Scaling Symmetries in Nature

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What to do with bad data

If the data values are quantities
plot it on a log:log plot.

If it still looks bad
fit a straight line to help the eye.

If that doesn’t help
reject all points that are 2-sigma from
the line

Make sure the slope of the line fits your
prejudice

Write the paper.
Kepler’s third Law

\[ T = \frac{2\pi r}{v} \Rightarrow T^2 \propto \frac{r^2}{v^2} \]

\[ v^2 \propto \frac{GM}{r} \]

\[ T^2 \propto r^3 \]
Why is scaling important?

The existence of a power law relationship spanning one or more decades of scales between two quantities is suggestive of a common cause.

The breakdown of the scaling relation is equally important since it tells us something about the driver for the association and causes us to look for a reason for the failure.

\[ D \, (\text{AU}) = 0.4 + 0.3 \times 2^N \]

The Titius-Bode “Law” for the radii of the orbits of the planets
Efstathiou et al. Measured to two point function for galaxy clustering and concluded that the break from the smaller scale power law could only be explained if $\Lambda \neq 0$

The discovery of \( \Lambda \)

It is argued here that the success of the cosmological cold dark matter (CDM) model can be retained and the new observations of very large scale cosmological structures can be accommodated in a spatially flat cosmology in which as much as 80 percent of the critical density is provided by a positive cosmological constant. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. This constant can also account for the lack of fluctuations in the microwave background and the large number of certain kinds of objects found at high redshift.

The cosmological constant and cold dark matter
Efstathiou, G.; Sutherland, W. J.; Maddox, S. J.
Rotation of Galaxies

Assume Newton’s law of gravity.

**Selection effect?**
Suppose that all galaxies in the sample have the same surface brightness and same mass to light ratio.

\[
v^2 \propto \frac{GM}{r} \Rightarrow M \propto rv^2
\]

\[
\frac{M}{r^2} = \text{const}
\]

\[
H \propto Mrv
\]

eliminate \(r\) and \(v\)

\[
H \propto M^{7/4}
\]

Tully-Fisher relationship

Correlation between the intrinsic brightness of a galaxy as measured in the 3.6 micron band, versus the maximum width of its rotation curve. (Sorce et al. 2015)

Calibration using Virgo Cluster

Superposition of 13 clusters to get relative distances

Slope corresponds to \( \mathcal{L} \propto V^{3.6} \)

Hubble constant = 74 ± 5 km s\(^{-1}\) Mpc\(^{-1}\)
G.I. Taylor’s analysis of A-bomb

**Figure 1.** Logarithmic plot showing that \( R^4 \) is proportional to \( t \).
The Sedov-Taylor blast wave

One of the most famous examples of scaling a physical problem is the similarity solution for a spherical blast wave created by suddenly releasing an energy $E$ in a medium of density $\rho_0$. The distance travelled by the blast is

$$r = \beta \left( \frac{E}{\rho_0} \right)^{1/5} t^{2/5}$$

which tells us the blast energy

$$E = \left( \frac{\rho_0}{\beta^5} \right) \frac{r^5}{t^2}$$

$\beta \approx 1.0 - 1.1$ describes the physics of the gas.

In 1950 Taylor published the calculation and estimated the strength of the first 1945 atomic explosion at around 20 kilotons of TNT. This was a highly classified number and some people were not pleased that this was published.
Moore’s Law – The number of transistors on integrated circuit chips (1971-2016)

Moore’s law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important as other aspects of technological progress – such as processing speed or the price of electronic products – are strongly linked to Moore’s law.

The data visualization is available at OurWorldInData.org. There you find more visualizations and research on this topic. Licensed under CC-BY-SA by the author Max Roser.
Kurzweil’s extension of Moore’s Law
Zipf’s law for word frequency

Lexical analysis is one of the main places where we see Zipf’s law taken relatively seriously. (Zipf `discovered’ this in linguistics)

This is a survey of the “plus” webpages where “the” is ranked 1 and “of” ranked 2, with “mathematics” ranked 51.
Zipf’s Law for languages

Analysis of word frequency from Wikipedias in 15 diverse languages. Esperanto is a made-up language so is particularly puzzling.
Zipf’s Law: size of cities

Zipf’s Law asserts that the frequencies of events are inversely proportional to their rank.

Hence with the population of cities the second ranked city \((r=2)\) would have half the population of the first ranked, while the third-ranked \((r=3)\) would have a third the population of the first ranked.

With such a relationship you could predict what is the size of the 26th largest city in some arbitrary country. What could possibly be the mechanism driving that?
Growth of a resource $Y(t) = Y_0 N(t)^\beta$

$N(t)$ is population growth, $Y(t)$ is wealth, pollution, ...
## Tracking city resources

<table>
<thead>
<tr>
<th>Property</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>Adj-$R^2$</th>
<th>Observations</th>
<th>Country-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>New patents</td>
<td>1.27</td>
<td>[1.25, 1.29]</td>
<td>0.72</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Inventors</td>
<td>1.25</td>
<td>[1.22, 1.27]</td>
<td>0.76</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Private R&amp;D employment</td>
<td>1.34</td>
<td>[1.29, 1.39]</td>
<td>0.92</td>
<td>266</td>
<td>U.S. 2002</td>
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<tr>
<td>&quot;Supercreative&quot; employment</td>
<td>1.15</td>
<td>[1.11, 1.18]</td>
<td>0.89</td>
<td>287</td>
<td>U.S. 2003</td>
</tr>
<tr>
<td>R&amp;D establishments</td>
<td>1.19</td>
<td>[1.14, 1.22]</td>
<td>0.77</td>
<td>287</td>
<td>U.S. 1997</td>
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<tr>
<td>R&amp;D employment</td>
<td>1.26</td>
<td>[1.18, 1.43]</td>
<td>0.93</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Total wages</td>
<td>1.12</td>
<td>[1.09, 1.13]</td>
<td>0.96</td>
<td>361</td>
<td>U.S. 2002</td>
</tr>
<tr>
<td>Total bank deposits</td>
<td>1.08</td>
<td>[1.03, 1.11]</td>
<td>0.91</td>
<td>267</td>
<td>U.S. 1996</td>
</tr>
<tr>
<td>GDP</td>
<td>1.15</td>
<td>[1.06, 1.23]</td>
<td>0.96</td>
<td>295</td>
<td>China 2002</td>
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<td>GDP</td>
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<td>[1.09, 1.46]</td>
<td>0.64</td>
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<td>EU 1999–2003</td>
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<tr>
<td>GDP</td>
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<td>[1.03, 1.23]</td>
<td>0.94</td>
<td>37</td>
<td>Germany 2003</td>
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<tr>
<td>Total electrical consumption</td>
<td>1.07</td>
<td>[1.03, 1.11]</td>
<td>0.88</td>
<td>392</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>New AIDS cases</td>
<td>1.23</td>
<td>[1.18, 1.29]</td>
<td>0.76</td>
<td>93</td>
<td>U.S. 2002–2003</td>
</tr>
<tr>
<td>Serious crimes</td>
<td>1.16</td>
<td>[1.11, 1.18]</td>
<td>0.89</td>
<td>287</td>
<td>U.S. 2003</td>
</tr>
<tr>
<td>Total housing</td>
<td>1.00</td>
<td>[0.99, 1.01]</td>
<td>0.99</td>
<td>316</td>
<td>U.S. 1990</td>
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<tr>
<td>Total employment</td>
<td>1.01</td>
<td>[0.99, 1.02]</td>
<td>0.98</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Household electrical consumption</td>
<td>1.00</td>
<td>[0.94, 1.06]</td>
<td>0.88</td>
<td>377</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>Household electrical consumption</td>
<td>1.05</td>
<td>[0.89, 1.22]</td>
<td>0.91</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Household water consumption</td>
<td>1.01</td>
<td>[0.89, 1.11]</td>
<td>0.96</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Gasoline stations</td>
<td>0.77</td>
<td>[0.74, 0.81]</td>
<td>0.93</td>
<td>318</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Gasoline sales</td>
<td>0.79</td>
<td>[0.73, 0.80]</td>
<td>0.94</td>
<td>318</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Length of electrical cables</td>
<td>0.87</td>
<td>[0.82, 0.92]</td>
<td>0.75</td>
<td>380</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>Road surface</td>
<td>0.83</td>
<td>[0.74, 0.92]</td>
<td>0.87</td>
<td>29</td>
<td>Germany 2002</td>
</tr>
</tbody>
</table>

Data sources are shown in SI Text. CI, confidence interval; Adj-$R^2$, adjusted $R^2$; GDP, gross domestic product.

The properties (resources) that are tracked by the model...
The equations for city evolution

The equations:

\[ dY = RN + E \frac{dN}{dt} \]

\[ \frac{dN}{dt} = \left( \frac{Y_0}{E} \right) - \left( \frac{R}{E} \right) \frac{dN}{dt} \]

\[ N(t) = \left[ \frac{Y_0}{R} + \left( N(0)^{1-\beta} - \frac{Y_0}{R} \right) \exp \left[ -\frac{R}{E}(1 - \beta)t \right] \right]^{1/(1-\beta)} \]

Are dominated by the choices \( \beta < 1, \beta = 1, \beta > 1 \)

R: quantity of resource per unit time to maintain individual

E: resource it takes to add a new individual

E/R is the timescale for adding an individual to the city
How cities may evolve

Evolution patterns

<table>
<thead>
<tr>
<th>Scaling exponent</th>
<th>Driving force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta &lt; 1$</td>
<td>Optimization, efficiency</td>
</tr>
<tr>
<td>$\beta &gt; 1$</td>
<td>Creation of information, wealth and resources</td>
</tr>
<tr>
<td>$\beta = 1$</td>
<td>Individual maintenance</td>
</tr>
</tbody>
</table>
Damuth relationship (2011)

Population density vs body mass

Data from 370 species.
Each point is one species.

Dawkins was highly critical of this, arguing that people are creating plots that support some particular point of contention. In this case that concerns the mechanism by which such a relationship could come about.

\[ \log D = -0.75 \log W + 4.23, \quad (r = -0.86) \]
metabolic power vs mass
Metabolic rate vs mass

A graph showing the relationship between metabolic rate (in kcal/h; log scale) and mass (in g; log scale). The graph includes data points for unicellular organisms, poikilotherms (cold-blooded organisms), and homeotherms (warm-blooded organisms). The relationship is linear on a log-log scale. The text notes that 1 kcal/h is equivalent to 1.162 watts.
Brain mass vs body mass

Brain mass is approximately proportional to body mass raised to the power of 3/4.
The scientists who developed the scaling theory took clues from naturally occurring networks that carry life-sustaining fluids in organisms in which each small part is a replica of the whole. No matter how big the organism, the ends of these fractal networks are always the same size, since individual cells are of similar size in all organisms.
Anscombe’s quartet

Regression line: $y = 3.00 + 0.500 \times$

Mean $x = 9$
Sample $x$ variance = 11
Mean $y = 7.5$
Sample $y$-variance = 4.125
Correlation $x,y = 0.816$

Moral:
Look at the data!
Manifestly fake (for fun)
The moral of the tale

The correlation between these two variables is 0.992

**Moral of the tale:** correlations does not imply causality
Nor does it indicate common cause.
Press promotion of dubious science
Improving a correlation

2-sigma rejection helps make a correlation more convincing

T. Westling, University of Helsinki HECER: Discussion paper 335 July 2011