagram. In those areas the Coulomb blockade will prevent the tunneling of the electrons. The n *values are the number of excess electrons on the island.*

Another way to picture the SET's performance is to plot the value of the threshold voltage against the gate voltage for different charging states of the island, as depicted in figure 5. According to this picture (also called the stability diagram of the SET) tunneling will be blocked by the Coulomb blockade within the rhombic shaped regions in the DS-voltage vs. gate voltage plain. The diagram linear relations present two slopes: $\left(\frac{\Delta V_{DS}}{\Delta V_G}\right) = +\frac{2C_G}{2C_R + C_G}$ (for the relations that increase linearly with the increase of V_G) and $\left(\frac{\Delta V_{DS}}{\Delta V_G}\right) = -\frac{2C_G}{2C_L + C_G}$ (for the relations that decrease linearly with the increase of V_G). If the parameters of the junctions are equal then the diagram will be symmetric and the slopes will be equal to: $\left(\frac{\Delta V_{DS}}{\Delta V_G}\right) = \pm\frac{2C_G}{C_\Sigma}$ respectively.

2.3 Parameters that Influence the Performance of the SET

The performance of the single-electron transistor is not solely determined by the voltages applied to the source, the drain and the gate electrodes, but also by other parameters such as temperature and external charges.

Thermal energy is a restriction for the proper working of the SET. The thermal energy ($=k_BT$) must be much lower than the electrostatic energy for the SET to operate in the manner it is supposed to, so the SET requires: $E_C >> k_BT$. This means that the SET must often be cooled down to very low temperatures, sometimes in the order of milli-Kelvin.

The presence of thermal energy will 'smooth out' the IV characteristics of the SET. The transitions from zero current to maximal current by varying the voltage (either the DS or the gate voltage) will not occur abruptly but the transition will occur gradually (see figure 6). The higher the temperature the less abrupt the transitions will be. At finite temperatures it is also impossible to reduce the current to zero if a non-zero V_{DS} is applied. The thermal energy aids the tunneling of the electrons and even when the Coulomb blockade is active some current (although very low) still will flow. It is important to note that quantum-mechanical effects (especially higher order tunneling effects such as co-tunneling) will also cause small deviations in the model described above, even at absolute zero temperature.

Increasing the charging energy ($e^2/2C_\Sigma$) in the SET will render possible to operate the SET at higher temperatures. This is achieved by lowering the total capacitance of the device since the electrostatic energy is inversely proportional to it ($E_C = q^2/C_\Sigma$). If the capacitance of the island is scaled down to a very low value (in the order of 10^{-18} F) the SET can be operated even at room temperature.

Another important parameter that affects the working of the SET is the presence of 'external charges', charges that are not on the SET's island but nearby it. Sometimes these charges can be a serious problem, causing an uncontrolled drift of the transistors threshold voltage. These can also be time-dependent, appearing as some king of noise. However this effect on the SET is not always negative: in fact, due to this phenomenon the SET can be used as a measurement device for charges and often with surprisingly high sensitivity and reliability.

The switching time of the SET is also a very important parameter. If we want the SET to operate as a switch or an amplifier, it will be important that the switching time of the transistor is as short as possible. The switching time τ_T is determined by the total capacitance of the device C_Σ and the tunneling resistance R_T:

$$\tau_T = R_T C_\Sigma \quad \textbf{(8)}$$

Due to the Heisenberg principle the switching time has a minimum value, that depends on the charging energy of the device, E_C:

$$E_C \tau_T \geq h \quad \textbf{(9)}$$

The tunneling resistance must be high enough to stop higher order tunneling processes (like co-tunneling), so:

$$R_T >> 2h/q^2 \quad \textbf{(10)}$$

which means that the tunneling resistance must be much higher than the so-called von-Klitzing resistance $R_K = h/q^2 = 25.8k\Omega$. Thus the SET carries a very high impedance than other transistor devices, yet since the capacitance of the device are also very small the switching time can be reduced into the order of a few nanoseconds. In practice the switching time is not as short as conventional transistors.

3. Ultra-sensitive Charge Measurements with the SET

3.1 The SET as electrometer

The single-electron transistor can be used as an amplifier. This enables the SET to perform, among other things, charge measurements, as it is possible to perform them with a common FET amplifier. Devoret and Schoelkopf (Nature vol. 406, p1039, 2000) describe the SET as a candidate for ultra-low-noise analog applications, such as the ultra-sensitive measurement of charges or quantum systems such as quantum bits (Qubits), with a charge sensitivity that is only a factor 10 higher than the theoretical limit. This, if true, can result to be a very intriguing property of the SET for different fields of modern physics.
The SET, as electrometer, can be operated in two different modes: DC and RF mode. The Set-up of the SET in the DC mode is pictured in figure 6.

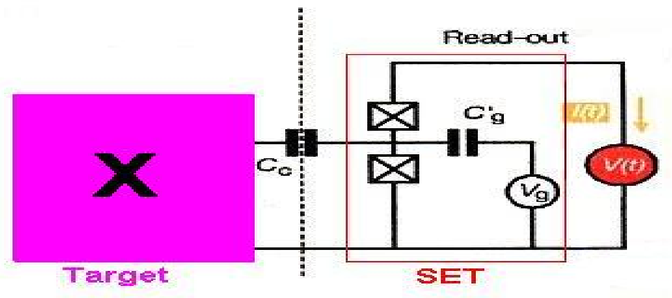

Fig. 6: The SET electrometer in DC mode (Source: Edited from Devoret & SchoelKopf, Nature 2000)

In the picture the SET is connected through a capacitance C_c to the target we wish to measure the charge from. The nature of this target is various. It could be a circuit, a quantum dot or even a molecule. The capacitance C_c is not necessarily an actual capacitor that is built into the circuit. If the target we would like to examine is for example a molecule or a quantum dot, the target will simply be built very near to the island of the SET (1μm or even less). This will result into an intrinsic capacitance between the SET and the target, since the charge of the target will influence the charge on the island. This makes possible to use the SET to measure charges in targets that would be very difficult to operate on with other electrometers. This is another reason why the SET appears to be such a good candidate as sensitive electrometer. These considerations are not only valid for the DC-SET, but also for the RF-SET.
The actual quantity measured when the SET is operated in the DC-mode is the DC-current (or equivalently the conductance) through the device. If the target one wishes to examine experiences a change in its charge, the island of the SET will feel this change through the capacitance C_c. This could be seen as a second gate charge, which will influence the threshold voltage of the device. The change will then be reflected in the value of the current through the SET. In figure 7 below the dependence of the current through the SET is put out against the value of the gate voltage at finite temperatures. Setting the drain-source and the gate voltage at values such that the current it is at its half maximum (in the picture given as a small circle) will give the highest sensitivity. The *dI/dV* slope at that point is at maximum, hence even small changes in the threshold voltage will cause relatively large changes in the current value.

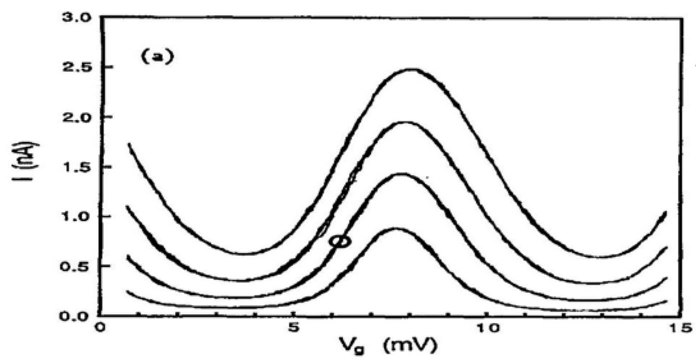

Fig. 7: I/Vg-characteristic of a SET at 200mK temperature. Since the I/V-characteristics were taken finite temperature the current does not go from zero to its maximum value abruptly but gradually. From bottom to top curve the characteristics were taken at different DS-voltages: 98, 154, 205 and 256 μV respectively. The small cicle gives the point in the curve where dI/dV is maximal (at the half maximum of the current). (Source: Martinis et. al., Appl. Phys. Lett. 1992)

The RF-SET has a more complicated set-up. It is shown in figure 8. In this set-up the SET is not only connected to the target through capacitance C_c but also to a LCR resonant circuit. The quantity measured in the RF mode is usually the damping of the resonant circuit or reflected power by the SET, rather than measuring the current through the SET directly. The reflected power depends on the conductance of the SET. The reflected power is equal to $P = \left(\frac{Z_{LCR} - Z_0}{Z_{LCR} + Z_0}\right)^2$, where Z_0 is the coax impedance and (at resonance) $Z_{LCR} = \frac{L_r G_{SET}}{C_r}$. Here L_r and C_r are the inductance and the capacitance and the LCR-circuit respectively and G_{SET} is the conductance of the SET. From these relations one can calculate the conductance of the SET from the power reflected by the SET, measuring it in the LCR-circuit. This set up offers the advantage that when operating the device at high frequencies the Flicker noise (which I'll discuss in the next section) is reduced to negligible values. At low f values, applying a feedback to the SET will also reduce the Flicker noise.

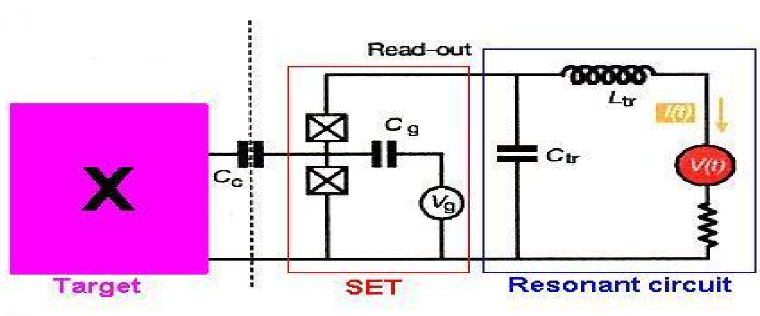

Fig. 8: The SET electrometer in RF mode (Source: Edited from Devoret & SchoelKopf, Nature 2000)

Another advantage of the RF-SET is that one can switch the device on and off very quickly, the turn-on being set by the damping time of the LCR-circuit, usually in the order of only a few tens of nanoseconds. This makes the RF-SET very suitable for conducting very fast measurements on a system. This is crucial for systems that might mix over time, hence they cannot be measured by the DC-SET.

3.2 Noise that can affect the SET

The SET experiences, like other amplifiers, different types of noise, which must be taken in consideration when analyzing the sensitivity of the device. We can describe any amplifier with a model. The model represents the inside of the box with elementary components that describe how it appears to the outside circuitry. A representation of a (linear) amplifier with two input leads and two output leads (which could be a SET) is given in figure 9.

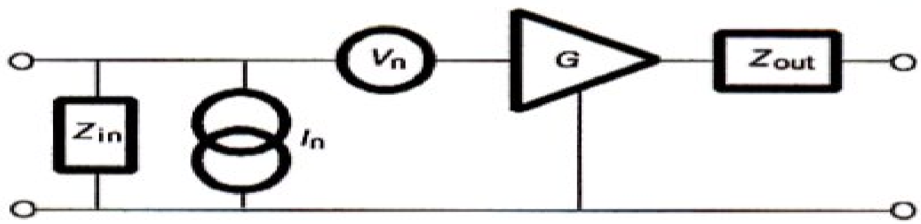

Fig. 9: Effective circuitry that describes a linear voltage amplifier with no feedback. (Source: Devoret & SchoelKopf, Nature 2000)

In the picture Z_{in} is the input impedance, which is infinite in an ideal device, Z_{out} is the output impedance, zero in an ideal amplifier, and a voltage gain G. V_n and I_n are the voltage and current noise respectively. These parameters are crucial for the sensitivity of the SET and it is important to find the sources that lead to them, so that one can find a way to minimize them to achieve the best sensitivity possible.
There are different types of noise that contribute to V_n and I_n. The first type of noise is thermal noise, also called Johnson-Nyquist. This kind of noise is generated by thermal motion of electrons in a resistive material. It is present in all circuits with a resistance (including the resistance given by power lines) and it is basically independent from the composition of the resistance itself. The mean square voltage for thermal noise is given by:

$$\left\langle V_T{}^2 \right\rangle = \frac{4hf\,\mathrm{Re}(Z)\Delta f}{\exp(hf/kT)-1} \qquad \textbf{(12)}$$

where k is the Boltzmann constant, h is Planck's constnat, T the absolute temperature, Z the (complex) impedance, f the frequency and Δf the bandwidth in Hz over which the noise is measured. Equation (15) is also often expressed in the form of an integral since the impedance is usually dependent on the frequency itself. The spectral density of the voltage thermal noise is the mean square voltage $\left\langle V_T{}^2 \right\rangle$ divided by the bandwidth. However if $kT/h >> f$ the expression for the average noise reduces to:

$$\left\langle V_T{}^2 \right\rangle = 4kT\,\mathrm{Re}(Z)\Delta f \qquad \textbf{(13)}$$

which is called the *Nyquist formula*.
This kind of noise depends directly on the temperature and the impedance. It is frequency independent is the impedance is a common resistance, in that case the spectral density is uniform. The Nyquist formula is only valid at low frequencies and/or high temperatures, since it predicts that the noise will increase with the frequency and that the mean square

noise will diverge at $\Delta f \rightarrow \infty$. This is called the ultraviolet catastrophe since at very high frequencies (above 100 GHz) the noise will actually tend to zero. This also depends on the impedance of the system, which tends to zero at very high frequencies. This happens even when in a circuit there are only resistances present, the circuit often also carries an intrinsic capacitance (especially if the dimension of the circuit are small), which will short out the circuit at very high frequencies. The normalized mean square average of the voltage from equation **(12)** is pictured in figure 10.

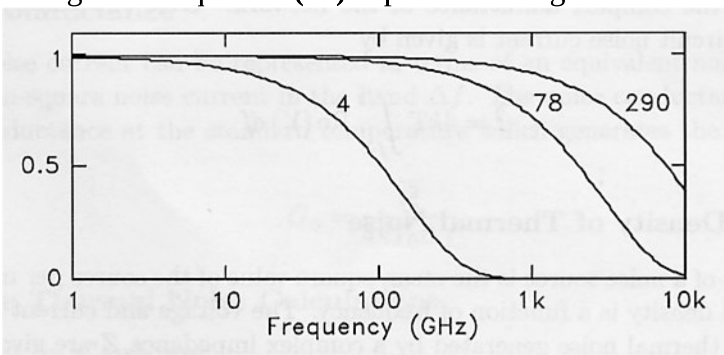

Fig. 10: Plot of the normalized (maximum value for the noise at each T was set equal to 1) thermal noise against the frequency at 4K, 78K and 290 K. (Source: Leach M. W. Jr., Prof. Leach Noise Potpourri)

The thermal noise is also depended on the real value of the impedance of the system. For a SET with two junctions with equal parameters the impedance is equal to:

$$Z(\omega) = \frac{R_\Sigma + i\omega R^2 C_\Sigma}{1 + i\omega R C_\Sigma - \omega^2 R^2 (C^2 + CC_G)} \quad \textbf{(14)}$$

where R and C are the resistance and capacitance of one junction respectively, R_Σ and C_Σ the total resistance and capacitance respectively, C_G the gate capacitance and ω the angular frequency. The real value of the impedance will then have this form:

$$\mathrm{Re}(Z(\omega)) \frac{\frac{A}{\omega^2} - B}{\frac{1}{\omega^2} + C\omega^4 + D} \quad \textbf{(15)}$$

where A, B, C and D are constants dependent on the values of the resistance and the capacitances. From this relation we can see that at zero frequency the value of the impedance will be equal to A (which is equal to the total resistance) and that at very high frequencies it goes to zero decreasing with a factor $1/\omega^4$. This will contribute to the disappearance of the thermal noise at high frequencies, as mentioned before.

The second type of noise is Flicker noise or *1/f*-noise, so called because it is proportional to $1/f^{\,n}$ where $n \cong 1$. This type of noise still remains ill-understood, however various probable explanations have been given. It arises from the so-called offset charge which is determined by several reasons. In conductors it arises from fluctuations in the mobility of the charge carriers. This kind of noise is also produced by differences in the work-functions of the conductors at the two sides of a contact or a tunneling junction. In the SET this might be a source of noise if the island is constituted by a different metal or semiconductor than the drain and source electrodes. Charges trapped in oxides can also cause flicker noise. This is especially important for the SET, since the tunneling junctions that constitute it are a thin layer of oxide sandwiched between two pieces of metal (often aluminum). The mean square voltage noise of this kind of noise is given by:

$$\left\langle V_F{}^2 \right\rangle = \frac{K_f V^2 \Delta f}{f^n} \quad \textbf{(16)}$$

In these expressions K_f is the Flicker noise coefficient (a constant), f the frequency with exponent $n \cong 1$.

The third noise source is Shot noise. This type of noise is generated by the random emission of electrons or by random tunneling of electrons across a potential barrier. This randomness has quantum mechanical nature, since the electrons at tunneling junctions have to "choose" between the two sides of the barrier. This type of noise, as we will see later, is crucial for the sensitivity of the SET as charge sensing device and eventually limits the ultimate sensitivity of the SET. Poisson processes can describe the tunneling events, satisfying three properties. First the numbers of changes in non-overlapping intervals are independent for all interval. Second the probability of exactly one change in a sufficiently small interval $h=1/m$ is $P=\nu h$, where ν is the probability of one change and m is the number of trials and third the probability of two or more changes in a sufficiently small interval h is essentially zero. The shot noise can be seen as random peaks in the current, the height of the current spike is equal to $q/\Delta t$, where q is the tunneled charge (usually equal to e) and Δt the time interval during which the tunneling occurs. The peaks can be approximated to random rectangular pulses of width Δt. The spectral densities and the mean square noise for the current and the voltage noise can be calculated from the auto-correlation functions of the current and the voltage. The auto-correlation $\varphi(\tau)$ function is defined by:

$$\varphi(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} \lambda(t)\lambda(t+\tau)dt \quad \textbf{(17)}$$

here $\lambda(t)$ is a function of time such as the voltage or the current in a device and T is a time interval. From the auto-correlation one can derive (I will not do it here) the *Shottky formula* which gives the average mean square current noise:

$$\left\langle I^2 \right\rangle = 2qI_{DC}\Delta f \quad \textbf{(18)}$$

where q is the charge of an electron and I_{DC} the DC current through a device. Equation **(18)** is only valid for one single tunneling junction.

Other kind of noises, such as Burst noise and Interference noise, can disrupt the sensitivity of the SET, yet they depend on manufacturing defects or poor set up of the devices. Taking considerate precautions in building and setting up a device these kinds of noises should be negligible, however one must always be very careful in neglecting them, since there is always the risk one may make faulty assumptions.

3.3 The Sensitivity of the SET

The SET sensitivity as an electrometer is limited by noise. The higher the noise inside a device the lower it's sensitivity will be. Signals with amplitude lower than the amplitude of the noise produced by the device will be lost in the noise itself and remain undetected. Three types of noise are the principal source of sensitivity limitation in the SET: thermal, Flicker and shot noise, described in the previous section.

Operating the SET at very low temperatures allows us ignore thermal noise. As it can be seen from equation (15) the mean square average of the thermal voltage noise is directly proportional to the temperature and the (real) impedance of the device. Reducing the temperature to very low values (*1K>*) will also reduce this kind of noise to values that are negligible. Low temperatures are also crucial for the working of the SET itself as it was discussed previously.
Flicker noise is inversely proportional to the frequency f a device is operated. This can be seen from the formula for the mean square average voltage in equation **(16)**. This kind of noise is produced by the offset charge, which is determined by the reasons we discussed above. The Flicker noise fluctuations can be very large, severely hindering the sensitivity of an amplifier. For low frequencies (*<100kHz-1MHz*) this kind of noise can be corrected by using a feedback in the device. The feedback controls the value of the current or conductance through the SET deviates from the half maximum (where the sensitivity is at maximum, see figure 7) the feedback sets it back to it. However, at high frequencies (*>1MHz*) there is no control over the offset charges, since the frequency is too high and the feedback cannot keep up with the corrections needed. This imposes a sever limitation to the RF-SET, yet if one goes to very high frequencies, higher than *100MHz* or, even better, *1GHz*, the Flicker noise will be reduced to very low values that can even be neglected. The RF-SET then has truly an advantage, since by operating it at sufficiently high frequencies the Flicker noise does not need to be corrected anymore but it just can be ignored. One must always keep in mind though that there is a 'danger zone' where the Flicker noise is a significant limitation of the sensitivity and the frequency is too high to apply any corrections. This is also the reason why the SET will probably not replace the FET in logical circuits.
Taking these considerations Devoret and Schoelkopf (Nature, 2000) calculated the noise spectral densities for the current (S_I) and the voltage (S_V). They considered a RF-SET working at low temperatures and at high frequencies so they could neglect contributions from the Johnson and the Flicker noises. They took in consideration only the shot noise as noise source, considering the tunneling events that give rise to this type of noise as Poisson processes with correlations. It was not possible to me to derive the relationships for the spectral densities of the current and voltage noises from the Shottky formula (equation **(18)**) or otherwise. From the spectral densities they calculated different sensitivities for the SET at optimal conditions. I will focus the charge sensitivity δQ. The charge sensitivity δQ gives the value of the charge noise that comes from the device, hence this is the smallest value of the charge one can measure. Smaller values will simply disappear into the noise and will not be detected. The charge sensitivity can be derived from the spectral density of the voltage noise S_V (in V^2/Hz). This is the mean square noise of the voltage per unit bandwidth. This gives the noise that is added by the amplifier (the SET) to the output signal. Taking the square of the spectral density and dividing it by the input impedance Z_{in} (in Ω) will give the fluctuations in current δI (per unit bandwidth) added to the output signal by the amplifier. Dividing this value by the frequency ω (in *Hz*) will give the value of the charge sensitivity (since $\delta Q = \delta I \omega$), hence we will come to the relation for the charge sensitivity: $\delta Q = \sqrt{S_V} / |\omega Z_{in}(\omega)|$. The charge sensitivity is a positive value, which is why the absolute value of ωZ_{in} is taken. If we look at the units of the quantities used we find that the value of the charge sensitivity is given

in $e/\sqrt{Hz}$ where *e* is the charge of one electron (converting Coulomb *C* into elementary charges *e*).

Devoret and Schoelkopf calculated values for the charge sensitivity and the noise energy for an optimal situation. These optimal values are calculated for a SET amplifier where one can neglect any contribution from the thermal and Flicker noise (as we stated before). They also neglected any higher order tunneling phenomena (such as co-tunneling) by assuming a strong Coulomb blockade, a high tunnel resistance ($R_T >> R_K$) and equal junctions capacitances (identical junctions) The capacitances were set to be very small for an optimized device to be very small (1 fF). Under these conditions the value calculated by the authors was equal to:

$$\delta Q_{Optimal} \leq 1.7 \times 10^{-6} e/\sqrt{Hz} \qquad \textbf{(19)}$$

A paper by Schoelkopf et. al. (Science, 1998) obtained experimental values for the charge sensitivity and the noise energy. For a still-unoptimized device operated at frequencies of above 100 *Mhz* and *45 mK* the value is:

$$\delta Q_{Experiment} = 1.2 \times 10^{-5} e/\sqrt{Hz} \qquad \textbf{(20)}$$

We see that for the charge sensitivity there is only a factor 10 difference from the calculated optimal value. The experimental values are thus pretty close to the values that were calculated theoretically. Although the calculated values still could not be reached experimentally, they still were very close to them. The authors of Schoelkopf et. al. also explicitly say that the device still could have been improved and was not optimized yet. Values for the charge sensitivity and energy noise could be improved even further by improving and eventually optimizing the experimental devices.

The sensitivity of the SET is highlighted if we compare then with the value for best performance of the FET, which is only: $\delta Q_{FET} = 10^{-2} e/\sqrt{Hz}$ (Mar et. al., Appl. Phys. Lett. 1994). The sensitivity of the unoptimized SET-electrometer is thus a factor 1000 better than the optimal value of the FET.

4. Conclusions

The single-electron transistor is a very interesting device that can basically take over the same functions of nowadays transistors, yet the way it functions and operates is very different than field-effect transistors. In chapter 3 we have reviewed the application of the SET as an electrometer and its sensitivity. According to the article from Devoret and Schoelkopf (Nature 2000) the sensitivity of the SET-electrometer is very high and they calculated a limit for an optimized device, $1.7\times10^{-6}e/\sqrt{Hz}$. The conditions they apply for this 'optimal device' are not unrealistic. The values chosen for the capacitances and the resistances taken can be easily obtained or modified by modern lithography techniques. The value of the Coulomb blockade parameter they set can be also be simply modified by changing the values of the gate and drain-source voltage. They also assume that the optimal SET-electrometer is kept at reasonable temperatures, where the interference from the Johnson-Nyquist noise is negligible. This is often achieved in practice since experimentalist can now reach temperatures far below *1K*. They also assumed that the mean value of the off-set charge to be equal to zero as long as the device operated at high enough frequencies, since the noise produced by the off-set charge scales with *1/f*.

Comparing the value that was calculated, $1.7\times10^{-6}e/\sqrt{Hz}$, with an experimental result, $1.2\times10^{-5}e/\sqrt{Hz}$ (Schoelkopf *et. al.* Science 2000), there is only a factor 10 that divides them and the authors that give the experimental result also imply that their device still was not optimized and could be improved further. The calculated optimal value for the SET is most probably a feasible reality and not simply a value that can only be reached theoretically. The SET is thus an excellent candidate as ultra-sensitive electrometer. It is certainly a strong improvement from the field-effect transistor, since the optimal value for the FET is 1000 times worse than the experimental value determined for the SET.

The ultra-sensitivity of the SET as electrometer means it is a perfect tool to measure charges in the most varied systems, especially nanoscale systems and even single molecules. It will also be possible for the SET electrometer to measure quantum systems such as qubits. Although the SET might not replace the FET in future electronic and logical systems it will be a very valuable tool for the investigation of the quantum computer and other systems such as molecules like fullerenes or proteins that can carry charges. These systems can be easily studies by the SET, simply by placing them very close to the SET's island as it was discussed in section *3.1*. The possibilities of the SET as ultra-sensitive electrometer are very promising and they are still to be fully investigated in future research.

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