The symmetries of QCD
(and consequences)

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Understand nature in terms of fundamental building blocks
The Rumsfeld Classification

“As we know, there are known knowns.
There are things we know we know.

We also know, there are known unknowns.
That is to say we know there are some things we do not know.

But there are also unknown unknowns,
the ones we don’t know we don’t know.”

– Donald Rumsfeld, U.S. Secretary of Defense, Feb. 12, 2002
Some known knowns

Quantum ChromoDynamics - the theory of quarks and gluons

- Embarassingly successful
- Precision tests of SM can reveal new physics - more important than ever!

Why focus on the 4.6%?

- QCD is the only experimentally studied strongly-interacting quantum field theory - highlights many subtleties.
- A paradigm for other strongly-interacting theories in BSM physics.
- There are still puzzles and surprises in this well-studied arena.
What is QCD?

A gauge theory for SU(3) colour interactions of quarks and gluons: Color motivated by experimental measurements; not a “measureable” quantum number. Lagrangian invariant under colour transformations.

Elementary fields: Quarks

\[ (q_\alpha)^a_f \]
- colour: \( a = 1, \ldots, 3 \)
- spin: 1/2 Dirac fermions
- flavour: \( f = u, d, s, c, b, t \)

Gluons

\[ A^a_\mu \]
- colour: \( a = 1, \ldots, 8 \)
- spin: 1 bosons

\[ \mathcal{L} = \bar{q}_f \left( i \gamma^m u D_\mu - m_f \right) q_f - \frac{1}{4} G^a_{\mu \nu} G^{a}_{\mu \nu} \]

with

\[ G^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \]

and

\[ i \gamma^\mu D_\mu q = \gamma^\mu \left( i \partial_\mu + g A^a_\mu t^a \right) q \]

- Generates gluon self-interactions - might expect consequences!
Elucidating the effect of the self-interactions: QED vs QCD

In QED: coupling runs and bare $e^-$ is screened at large distances - reducing
Same but different for QCD with anti-screening from gluon interactions dominates!

Asymptotic freedom

Coupling small at high energies - energetic quarks are (almost) free.

Nobel prize 2004 for Gross, Politzer and Wilczek.
The interaction between quarks, between gluons and between quarks and gluons grows rapidly with separation.

At hadronic scales, $r \sim 0.2\text{fm} \sim \frac{1}{4} r_{\text{proton}}$ it is huge! A perturbative expansion not appropriate...
Confinement: a consequence of QCD interactions

- Quarks and gluons are not observed in nature - only in bound states (colour singlets)
- Confinement not yet derived from QCD
- Verified by numerical simulation from first principles

- Strong force overcomes repulsive EM force to “glue” protons in the nucleus.
Gluons and QCD

- QCD describes structure and interactions of nuclear matter
- Without gluons there are no protons, no neutrons and no atomic nuclei
- Gluons dominate the structure of the QCD vacuum

- Essential features of QCD (asymptotic freedom, chiral symmetry breaking, colour confinement) are driven by gluons.
- Unique aspect for QCD is gluon self interactions.
- 99% of mass of visible universe arises from glue.
- Half the nucleon momentum is carried by gluons.
Some symmetries of QCD

- The structure of QCD fully defined by the requirement of invariance under **local gauge transformations**: ie physical content unchanged if quarks and gluons transform under colour SU(3).

- **Isospin symmetry**: a global transformation - SU(2) rotation in flavour space (QCD interactions are flavour-blind). Acts on up and down quarks and $\mathcal{L}$ is invariant for identical or vanishing masses.

- **Chiral Symmetry**: For massless QCD, left and right handed pieces of $\mathbf{L}$ are separately invariant. A global symmetry. $\psi = \psi_R + \psi_L$, $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)\psi$:

$$\psi(x) \rightarrow \exp\left(-i \sum_{b=1}^{8} a_b t_b \gamma^5\right) \psi(x)$$

![Right-handed and Left-handed Chiral Symmetry](attachment:image.png)
• **Spontaneous Chiral Symmetry Breaking**: chiral symmetry broken by properties of the QCD vacuum (cf Higgs mechanism). The (approximately massless) pions are the Goldstone bosons of the broken symmetry.

• QCD vacuum state is not invariant under the same symmetries as $\mathcal{L}$

• Chiral condensate is an order parameter

\[ \langle q\bar{q} \rangle = \begin{cases} 
  \neq 0 & \text{(broken phase)} \\
  0 & \text{(unbroken)}
\end{cases} \]

• Many others: flavour, isospin, ...
**Dynamical mass generation through non-linear interactions**

Very little to do with Higgs!

Massless gluons and almost massless quarks interact - generating most of the mass of nucleons

Proton: \textbf{uud}

- \( m_u = 2.3^{+0.7}_{-0.5} \text{ MeV}/c^2 \)
- \( m_d = 4.8^{+0.7}_{-0.3} \text{ MeV}/c^2 \)
- \( M_p = 938.3 \text{ MeV}/c^2 \)
- \( M_\pi = 130 \text{ MeV}/c^2 \)

- The pion is the Goldstone boson of dynamical chiral symmetry breaking.
- Only 1\% of the proton’s mass comes from the constituent quarks’ intrinsic masses.
- The proton is an emergent (long-range) phenomena resulting from the collective behaviour of quarks and gluons - QCD!
QCD: A PRACTICAL TOOL FOR UNDERSTANDING MATTER?

There are two regimes:

**Deep inside the proton**
- at short distances quarks behave as free particles
- weak coupling

⇒ perturbation theory works

**At “observable” distances**
- at long distance (1fm) quarks confined
- strong coupling

⇒ perturbation theory fails: nonperturbative approach needed.
How do quarks behave? Quantum chromodynamics offers the theory but requires enormous computing power to do the math.

from Science & Technology Review
Observable properties of QCD from numerical simulation
Does it work?
Life on a Lattice

Start from the QCD Lagrangian:

\[ \mathcal{L} = \bar{\Psi} \left( i \gamma^\mu D_\mu - m \right) \Psi - \frac{1}{4} G^a_\mu \nabla G_{\mu \nu}^a \]

- Gluon fields on links of a hypercube; quark fields on sites.
- Approaches to fermion discretisation - Wilson, Staggered, Overlap.
- Derivatives \( \rightarrow \) finite differences

Observables determined from finite-dimensional (Euclidean) path integrals of the QCD action

\[ \langle O \rangle = \frac{1}{Z} \int D[U, \bar{\psi}, \psi] e^{-S[U, \bar{\psi}, \psi]} \]

borrowing ideas from statistical mechanics.
Scale of theory tools

Models

χPT  \[ Q^2 \]  OPE  pQCD

Lattice QCD
**The lattice simulation landscape**

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from K. Jansen et al
THE LIGHT HADRON SPECTRUM I

BMW collaboration: a precision realisation of the low-energy spectrum of QCD
THE LIGHT HADRON SPECTRUM II

BMW: first principles demonstration of the correct neutron-proton mass difference
QCD ADMITS A RICH AND EXOTIC SPECTRUM

Mesons/Baryons

Molecules/Multiquarks

Hybrids

Glueballs

+ Effects due to the complicated QCD vacuum
PREDICTIONS: THE GLUEBALL SPECTRUM

Morningstar & Peardon, 1999
PREDICTIONS: (EXOTIC) HYBRID MESON IN CHARMONIUM

Hadron Spectrum Collaboration, 2012
Some Current Challenges in QCD
GOING BEYOND SIMPLE BOUND STATES: RESONANCES IN A EUCLIDEAN THEORY

- On lattice volumes extract the spectrum. Lüscher formalism (1991) allows to deduce phase shift information

\[
\det \left[ \cot \delta(E_n^*) + \cot \phi(E_n, \vec{P}, L) \right] = 0
\]

- the more distinct spectrum points the better the phase shift picture
The XYZs

- The new strong exotic matter has been around for 15 years ... and we still don’t understand it..
- Will need precision lattice resonance studies including multi-hadrons and bound states ...
The QCD phase diagram:

Lattice can:
- Explore $T \geq 0, \mu = 0$ - fate of hadrons in medium

Lattice can’t:
- Simulate at $\mu \neq 0$
  - Fermion determinant is complex, no positive weight in path integral
  - Sign problem: Monte Carlo methods fail!
THE COST OF DOING BUSINESS

- Generating gauge ensembles at physical quark masses on large volumes requires millions of core hours.

\[
\text{Computational Cost} \sim c \times (\text{Number of configurations}) \times \left(\frac{1}{M_\pi}\right)^6 \times (L)^5 \times (a^{-1})^7
\]

- Once the gauge configurations are generated just have to invert the Dirac matrix $M$ to get the fermion propagators ... how hard can that be?

- Let’s calculate:
  - a lattice might have $24 \times 24 \times 24 \times 48 = 663,552$ sites
  - a fermion (quark) has 4 Dirac components
  - 3 colours in $SU(3)$
  - $\Rightarrow M$ is easily $10^6 \times 10^6!!$

- Requires high memory/network bandwidth, but also highly parallelisable
- Tera/Peta bytes of data is also generated!
QCD describes the properties of observed matter in terms of fundamental variables and their interactions.

Symmetries (and their breaking) have dramatic consequences.

Many open questions and unsolved problems - phenomenological and theoretical - remain.

Many knowns and unknowns to understand!
SUMMARY

- QCD describes the properties of observed matter in terms of fundamental variables and their interactions.

- Symmetries (and their breaking) have dramatic consequences.

- Many open questions and unsolved problems - phenomenological and theoretical - remain.

- Many knowns and unknowns to understand!

Thanks for listening!