

October 20, 2016

A Deterministic Interpretation of Quantum Mechanics

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H.J. Groenewold symposium on advances in semi-classical methods
in mathematics and physics

A theory attempting to explain the quantum mechanical nature of the laws of physics at small scales, starting from non-quantum mechanical dynamics

typically:

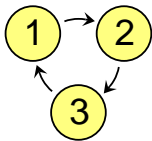
a set of cogwheels



Cogwheels will be time-reversal-invariant. At a later stage of this theory, *information loss* is introduced, leading to a new, and natural *arrow of time*.

But we begin with time-reversible situations . . .

A fundamental ingredient of a generic theory: a finite, periodic system: the *Cogwheel Model*:

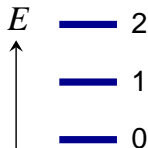
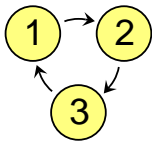


$$U(\delta t) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = e^{-iH\delta t} \quad U^3(\delta t) = \mathbb{I}$$

$$\delta t = 1, \quad U \rightarrow \begin{pmatrix} 1 & & \\ & e^{-2\pi i/3} & \\ & & e^{-4\pi i/3} \end{pmatrix}; \quad H \rightarrow 2\pi \begin{pmatrix} 0 & & \\ & \frac{1}{3} & \\ & & \frac{2}{3} \end{pmatrix}$$

In the original basis:

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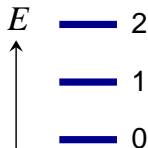
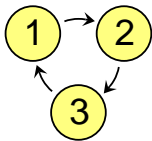


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In the original basis:

$$H = \frac{2\pi}{3} \begin{pmatrix} 1 & \kappa & \kappa^* \\ \kappa^* & 1 & \kappa \\ \kappa & \kappa^* & 1 \end{pmatrix}; \quad \kappa = -\frac{1}{2} + \frac{i\sqrt{3}}{6}; \quad \kappa^* = -\frac{1}{2} - \frac{i\sqrt{3}}{6}$$

This is an example of a *classical system* that can be mapped onto a purely quantum mechanical system.

The example is of course trivial, but can easily be generalised to much more complex models . . .

The converse can also be done:

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The example is of course trivial, but can easily be generalised to much more complex models . . .

The converse can also be done:

Consider a quantum mechanical system, and try to construct operators that are *beables*:

Beables are sets of operators that have the property that the time evolution law is a pure permutation:

$$U(t_i) = P_i$$

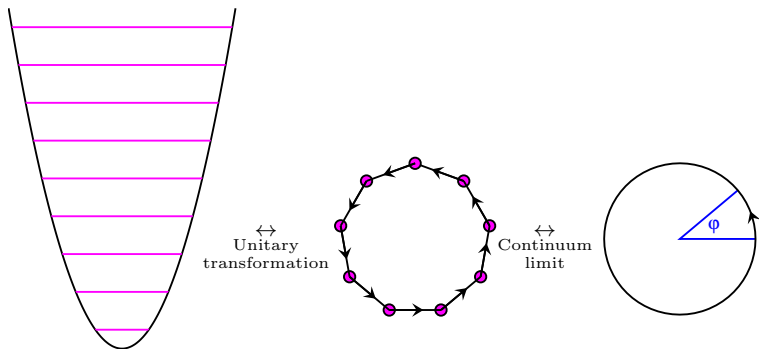
or, in the Heisenberg notation:

$$[\mathcal{O}(t_1), \mathcal{O}(t_2)] = 0 \quad \forall (t_1, t_2) \in \text{some large set of times } t_i$$

Examples:

- quantum harmonic oscillator =
classical point moving around circle
- chiral Dirac fermion =
infinite plane moving with speed of light
- the bulk of a quantized superstring =
string on a lattice in transverse space

The harmonic oscillator.



Quantum Oscillator

\leftrightarrow

Classical periodic system

Massless, chiral, non interacting “neutrinos” are *deterministic*:

Second-quantised theory: $H = -i \psi^\dagger \sigma_i \partial_i \psi$

First quantised theory: $H = \sigma_i p_i$

$$\frac{d}{dt} \mathcal{O}(t) = i[\mathcal{O}(t), H]$$

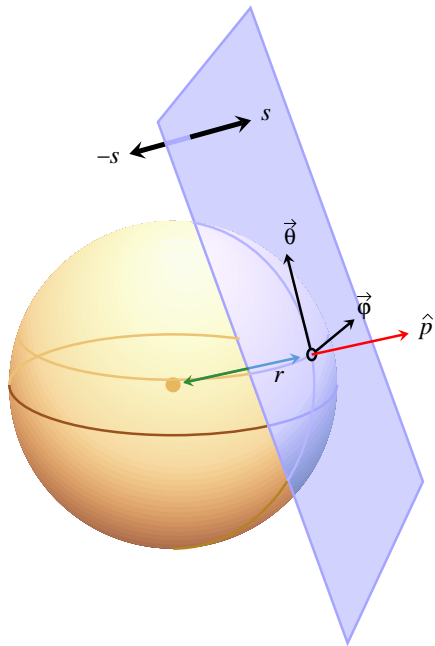
$$\text{Beables } \{\mathcal{O}_i^{\text{op}}\} = \{\hat{p}, s, r\}:$$

$$\hat{p} \equiv \pm \vec{p}/|p|, \quad s \equiv \hat{p} \cdot \vec{\sigma}, \quad r \equiv \frac{1}{2}(\hat{p} \cdot \vec{x} + \vec{x} \cdot \hat{p}).$$

$$|\hat{p}| = 1, \quad s = \pm 1, \quad -\infty < r < \infty$$

$$\frac{d}{dt} \hat{p} = 0, \quad \frac{ds}{dt} = 0, \quad \frac{d}{dt} r = s$$

These beables form a *complete set*



The neutrino sheet.

Beables: $\{\hat{p}, s, r\}$

The eigenstates of these operators span the entire Hilbert space.

Introducing operators in this basis, one can reconstruct the usual operators $\vec{x}, \vec{p}, \sigma_i$

Interesting mathematical physics:

$$x_i = \hat{p}_i \left(r - \frac{i}{p_r} \right) + \varepsilon_{ijk} \hat{p}_j L_k^{\text{ont}} / p_r + \frac{1}{2p_r} \left(-\varphi_i s_1 + \theta_i s_2 + \frac{\hat{p}_3}{\sqrt{1 - \hat{P}_3^2}} \varphi_i s_3 \right) \quad (1)$$

θ_i and φ_i are beables, functions of \hat{q} .

L_k^{ont} are generators of rotations of the sheet,
 $s_3 = s$, s_1 and s_2 are spin flip operators.

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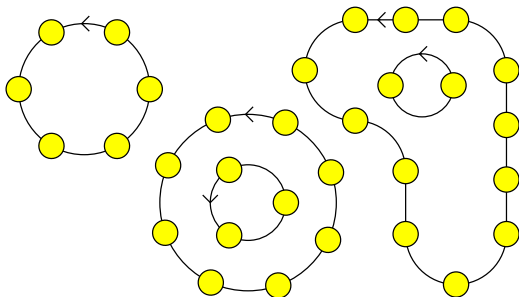
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The Hamiltonian of the first-quantised theory has no ground state, but, just as in Dirac's theory, the second quantised theory *does* have a ground state

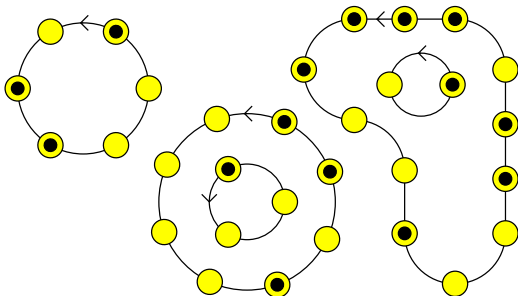
1st quantization



Hamiltonian with cut-off

$$H|\psi_i\rangle = h_{ij}|\psi_j\rangle$$

2nd quantization



Hamiltonian with cut-off

$$H|\psi_i\rangle = h_{ij}|\psi_j\rangle \longrightarrow H = \bar{\psi}_i h_{ij} \psi_j$$

Improves locality!

And H is bounded from below!

This we can do with non-interacting neutrinos, *not yet* with other fields.

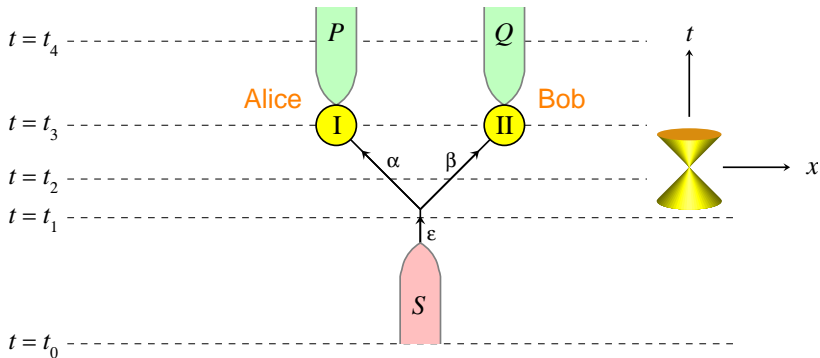
Future strategy: in principle, it may be possible to construct such a theory by replacing other 1st quantized particle systems with 2nd quantized ones.

Add interactions as small corrections: perturbative QFT.

Strategy for obtaining a CA that may lead to a perturbative QFT;

Note, that perturbative QFT are not mathematically perfect, but they can serve as satisfactory descriptions of a SM for elementary particles . . .

Why it is all wrong: Bell's theorem



In the Bell experiment, at $t = t_0$, one must demand that Alice's setting a and Bob's setting b , and the source c , have

3 - body correlations of the form

$$W(a, b, c) \propto |\sin(2(a + b) - 4c)| \quad (\text{or worse})$$

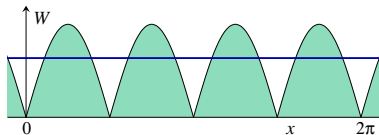
But Alice and Bob have *free will*. How can actions of free will be correlated to S at time $t = t_2 \ll t_3$?

Answer: **They don't have such free will:** *superdeterminism*.

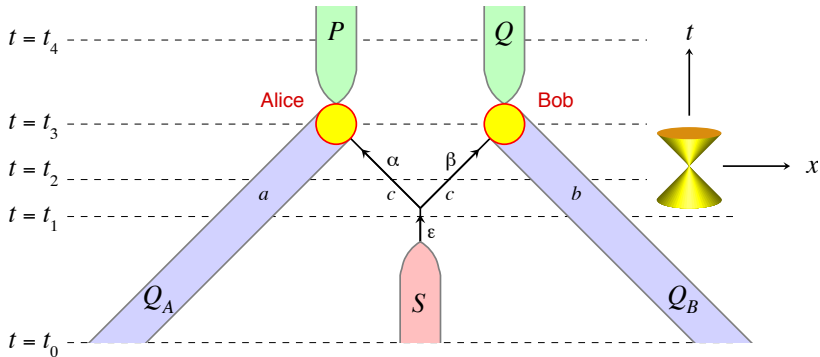
The Mouse-dropping objection:



Alice and Bob
count the mouse droppings ...

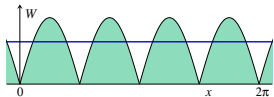


The Mousedropping function, $|\sin(2(a + b) - 4c)|$



Or, Alice and Bob may register fluctuations coming from quasars Q_A and Q_B .

Correlations in spacelike directions are very strong everywhere



Conspiracy ?

The *ontology conservation law*:

If we apply the “true” Schrödinger equation, we can be sure that:

- Original basis elements (ontic states) evolve into Original basis elements (ontic states) ;
- Superpositions evolve into superpositions.
- Neither Alice nor Bob can ever modify their state (applying “free will”) to turn an ontic state into a superposition.

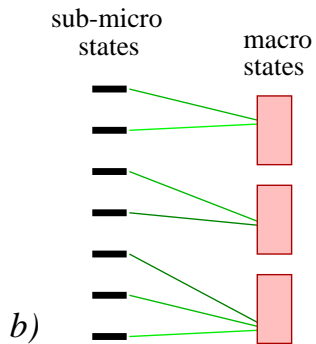
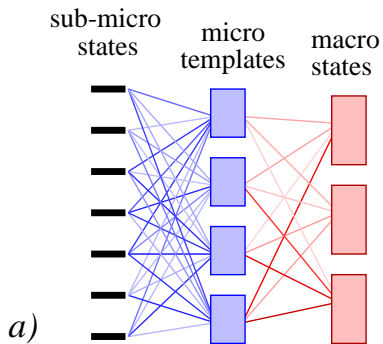
We should not worry about ‘conspiracy’. Ontology is conserved just like (or better than) angular momentum

Nature’s ontological states are strongly correlated (that’s not in contradiction with causality and/or locality)



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“About your cat, Mr. Schrödinger—I have good news and bad news.”



Problem with locality:

$$U(n\delta t) = e^{-iHn\delta t} \rightarrow$$

$$H\delta t = i \log U = \pi + \sum_{n=1}^{\infty} \frac{1}{in\delta t} \left(U(n\delta t) - U(-n\delta t) \right).$$

Except: vacuum state:

$$U(n\delta t)|0\rangle = |0\rangle \rightarrow U(n\delta t) = 1.$$

But: $H|0\rangle = 0$: contradiction

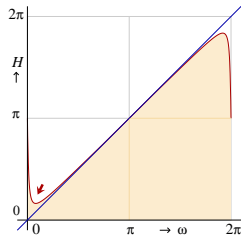
For states close to the vacuum, the expansion converges badly.

But, $U(n\delta t)$ connects n neighbours.

This means that, if $H = \sum_{\vec{x}} \mathcal{H}(\vec{x})$,
 $[\mathcal{H}(\vec{x}_1), \mathcal{H}(\vec{x}_2)] \rightarrow 0$ when $|\vec{x}_1 - \vec{x}_2| \neq 0$

Classical locality, but no quantum locality !

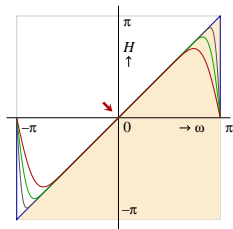
Possible remedy:



Second quantization: like with the neutrinos: first construct a classical theory for single particles.

Now, we allow $E > 0$ and $E < 0$ solutions.

The expansion for H then converges fast.



Therefore, the 1-particle Hamiltonian is local. To get positivity of the Hamiltonian, we *second quantize*: negative energy states are holes of antiparticles (as in Dirac).

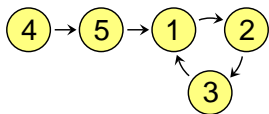
Take *interactions* to be weak; they are obtained by adding local disturbances in the deterministic evolution.

This may lead to effectively renormalizable QFT. The perturbation expansion does not converge, but the expansion parameter may turn out to be sufficiently small.

Time reversibility

So-far, theory was time reversible. The evolution operator $U(t)$ is then a pure permutator, and its representation in Hilbert space is unitary \Rightarrow The Hamiltonian is hermitean.

Black holes: non time-reversibility? Break down of *unitarity* ?

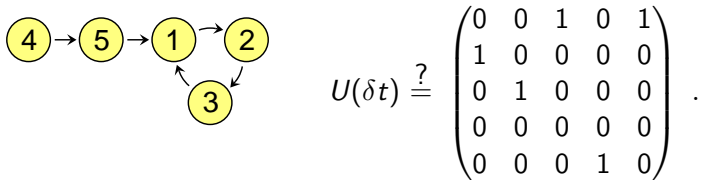


$$U(\delta t) \stackrel{?}{=} \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} .$$

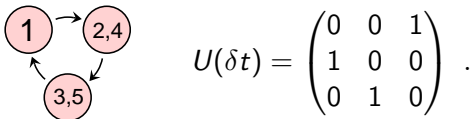
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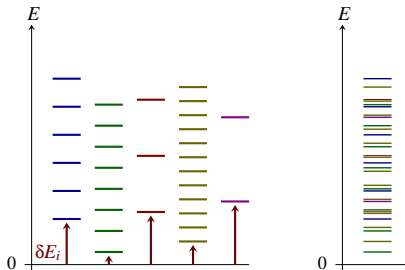
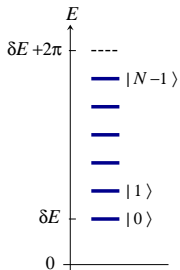
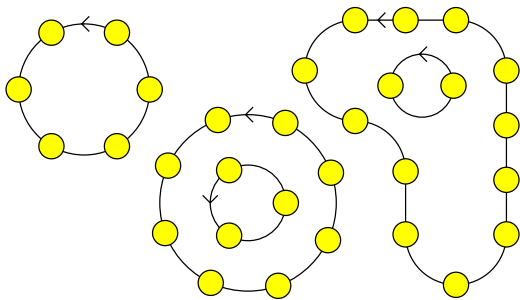
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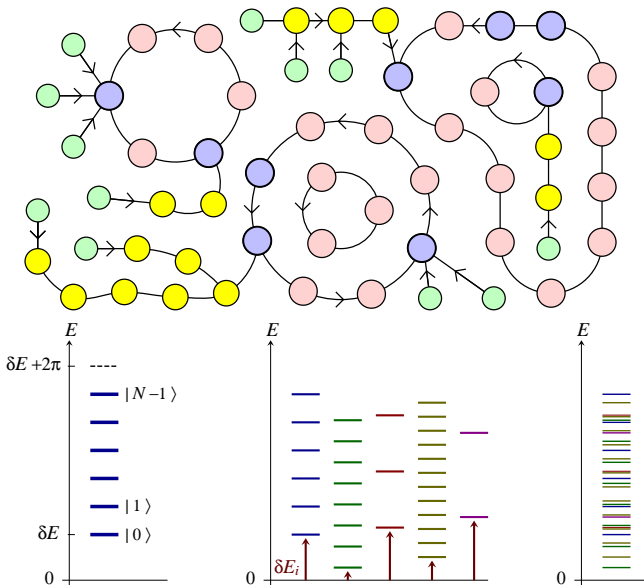
Introduce: info-equivalence classes: $(5) \approx (3)$, $(4) \approx (2)$



The generic, finite, deterministic, time reversible model:



The generic, finite, deterministic, time *non* reversible model:



The info-equivalence classes act as local gauge equivalence classes

Maybe they *are* local gauge equivalence classes!

By construction, these equivalence classes are time-reversible.

So, *in spite of info-loss*, the quantum theory will be time-reversible:
PCT invariance in QFT.

The classical, ontological states are not time reversible!

Therefore, the classical states carry an explicit arrow of time!

The quantum theory does not!

But conceivable, one first has to add the gravitational force . . .

Changes everything !

Gravitation as a local gauge theory for diffeomorphisms.

Could diffeomorphism classes be info equiv classes?

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Changes everything !

Gravitation as a local gauge theory for diffeomorphisms.

Could diffeomorphism classes be info equiv classes?

YES!

Such a theory may help explain why the cosm coupling const, Λ , is small, and why space is *globally flat* ($k \approx 0$):

“The ontological theory has a flat coordinate frame”

G. 't Hooft, The Cellular Automaton Interpretation of Quantum Mechanics,

Fundamental Theories of Physics, Vol. **185**,
Springer International Publishing, 2016.

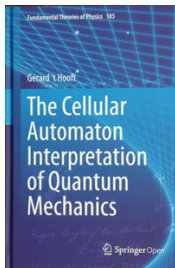
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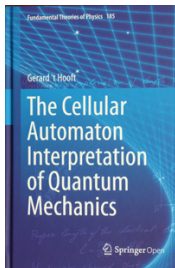
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THE END