

# Regional and Interregional IO Analysis

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## 1. Foreword and introduction

This chapter is written as an addendum to McCann (2001), which is used in Groningen as the main Bachelor textbook, and as an economic explanation of the InterRegional Input-Output Software (IRIOS) package.<sup>1</sup> Section 2 explains why the use of location quotients when constructing regional IO tables, discussed in appendix 4.2 of McCann, produces a systematic overestimation of regional IO multipliers. Section 3 summarizes the basic demand-driven IO quantity model for a closed national economy, discussed in appendix 4.3 of McCann. The demand-driven quantity model constitutes the theoretical basis of the IRIOS package and provides the formula for its so-called backward linkages. Section 4 introduces the disaggregation of the closed national IO model into an interregional IO model, which constitutes the actual basis of the IRIOS package. Section 5 explains how endogenizing parts of final demand can extend the interregional model further. This is illustrated by means of the type II interregional IO model with partly endogenous private consumption expenditures. Section 6 concludes with some notes on IRIOS and its application in multiplier and impact analysis. Finally, Appendix A adds the cost-push price model as the dual of the demand-driven quantity model, and Appendix B summarizes the alternative supply-driven IO quantity model and its complement, the demand-pull IO price model, and compares the plausibility of their assumptions with those of the standard IO quantity and price model. The supply-driven quantity model provides the base for the forward linkages calculated in IRIOS, but should not be used for other purposes.

## 2. Location Quotients and the construction of regional IO tables

There is an extensive literature on the construction of regional and interregional IO tables (Boomsma & Oosterhaven, 1992; Lahr, 1993). Here, we discuss only one point of general interest and warning.

When survey-based regional IO tables are absent, many researchers use location quotients (LQ's) for a non-survey estimate of the intra-regional input coefficients (called expenditure coefficients in McCann). However, all location quotient methods produce a systematic *overestimation* of intra-regional transactions and a directly related systematic

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<sup>1</sup> IRIOS is freely downloadable at [www.REGroningen.nl/irios](http://www.REGroningen.nl/irios). Technical details on using the software package are given in the Manual (Stelder, Oosterhaven & Eding, 2000), which is also downloadable at [www.REGroningen.nl/irios](http://www.REGroningen.nl/irios).

underestimation of regional imports and exports. To understand why, it is important to recognize that the *intra-regional input* coefficient of sector  $j$  in region  $r$  ( $a_{ij}^{rr}$ ) is in fact the product of a *regional technical* coefficient, which defines sector  $j$ 's requirements of inputs  $i$  from all over the world ( $a_{ij}^{\bullet r}$ ), and an *intra-regional trade* coefficient, which defines the share of these worldwide inputs that originates from the own region ( $t_{ij}^{rr}$ , also called regional self-sufficiency ratio or regional purchase coefficient, RPC). The interregional input coefficients of sector  $j$  in region  $r$  ( $a_{ij}^{sr}$ ) are defined analogously as the product of the same technical coefficient and an *interregional trade* coefficient, which defines the share of these worldwide inputs that originates from region  $s$  ( $t_{ij}^{sr}$ , also called interregional import ratio). Thus:

$$a_{ij}^{rr} = t_{ij}^{rr} a_{ij}^{\bullet r} \quad \text{and} \quad a_{ij}^{sr} = t_{ij}^{sr} a_{ij}^{\bullet r}, \quad \text{with} \quad \sum_s t_{ij}^{sr} = 1 \quad \text{and} \quad \sum_s a_{ij}^{sr} = a_{ij}^{\bullet r} \quad (1)$$

Location quotient methods are basically used to estimate  $t_{ij}^{rr}$ . When  $LQ_i \geq 1$ , i.e. when a sector is overrepresented in a certain region, it is assumed that the region is self-sufficient in  $i$  and  $t_{ij}^{rr}$  is set equal to 1. Herewith, gross imports of  $i$  are set equal to zero, while the difference between local production and local demand is exported. When  $LQ_i < 1$  it is assumed that  $t_{ij}^{rr}$  is equal to  $LQ_i$  and that the difference between local production and local demand is imported, while the gross exports of  $i$  are set equal to zero.

The problem with location quotient methods is that they only help to provide unbiased estimates of regional *net* exports (for sectors with an  $LQ > 1$ ) or regional *net* imports (for sectors with an  $LQ < 1$ ). They do not help to get good estimates of regional *gross* imports and *gross* exports, which are needed to get an unbiased estimate of intra-regional transactions. Consequently, LQ-methods are only helpful in constructing regional IO tables when there is no cross-hauling of comparable commodities.<sup>2</sup> But that is very seldom: almost all regions simultaneously import and export the 'same' products, at least at the usual level of some 30-60 sectors and for reasonable diversified regional economies. Only at a very fine level of commodity detail and/or when very specialized regions are studied one may expect negligible levels of cross-hauling.

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<sup>2</sup> The fourth condition for an accurate estimate McCann (2101, p.167) therefore relates gross regional trade and not to net regional trade.

In fact, estimating the regional trade coefficients  $t_{ij}^{rr}$  and  $t_{ij}^{sr}$  from other regional IO tables or using any kind of unreliable but unbiased survey data is superior to using systematically biased LQ-methods (see Oosterhaven, 2005, for a further discussion and application of non-survey methods).

### 3. The Leontief quantity model

The standard Leontief quantity IO model will be denoted here with  $i$  intermediate sectors (i.e. industries),  $q$  final sectors (i.e. final demand categories) and  $p$  primary sectors (i.e. value added categories). We will use the following standard symbols:

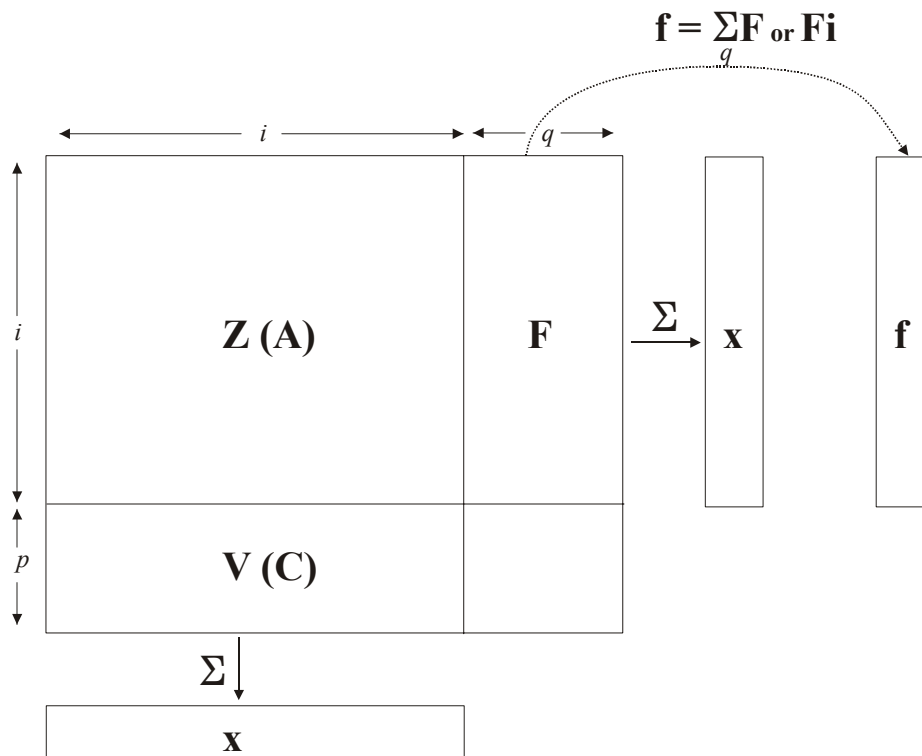
$x$  = total output = total input

$z$  = intermediate demand = intermediate supply (deliveries between firms)

$f$  = final demand (in McCann app. 4.3 the symbol  $d$  is used). The most common final demand categories are household consumption, investment and foreign exports.

$v$  = primary inputs. The most common primary input categories are imports, wages and capital income.

**Figure 1. National or one-regional IO table**



As figure 1 shows, the matrix notation (small bold cases for vectors and capital bold cases for matrices) and dimensions are as follows:

$\mathbf{i}$  = column vector of the appropriate size with ones, i.e. a summation vector.<sup>3</sup>

$\mathbf{x}$  =  $i$  - vector of output  $\{x_1, x_2, \dots, x_i\}$

$\mathbf{Z} = i \times i$  - matrix of intermediate demand  $\{z_{11}, z_{12}, \dots, z_{1i}$   
 $z_{21}, z_{22}, \dots, z_{2i}$   
 $\dots$   
 $z_{i1}, z_{i2}, \dots, z_{ii}\}$

$\mathbf{F} = i \times q$  - matrix of final demand.

As shown in figure 1, the horizontal summation over all  $q$  final demand categories of  $\mathbf{F}$ , in matrix notation written as  $\mathbf{F} \mathbf{i}$ , gives the total  $i$  – vector of final demand, which will be denoted as  $\mathbf{f}$ .

$\mathbf{V} = p \times i$  - matrix of primary costs, with  $\mathbf{i}' \mathbf{V} = \mathbf{v}'$ .

If the cells of  $\mathbf{Z}$  are divided by their column totals from  $\mathbf{x}$ , we get the  $i \times i$  matrix of input coefficients  $\mathbf{A}$ , i.e.  $a_{ij} = z_{ij} / x_j$ . The same holds for the matrix of primary input coefficients  $\mathbf{C}$ , which is calculated as  $c_{ij} = v_{ij} / x_j$ . Note that the column totals of  $\mathbf{A}$  and  $\mathbf{C}$  together add up to 1, i.e.  $\mathbf{i}' \mathbf{A} + \mathbf{i}' \mathbf{C} = \mathbf{i}'$ .

First, as shown in figure 1, the *accounting identities* give the definition of total output across the rows of the IO table:

$$\mathbf{x} = \mathbf{Z} \mathbf{i} + \mathbf{F} \mathbf{i} = \mathbf{Z} \mathbf{i} + \mathbf{f} \tag{2}$$

Second, the *behavioral relations* explain the demand for intermediate and primary input inputs from the sectoral level of total output (= production), with input coefficients  $\mathbf{A}$  and primary cost coefficients  $\mathbf{C}$ :

$$\mathbf{Z} = \mathbf{A} \hat{\mathbf{x}} \quad \text{and} \quad \mathbf{V} = \mathbf{C} \hat{\mathbf{x}} \tag{3}$$

With:  $\hat{\mathbf{x}}$  = diagonal matrix made from the corresponding vector.

With the final *economic assumption* that supply (production) follows demand, the solution of (2)-(3) is simple:

$$\mathbf{x} = \mathbf{A} \mathbf{x} + \mathbf{f} \Rightarrow (\mathbf{I} - \mathbf{A}) \mathbf{x} = \mathbf{f} \Rightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{L} \mathbf{f} \tag{4}$$

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<sup>3</sup> The row totals of a matrix  $\mathbf{M}$  are written as  $\mathbf{M} \mathbf{i}$  and the column totals are written as  $\mathbf{i}' \mathbf{M}$ .

This is the core formula of the Leontief quantity model. It shows why the standard quantity model is called demand-driven as *exogenous final demand*  $\mathbf{f}$  determines *endogenous total output*  $\mathbf{x}$ , which in its turn determines *endogenous intermediate and primary inputs*  $\mathbf{Z}$  and  $\mathbf{C}$  according to (3).

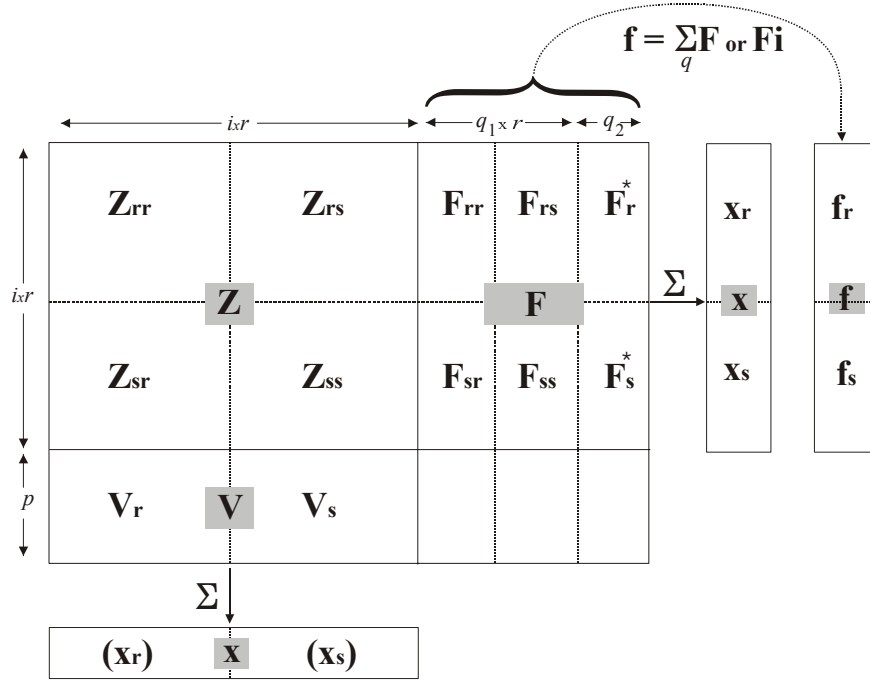
The  $i \times i$  matrix  $\mathbf{L}$  in (4) is the so-called *Leontief-inverse*, which equals:  $\mathbf{I} + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 + \dots$ . This so-called Taylor expansion of  $\mathbf{L}$  is the matrix equivalent of:  $1 + \frac{1}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^3 \dots = (1 - \frac{1}{2})^{-1} = 2$  (compare the other multipliers in McCann, ch.4). The expansion of  $\mathbf{L}$  indicates the direct effect  $\mathbf{I}$ , and the first and higher order indirect effects  $\mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \dots$  of a change in exogenous final demand. Note that the indirect effects on the supplying industries, i.e. per column of the IO table, represent the so-called *backward linkages* calculated in IRIOS.

Finally, in standard IO terminology the column totals of  $\mathbf{L}$  ( $\mathbf{l}' = \mathbf{i}' \mathbf{L}$ ) are known as the Leontief type I *production multipliers*. In words: the first entry  $l_1$  of  $\mathbf{l}'$  gives the total direct and indirect production effects in the whole economy (i.e. in all  $i$  sectors) of one unit of final demand for the products of sector 1. The second entry  $l_2$  gives the total direct and indirect production effects in the whole economy (i.e. in all  $i$  sectors) of one unit final demand of sector 2, etc. If, for example, sector 1 is agriculture, a value of 1,3 for  $l_1$  means that when final demand of agriculture is increased with 1 unit (say 1 million euro), output in all sectors together will increase with 1,3 million euro. Note that always  $l_i > 1$ , because the direct effect of the 1 million increase in final demand for agricultural products on the production of agriculture itself is included in  $l_i$ .

#### 4. The basis interregional IO model with two regions

Next, we discuss how including the economies of surrounding regions extends the basic model for a single open economy  $r$  into an interregional IO model. For simplicity's sake we will only discuss adding one region  $s$  that represents all other regions (see figure 2).

**Figure 2. Interregional IO-table for two regions ( $r = 2$ )**



The two subscript  $r$  and  $s$  subsequently indicate the *region of origin* and the *region of destination*, i.e. the  $i \times i$  matrix  $Z_{rr}$  represents the intermediate flows between firms within region  $r$ , and  $Z_{rs}$  represents the intermediate exports from firms in region  $r$  to firms in region  $s$ . If there is only one regional index of a variable, for example production  $x$ ,  $x_r$  is simply the  $i$ -vector of production in region  $r$ . Analogously, there is only one matrix  $V_r$  and  $V_s$ . Total final demand for products from firms in region  $r$  is  $f_r$  and total final demand for products from firms in region  $s$  is  $f_s$ .

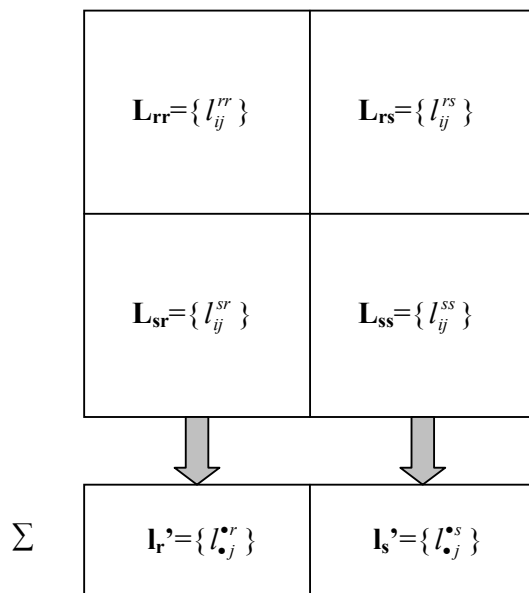
For final demand the interregional table is a little more complicated. First, some final demand categories, like household consumption and investment, have a region of origin and a region of destination. These  $q_1$  categories are represented by the interregional matrices  $F_{rr}$ ,  $F_{rs}$ ,  $F_{sr}$  and  $F_{ss}$ . For example, household consumption column in  $F_{rs}$  denotes the consumption of goods produced in region  $r$  consumed by households in region  $s$ . Second, some  $q_2$  final demand categories do not have region  $r$  or  $s$  as their destination but go somewhere else. The most common category of this type is foreign exports. These demand categories are represented by  $F_r^*$  and  $F_s^*$ . As shown in figure 2, the total final demand  $f_r$  for the products of firms in region  $r$  thus equals the combined row sum of  $F_{rr}$ ,  $F_{rs}$  and  $F_r^*$ .

In order to see the similarity with the one-region model in figure 1, the grey boxes with  $\mathbf{Z}$ ,  $\mathbf{F}$ ,  $\mathbf{V}$ ,  $\mathbf{x}$ , and  $\mathbf{f}$  in figure 2 indicate that the most simple way to look at the mathematics of an interregional IO model is to review the solution in (4):

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{L} \mathbf{f} \quad (4a)$$

The only difference is that now in this two-region case the dimensions of  $\mathbf{x}$  and  $\mathbf{f}$  are  $2i$  and the dimensions of  $\mathbf{I}$ ,  $\mathbf{A}$  and  $\mathbf{L}$  are  $2i \times 2i$  (see figure 3 for the Leontief-inverse). Again, the column sums  $\mathbf{1}'$  of the Leontief inverse  $\mathbf{L}$  represent the *production multipliers*, where - if we take again the example of sector 1 being agriculture -  $l_1$  is the total direct and indirect effect on production in the whole economy, i.e. *all sectors in all regions*, of one unit increase of final demand for agricultural product *from region 1*. Analogously,  $l_{i+1}$  is the total direct and indirect effect on production in the whole economy, i.e. *all sectors in all regions*, of one unit increase of final demand for agricultural product *from region 2*.

**Figure 3. The interregional production multipliers**

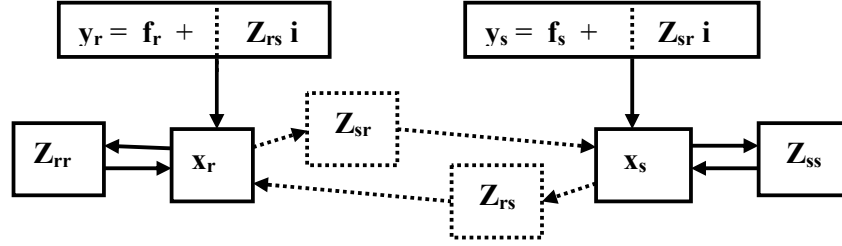


Of course, an interregional IO model will primarily be built to look what happens inside the Leontief-inverse. Therefore, an interregional representation of the model is shown in Figure 4. The bold lines and boxes summarize the casual structure of two independent single-region models for the regions  $r$  and  $s$ . The dotted separation in the two top boxes indicates which part of demand remains exogenous and which



part is made endogenous by adding the two dotted boxes that link the two independent single-region models into one interregional model. For reasons of simplicity, we will assume here that the two regions  $r$  and  $s$  together form a *closed economy*, i.e. there are no foreign exports (there is no final demand  $\mathbf{F}^*$  going elsewhere as mentioned in figure 2).

**Figure 4. The causal structure of the interregional IO model extension**



In this simple case, from the viewpoint of region  $r$ , the intermediate flows  $\mathbf{Z}_{rs}$  from its own firms to firms in regions  $s$  are actually (domestic) exports and thus should be considered as part of its total final demand, here denoted as  $\mathbf{y}_r$ . In a single-region model this  $\mathbf{y}_r$  would be exogenous. Figure 4 shows that this exogenous final demand for a single region  $\mathbf{y}_r$  equals the sum of the rest of the final demand  $\mathbf{f}_r$  for products from region  $r$  (from consumers, investors and government, C+I+G, in the own region and in region  $s$ ) and its total intermediate exports  $\mathbf{Z}_{rs} \mathbf{i}$  to firms in region  $s$ . Thus, of total exports, part is sold as intermediate output to businesses in  $s$  and part is sold to local final demand in  $s$ .

In the basic interregional IO model, the intermediate exports  $\mathbf{Z}_{rs}$  become endogenous as imports into region  $s$  by making it dependent on the output  $\mathbf{x}_s$  of the sectors in the destination region, whereas the rest of final demand  $\mathbf{f}_r$  remains exogenous. The same happens in region  $s$  where the export of intermediate output  $\mathbf{Z}_{sr}$  becomes endogenous by making it dependent on sectoral level of production  $\mathbf{x}_r$  in the destination region, whereas the rest of final demand  $\mathbf{f}_s$  again remains exogenous. In formulas:

$$\mathbf{x}_r = \mathbf{Z}_{rr} \mathbf{i} + \mathbf{Z}_{rs} \mathbf{i} + \mathbf{f}_r = \mathbf{A}_{rr} \mathbf{x}_r + \mathbf{A}_{rs} \mathbf{x}_s + \mathbf{f}_r \quad (5)$$

$$\mathbf{x}_s = \mathbf{Z}_{sr} \mathbf{i} + \mathbf{Z}_{ss} \mathbf{i} + \mathbf{f}_s = \mathbf{A}_{sr} \mathbf{x}_r + \mathbf{A}_{ss} \mathbf{x}_s + \mathbf{f}_s \quad (6)$$

The first equalities of (5)-(6) show the accounting identities across the rows of the underlying bi-regional IO table. The last part of (5)-(6) shows how the product of the intra- and interregional input coefficients and the column totals of the bi-regional IO table determine intermediate demand.

When the corresponding terms are put together into single vectors and single matrices (5)-(6) can be summarized and solved, given again the standard Leontief formula:

$$\mathbf{x} = \mathbf{Z} \mathbf{i} + \mathbf{f} = \mathbf{A} \mathbf{x} + \mathbf{f} \Rightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{L} \mathbf{f} \quad (7)$$

$\mathbf{L}$  is now equal to the *interregional Leontief-inverse* shown in figure 3. Solving the original (5)-(6) step-by-step<sup>4</sup> gives the following disaggregated version of (7):

$$\mathbf{x}_r = \{\mathbf{I} - \mathbf{A}_{rr} - \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{A}_{sr}\}^{-1} \{\mathbf{f}_r + \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{f}_s\} \quad (7a)$$

$$\mathbf{x}_s = \{\mathbf{I} - \mathbf{A}_{ss} - \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{A}_{rs}\}^{-1} \{\mathbf{f}_s + \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{f}_r\} \quad (7b)$$

Thus, total output (in each region) is determined by the level of its own final output as well as by that of the other region. For region  $r$ , the first term in (7a) describes the *intra-regional impacts* of its own final demand  $\mathbf{f}_r$  and the second terms describe the *interregional spillovers* of final demand  $\mathbf{f}_s$  of the second region.

To better understand the difference between the single-region IO model and its interregional extension; we have a closer look at the intra-regional impact of the own final demand on the own region. In the single-region model, it equals  $(\mathbf{I} - \mathbf{A}_{rr})^{-1}$ , and in the interregional model the same effect equals  $\{\mathbf{I} - \mathbf{A}_{rr} - \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{A}_{sr}\}^{-1}$ . Clearly, the first impact is smaller than the second. The reason that the interregional multipliers are larger is that they also take into account the *interregional feedbacks* of the own final demand via the own imports on the production of the other region, and the subsequent effect of that region's increase in production on that region's imports which equal the now endogenous part of the own region's exports. The larger multipliers are, however, compensated by the fact that the size of exogenous demand has become smaller than that of single-region model, because  $\mathbf{f}_r = \mathbf{y}_r - \mathbf{Z}_{rs} \mathbf{i}$  (see (5)-(6) and figure 4).

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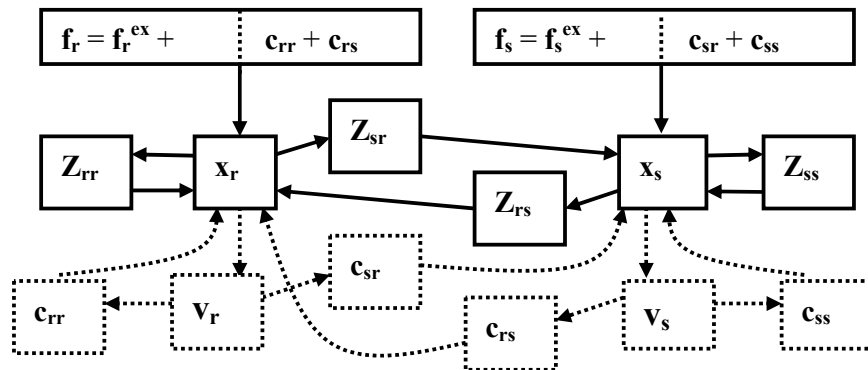
<sup>4</sup> A good exercise in matrix algebra is to check this yourself. First, solve (5) for  $\mathbf{x}^R$ . Then substitute this in (6) and solve that for  $\mathbf{x}^S$ . Finally, do the same in the reverse order to solve for  $\mathbf{x}^R$ .

To summarize: the interregional IO model adds interregional spillovers and interregional feedbacks to the single-region IO model, with the following relation:  $\text{feedback}^{\text{RR}} = \text{spillover}^{\text{RS}} * \text{intra-regional effect}^{\text{SS}} * \text{spillover}^{\text{SR}}$  (see equation 7a). The basis interregional IO model enables us to calculate the regional and the interregional impacts of any change in exogenous final demand in one consistent framework.

## 5. Extension of the base model with a consumption function

Although important from a methodological point of view, the interregional extension still leaves the most important component of local final demand, namely private consumption, unexplained. Investments and government expenditures may also be endogenised, but here we will only discuss how to endogenise the larger part of private consumption demand. Figure 5 shows the nature of this extension. The bold lines and boxes summarize the causal structure of the so-called *Type I interregional IO model* from Figure 4. The dotted separation in the two top boxes again indicates which part of final demand remains exogenous and which part now becomes endogenous by adding the six dotted boxes lower on in figure 5. This further extension results in the so-called *Type II interregional IO model*.

**Figure 5. The causal structure of the endogenous consumption extension**



In the standard interregional IO model all local consumption demand of region  $r$  (for locally produced consumption goods as well as for imported consumption goods) is exogenous. The same holds for the local consumption demand of region  $s$ . Some part of local consumption demand, however, will be *directly dependent* on the size and growth of the own regional value added. Some other part will be influenced

indirectly by the *interregional redistribution* of regional value added through capital income flows of multiregional firms and through central government's social security and taxation schemes.<sup>5</sup> Here we will only add the direct relation between regional value added ( $v_r$ ) and regional private consumption demand ( $c_{rr}$  and  $c_{sr}$ ) for region  $r$ , and of course analogously for region  $s$ .

To better understand the nature of the extension we will first discuss the determination of the endogenous part of the consumption of *households living in region  $r$*  in normal algebra:

$$c_{ij}^{rr} = q_{ij}^{rr} x_j^r = t_{ic}^{rr} p_i^r (1 - s^r)(1 - t^r) w_j^{rr} x_j^r, \quad \forall i, j \quad (8a)$$

$$c_{ij}^{sr} = q_{ij}^{sr} x_j^r = t_{ic}^{sr} p_i^r (1 - s^r)(1 - t^r) w_j^{sr} x_j^r, \quad \forall i, j \quad (8b)$$

$$c_{ij}^{rs} = q_{ij}^{rs} x_j^s = t_{ic}^{rs} p_i^r (1 - s^r)(1 - t^r) w_j^{rs} x_j^s, \quad \forall i, j, \text{ commuters from } R \text{ to } S \quad (8c)$$

$$c_{ij}^{ss} = q_{ij}^{ss} x_j^s = t_{ic}^{ss} p_i^r (1 - s^r)(1 - t^r) w_j^{ss} x_j^s, \quad \forall i, j, \text{ commuters from } R \text{ to } S \quad (8d)$$

In (8), the typical coefficient  $q_{ij}^{sr}$  indicates the amount of goods produced by sector  $i$  in region  $s$  that is consumed by households from  $r$  that are working in sector  $j$  in the own region  $r$  per unit of production of  $j$  in  $r$ , whereas  $q_{ij}^{ss}$  indicates the same amount but then bought by commuters living in  $r$  but working in  $s$  per unit of production of  $j$  in  $s$ . Hence, if commuting is zero, as in Figure 5, (8c) and (8d) disappear.

The matrix  $\mathbf{Q}_r$  with the four sets of  $q$ -coefficients from (8) may be built up empirically as the product of the following series of coefficients:

$w_j^{rs}$  = labor income coefficient, indicating gross labor income earned in  $j$  in  $s$  by commuters from  $r$  per unit of production of  $j$  in  $s$ ,

$t^r$  = average tax rate, assumed to be equal over all sectors of employment,

$s^r$  = household saving ratio, assumed to be equal over all sectors of employment,

$p_i^r$  = consumption package coefficient, indicating how much of total private consumption in  $R$  is spend on products of sector  $i$  worldwide,

$t_{ic}^{sr}$  = consumption trade coefficient, indicating how much of total consumption of products  $i$  in region  $r$  originates from region  $s$ .

The endogenous part of private consumption by households living in  $r$  can be summarized in matrix algebra:

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<sup>5</sup> This is described in so-called social accounting models (SAM's, see Pyat & Round, 1979).

$$\begin{vmatrix} c_{RR} + c_{RS} \\ c_{SR} + c_{SS} \end{vmatrix} = \begin{vmatrix} Q_{RR} & Q_{RS} \\ Q_{SR} & Q_{SS} \end{vmatrix} \begin{vmatrix} x_R \\ x_S \end{vmatrix}, \text{ for region } r, \text{ in short: } \mathbf{c}_r = \mathbf{Q}_r \mathbf{x} \quad (9a)$$

$$\text{and by analogy for region } s: \mathbf{c}_s = \mathbf{Q}_s \mathbf{x} \quad (9b)$$

When commuting from  $r$  to  $s$  is zero,  $\mathbf{Q}_{rs}$  and  $\mathbf{Q}_{ss}$  will be zero in  $\mathbf{Q}_r$  and also  $\mathbf{c}_{rs}$  and  $\mathbf{c}_{ss}$  in  $\mathbf{c}_r$ . When commuting from  $s$  to  $r$  is zero,  $\mathbf{Q}_{sr}$  and  $\mathbf{Q}_{rr}$  will be zero in  $\mathbf{Q}_s$  and thus also  $\mathbf{c}_{sr}$  and  $\mathbf{c}_{ss}$  in  $\mathbf{c}_s$ .

The solution of the Type II interregional IO model is simple, but first the final demand of the Type I model needs to be split up into endogenous consumption demand and remaining exogenous final demand (see also Figure 6):

$$\mathbf{f} = \mathbf{f}^{\text{ex}} + \mathbf{c}^R + \mathbf{c}^S \quad (10)$$

Then substitution of (9) into (10), and (10) into (7) gives:

$$\mathbf{x} = \mathbf{A} \mathbf{x} + \mathbf{Q}_r \mathbf{x} + \mathbf{Q}_s \mathbf{x} + \mathbf{f}^{\text{ex}} \Rightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A} - \mathbf{Q}_r - \mathbf{Q}_s)^{-1} \mathbf{f}^{\text{ex}} = \mathbf{L}^* \mathbf{f}^{\text{ex}} \quad (11)$$

$\mathbf{L}^*$  is now the *Type II interregional Leontief-inverse*, indicating the direct, the indirect and the so-called induced interregional impacts of any change in the remaining exogenous demand in region  $R$  or  $S$ . Again the multipliers have become larger, but this is compensated by a smaller exogenous demand  $\mathbf{f}^{\text{ex}}$  compared to the type I exogenous demand  $\mathbf{f}$ , as shown in (11).

## 6. Some comments on the options of IRIOS

By default, IRIOS works with the standard demand-driven Leontief model, which is referred to as the “basic model” in various IRIOS windows:

$$\mathbf{V} = \mathbf{C} \mathbf{x} = \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{F}. \quad (12)$$

An IO table used by IRIOS contains gross production  $\mathbf{x}$  and a primary cost matrix  $\mathbf{V}$ , which are related through the primary cost coefficients’ matrix  $\mathbf{C}$  that is obtained by column-wise division of  $\mathbf{V}$  by gross production  $\mathbf{x}$ . The rows of  $\mathbf{V}$  are referred to as “variables” in the IRIOS menus.

Any other information that is not part of a standard IO table can be added to the model as an **Extra variables** by extending matrix **C** to **C\*** and **V** to **V\***. Adding a vector of employment coefficients to **C** is the most obvious example, but one can also think of a vector of CO<sub>2</sub> coefficients for environmental implementations. It is important to understand that these extra variables mostly have a dimension (number of jobs or tons of CO<sub>2</sub>) that is different from that of the IO table used (euros of a certain year). Consequently, the extension **C\*** will have also a specific dimension, whereas the basic **C** will be dimensionless. Thus, when **C\*** relates to for instance an older year than the matrix with exogenous impulses **F**, the calculated impacts **V** will be incorrect. Before calculating impacts either **F** should be deflated or **C\*** should be inflated. The standard **V** or you own extended **V\*** contains endogenous variables of the model *without a feedback to output*.

Next, in addition, you can extend the model to produce any kind of type II, III, IV etc. multipliers by specifying **Extra relations** *with a feedback to output* (these are always referred to as “relations” in the IRIOS menus):

$$\mathbf{V} = (\mathbf{C} + \mathbf{Q}_c) \mathbf{x} = (\mathbf{C} + \mathbf{Q}_c) (\mathbf{I} - \mathbf{A} - \mathbf{Q})^{-1} \mathbf{F}^{\text{ex}} \quad (13)$$

How the **Q**-matrix of the extra relation in (13) may be constructed has been discussed in the last section. Besides, a **Q<sub>c</sub>**-matrix needs to be specified for the following reason. Since a variable is defined as a row with coefficients **c** per unit of gross production, variable **v** in the basic model may be computed as  $\mathbf{v} = \mathbf{c} (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f}$ . However, in a model extended with a relation **Q**, the link between a variable **v** and a relation **Q** should be specified. This link is specified by the **Q<sub>c</sub>**-matrix and consists of *k* rows (for each variable *k*) and *i* x *r* columns. For example, if **Q** represents endogenous private consumption, **Q<sub>c</sub>** should inter alia contain the foreign imports of consumer goods or the CO<sub>2</sub>-emissions by households per unit of output.

With the extra variables and the extra relations the IRIOS model solution will have the following general form:

$$\mathbf{V}^* = (\mathbf{C}^* + \mathbf{Q}_{c1} + \mathbf{Q}_{c2} + \mathbf{Q}_{c3} + \dots + \mathbf{Q}_{cn}) (\mathbf{I} - \mathbf{A} - \mathbf{Q}_1 - \mathbf{Q}_2 - \mathbf{Q}_3 - \dots - \mathbf{Q}_n)^{-1} \mathbf{F}^{\text{ex}}. \quad (14)$$

The potential size of the exogenous final demand matrix **F<sup>ex</sup>** of course depends on the amount of closure of the IO model, as indicated in the last two sections.

In order to compute **Impacts** and/or **Projections** with the basic model or with the extended model, various exogenous impulses may be specified to fill the  $\mathbf{F}^{\text{ex}}$  matrix. IRIOS then produces the results according to the generalized equation (14), but with the option to switch each variable and/or each relation and/or each impulse on or off. Besides, IRIOS also produces an array of descriptive statistics and multipliers. The **Multipliers** that can be calculated with IRIOS include the traditional output multipliers as well as the multipliers for the endogenous variables, both in their *regular* format, i.e. per unit of exogenous final demand, and in their *standardized* or normalized format, i.e. per exogenous unit of the variable (employment/income/CO<sub>2</sub>) to which the multiplier relates. Standardized multipliers are thus defined as the total endogenous employment/income/CO<sub>2</sub> per unit of direct exogenous employment/income/CO<sub>2</sub> or as:

$$\text{Standardized multiplier} = \text{Regular multiplier} / \text{Direct coefficient.}$$

## Appendix. Additional IO models

### A. The Leontief price model

Here the standard IO price model, also called the *cost-push IO price model* will be discussed. Recall the standard IO quantity model from section 3:

$$\mathbf{x} = \mathbf{Z} \mathbf{i} + \mathbf{F} \mathbf{i} = \mathbf{Z} \mathbf{i} + \mathbf{f} \quad (2)$$

$$\mathbf{Z} = \mathbf{A} \hat{\mathbf{x}} \quad \text{and} \quad \mathbf{V} = \mathbf{C} \hat{\mathbf{x}} \quad (3)$$

$$\mathbf{x} = \mathbf{A} \mathbf{x} + \mathbf{f} \Rightarrow (\mathbf{I} - \mathbf{A}) \mathbf{x} = \mathbf{f} \Rightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{L} \mathbf{f} \quad (4)$$

The standard IO price model consists of two sets of equations. First, the accounting identities for the values of sectoral total inputs (= total cost), which are defined along the columns of the IO table:

$$\mathbf{p} \hat{\mathbf{x}} = \mathbf{p}' \mathbf{Z} + \mathbf{p}_v' \mathbf{V} \quad (15)$$

where:

$\mathbf{p}$  = I-vector with (index)prices for total sectoral output,

$\mathbf{p}_v$  = P-vector with (index)prices for primary inputs per category.

Second, just like the quantity model, the price model assumes constant input coefficients. Substitution of (3) in (15) and post-multiplication by  $\hat{\mathbf{x}}^{-1}$  shows that the unit costs of production per sector equals the weighted average of the prices for the intermediate and primary inputs:

$$\mathbf{p}' \hat{\mathbf{x}} = \mathbf{p}' \mathbf{A} + \mathbf{p}_v' \mathbf{C} \quad (16)$$

Note that the sum of the weights for each industry indeed equals one:  $\mathbf{i}' \mathbf{A} + \mathbf{i}' \mathbf{C} = \mathbf{i}'$ , as the input accounting identities in (16) are measured in unit price indices of the base-year.

The essence of the Leontief price model lies in the causal relationships between these prices. The primary input prices (equal per IO row) are assumed to be exogenous, whereas the prices for total output (also equal per IO row) are endogenously determined by the solution of (16):

$$\mathbf{p}' = \mathbf{p}_v' \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{p}_v' \mathbf{C} \mathbf{L} \quad (17)$$



Primary input prices  $\mathbf{p}_v'$  change exogenously. The size of the direct effect of such changes on endogenous unit cost (= output prices  $\mathbf{p}'$ ) is determined by the primary input cost shares  $\mathbf{C}$ . Next, the prices of intermediate inputs rise as firms compensate their unit cost increases by an increase in their output prices that is passed on to all customers along the rows of the IO table. This again causes output prices to rise according to the intermediate input cost shares  $\mathbf{A}$ ; the cumulative effect of this cost-push process is described by the Leontief-inverse in (17). Note that the price multipliers  $\mathbf{C} \mathbf{L}$  are calculated in IRIOS as primary input multipliers (see equation 14)

To clarify the relation between the quantity and the price version of the Leontief model (17) is post-multiplied by  $\mathbf{y}$ , which gives:

$$\mathbf{p}' \mathbf{y} = \mathbf{p}_v' \mathbf{C} \mathbf{L} \mathbf{y} = \mathbf{p}_v' (\mathbf{V} \mathbf{i}) \quad (18)$$

On the one hand, (18) expresses the *equality in value* between total final output and total primary inputs. On the other hand, (18) emphasizes the independency (or better duality) of the quantity model and the price model. Although their solutions are linked in (18), their variables are determined independently, namely,  $\mathbf{v}$  by  $\mathbf{y}$  in (3), and  $\mathbf{p}'$  by  $\mathbf{p}_v'$  in (17). Consequently, each IO market functions as in Figure 1. Prices and quantities move independently. Supply is perfectly price elastic, which means that there is infinite supply at the going market price, and demand is perfectly *inelastic* to price changes. Clearly, reality is a bit more complicated than that.

Figure 1. Markets in Leontief IO models

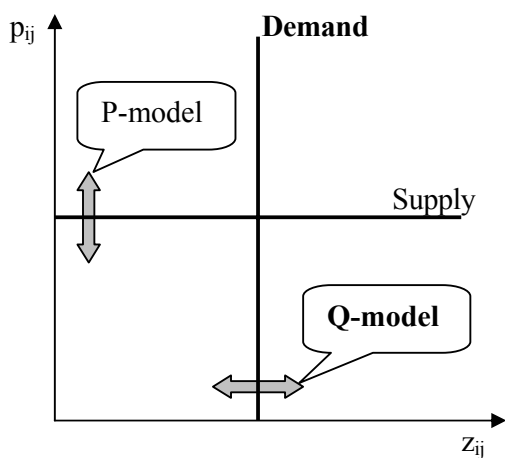
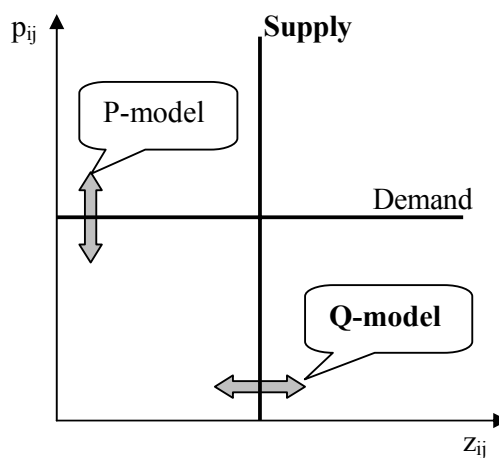


Figure 2. Markets in Ghoshian IO



## B. The alternative Ghoshian quantity and price model

In the *Ghoshian IO model* the homogeneity assumption is made for all columns of the IO table, instead of for all rows as in the Leontief model. Its mathematics is equally simple, but also opposite to that of the standard model. Instead of accounting identities for total output, it starts with accounting identities for total *input*, across the columns of the IO table, again with all (index)prices set to one in the base year:

$$\mathbf{x}' = \mathbf{i}' \mathbf{Z} + \mathbf{i}' \mathbf{V} = \mathbf{i}' \mathbf{Z} + \mathbf{v}' \quad (19)$$

Next, instead of the constant intermediate and primary input coefficients, the Ghoshian model uses constant intermediate and final *output* coefficients, derived from the rows of the IO table, to determine the supply of intermediate and final outputs:

$$\mathbf{Z} = \hat{\mathbf{x}} \mathbf{B} \quad \text{and} \quad \mathbf{Y} = \hat{\mathbf{x}} \mathbf{D} \quad (20)$$

where:

$\mathbf{B}$  = IxI-matrix of constant intermediate output coefficients,

$\mathbf{D}$  = IxQ-matrix of constant final output coefficients.

The causal interpretation of (19)-(20) is also opposite. Instead of exogenous final demand, the Ghoshian model has *exogenous primary supply*, and instead of the solution for total outputs, the Ghoshian model produces a solution for total inputs:

$$\mathbf{x}' = \mathbf{v}' (\mathbf{I} - \mathbf{B})^{-1} = \mathbf{v}' \mathbf{G} \quad (21)$$

Finally, opposed to the Leontief solution for endogenous intermediate and primary inputs, total inputs from (21) are used to derive the Ghoshian solutions for *endogenous intermediate and final outputs*:

$$\mathbf{Z} = \mathbf{v}' \mathbf{G} \mathbf{B} \quad \text{and} \quad \mathbf{Y} = \mathbf{v}' \mathbf{G} \mathbf{D} \quad (22)$$

$\mathbf{G}$  is the so-called *Ghoshian-inverse*, which equals  $\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \dots$ . Opposed to the Leontief-inverse, the indirect part of this inverse ( $\mathbf{B}^1 + \mathbf{B}^2 + \mathbf{B}^3 + \dots$ ) gives an indication of the cumulative linkages of each sector with its customers (i.e. the processing

industries along the rows of the IO table). In IRIOS ( $\mathbf{G} - \mathbf{I}$ ) is used to calculate the so-called *forward linkages* across the rows of the IO table.

The price version of the supply-driven input-output model, also called the *demand-pull IO price model*, will not be discussed in detail (see Oosterhaven, 1996, for details). We will only show its solution and contrast it with the cost-push IO price model.

$$\mathbf{p} = (\mathbf{I} - \mathbf{B})^{-1} \mathbf{D} \mathbf{p}_y = \mathbf{G} \mathbf{D} \mathbf{p}_y \quad (23)$$

where:

$\mathbf{p}$  = I-vector of (index)prices for total sectoral input,

$\mathbf{p}_y$  = Q-vector of (index)prices for final output per category.

As opposed to the cost-push model where  $\mathbf{p}$  refers to the price for each sector's single homogeneous output,  $\mathbf{p}$  in (5) relates to the price for each sector's single homogeneous *input*. Furthermore, as opposed to the cost-push model where primary input was homogeneous across rows, here final output is homogeneous across each column of the IO table. The direction of causality also runs opposite to that of the cost-push price model. The single price for each (homogeneous) column with final output is assumed exogenous, whereas the prices for (primary) inputs per sector are now endogenous.

Solution (5) then describes the cumulative effects of changes in final output prices on the prices of (primary) inputs, such as labour and the use of capital. The final output shares in total production  $\mathbf{D}$  capture the direct effect of  $\mathbf{p}_y$  on unit revenues per sector. The increases in unit revenues are entirely passed on into the prices paid for the inputs. As the price of total inputs per sector thus increases, the prices of all intermediate inputs must also increase, not row-wise as in the cost-push model but column-wise. This means that in each column of the input-output table prices of all intermediate inputs change with that sector's increase in total input price. The cumulative effect of these intermediate price increases is described by the Ghoshian-inverse in (5).

Pre-multiplication of (5) by  $\mathbf{v}'$  clarifies the duality between the price and quantity versions of the Ghoshian model:

$$\mathbf{v}' \mathbf{p} = \mathbf{v}' \mathbf{G} \mathbf{D} \mathbf{p}_y = \mathbf{x}' \mathbf{D} \mathbf{p}_y = (\mathbf{i}' \mathbf{Y}) \mathbf{p}_y \quad (6)$$

Equation (6) shows, as in the Leontief case, how the solutions of the price and quantity models are linked in value terms, but are determined independently. Figure 2 shows this independent nature of the price and quantity version of the Ghoshian model on an individual IO market. Purchase prices move independently and are set by the buyers of output. Hence, firms maximize their revenues by holding their output coefficients constant, passing any increase in output prices (= unit revenues) on to their suppliers, from which they accept any quantity supplied, as all (intermediate and primary) inputs are perfect substitutes for each other!

To summarize: causality runs opposite in both sets of models. Loosely formulated: (final) demand drives the Leontief quantity model in which supply reacts with perfect (price) elasticity, whereas (primary) supply drives the Ghoshian quantity model with demand reacting with perfect (price) elasticity. In the latter case, this implies that cars can be driven without gas and factories can be run without essential inputs or even worse without labor. This was the prime reason why the supply-driven model was labeled as far more implausible than the demand-driven model (Oosterhaven, 1988). In the ensuing discussion (Gruver, 1989; Oosterhaven, 1989), the theoretical underpinnings of both IO quantity models were further clarified.

The causality and the assumptions involved in the price version of the Ghosh model are as implausible as those of the quantity version. In the cost-push IO price model all sectors set their output prices independently and pass all cost increases on to their purchasers, who do not react. In the demand-pull IO price model the opposite happens. All sectors set their input prices independently, passing any increase in revenue from rising output prices on to their suppliers, who do not react either. Hence, although the cost-push price model may not be entirely plausible, the demand-pull price model is much less plausible.

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