Chapter 5

Fluidised particles as turbulence promoters in ultrafiltration: Improvement of flux and rejection

Introduction

A serious problem in ultrafiltration processes is fouling of the membranes. This can be due to adsorption of solutes on outer membrane and inner pore surfaces, blocking of pores by rejected solutes or formation of a thick cake layer of precipitated solutes on top of the membrane. The latter type of fouling is predominantly caused by concentration polarisation occurring in the liquid boundary layer in front of the membrane. Concentration polarisation, and thereby fouling, depends on the mixing of the liquid feed. This mixing can be improved by various means. In cross flow filtration, the classical way is by increasing the cross flow velocity. Also static turbulence promoters have been used (Vatai 1995, Da Costa 1993, Copas 1974). However, these methods introduce a large pressure drop across the membrane modules, resulting in high energy consumption. An alternative is adding fluidised particles. These can do their job in two ways: mixing up the liquid boundary layer in front of the membrane and scouring of an already formed fouling layer by collisions. The potential of preventing fouling by using fluidised particles was shown by Meijer (Meijer 1984, 1986). He used them to prevent scale formation in tubes of heat exchangers. Fluidised particles were successfully applied to ultrafiltration processes by van der Velden et al. (Van der Velden 1976, van der Waal 1977) and Rios et al (Rios 1987). Van der Velden found a considerable increase in mass transfer in the boundary layer during hyperfiltration of salts, when varying the cross flow (and thereby the bed porosity) with a constant transmembrane pressure drop. He fluidised glass beads with diameters ranging from 0.4 - 2 mm. The increase in mass transfer could be predicted by an empirical mass transfer relation determined for fluid beds with non porous walls by Smith (Smith 1975). Van der Velden also did a few ultrafiltration
experiments using a high molecular weight polyethylene glycol (PEG4,000,000) to study improvement of flux under gelation conditions. In his work a major disadvantage of applying fluidised particles came to light. SEM pictures of the polymeric ultrafiltration membranes used showed severe damage caused especially by the bigger particles, those that removed the gel layer completely. From these observations it can be concluded that the membrane should be protected from impacts of particles by at least a small amount of fouling. We also encountered membrane damage under certain conditions. In the next chapter this will be discussed in more detail. Rios et al. (Rios 1987) used inorganic membranes and did not notice any damage of their membranes, despite the brittle nature of inorganic membranes. They filtered gelatine solutions and assumed to be working in a gel regime. They found considerable improvements on flux and modest changes in rejection of gelatine. In this study, the influence of fluidised particles on mass transfer in the boundary layer is determined with aqueous solutions of both a medium sized solute (PEG10000) and a large solute (Bovine Serum Albumin). This is done by measuring fluxes and rejections at various transmembrane pressure drops.

Theory

Ultrafiltration theory

Ultrafiltration is driven by a pressure gradient. This induces convective transport of bulk liquid to the membrane interface. At this interface some of the solutes are (partly) rejected. Their concentrations at the interface build up until they get high enough to give a diffusive flux back to the bulk solution. This back diffusion is balanced by the convective transport of solutes to the interface and the flux of the solutes through the membrane. The increase in concentration at the membrane interface is called concentration polarisation. The volume flux is described by the following equation (Van den Berg 1988):

\[ J = k \ln \left( \frac{C_w - C_p}{C_b - C_p} \right) \]  

(1)
where \( C_w, C_p, \) and \( C_b \) are the concentration of solute at the membrane interface, in the permeate stream and in the bulk respectively, \( k \) is the mass transfer coefficient in the boundary layer. The degree of separation of a solute from the solution is expressed in a rejection coefficient, which is defined as:

\[
R_{obs} = 1 - \frac{C_p}{C_b}
\]  

(2)

This \( R_{obs} \) is influenced by concentration polarisation effects. To correct for these, a true rejection, \( R_{true} \), is defined according to:

\[
R_{true} = 1 - \frac{C_p}{C_w}
\]  

(3)

These two can be interrelated with the help of equation (1):

\[
\ln \left( \frac{1 - R_{obs}}{R_{obs}} \right) = \frac{J}{k} + \ln \left( \frac{1 - R_{true}}{R_{true}} \right)
\]  

(4)

At low permeate fluxes \( R_{true} \) increases with increasing flux (and thus with increasing pressure drop). At sufficiently high fluxes \( R_{true} \) approaches a constant value, while \( R_{obs} \) decreases with increasing flux due to concentration polarisation effects. Under these circumstances, mass transfer coefficients can be obtained from a plot of \( \ln(1-R_{obs}/R_{obs}) \) vs. \( J \) at constant feed velocity (see appendix).

The above equation applies to solutions of uncharged components. Protein solutions are more complicated as electrical forces have to be taken into account. To describe these results we use the Maxwell-Stefan model as developed in chapter 4.

**Estimating mass transfer coefficients**

The mass transfer coefficient in the liquid boundary layer, \( k \), can be estimated from empirical mass transfer relations presented in literature. For empty tubes
operated in the turbulent regime the relation of Dittus-Boelter is often used (Gekas 1987):

\[ Sh = 0.023 Re^{0.8} Sc^{0.33} \]  (5)

For fluidised beds various empirical relations have been given for heat or mass transfer. In this chapter we compare the experimental results with the mass transfer relations determined by Smith and King (Smith 1975) and Yasunishi et al. (Yasunishi 1988). The relation of Smith for larger particles (diameter ratio \( d_p/D_t \) up to 0.06) is:

\[ Sh = \frac{1}{\varepsilon} 0.455 Re_h^{0.56} Sc^{0.33} \]  (6)

where \( Re_h \) is defined as

\[ \frac{D_t \varepsilon}{1 + 1.5(1 - \varepsilon) \frac{D_t}{d_p}} \]

The relation of Yasunishi relates the \( Sh \)-number directly to the rate of energy consumption in the fluidised bed:

\[ Sh = 0.13 \left( \frac{E_{diss}^{1/3} D_t^{4/3}}{m_t^{1/3} (\eta/\rho)} \right)^{3/4} Sc^{1/3} \]  (7)

An expression for the energy dissipated in fluidised beds is given in equation (8) below. The relation of Yasunishi holds for liquid-solid, gas-liquid and three phase systems.

**Energy dissipation**

We want to compare the effect of fluidised particles on mass transfer with empty tube cross flow operation. Therefore it is best to relate mass transfer coefficients in both systems to the energy input that is required. The energy
dissipation due to the presence of the fluidised particles is equal to:

$$E_{\text{diss,fb}} = \frac{\pi}{4} D_h^2 v_{sf} g H (1 - \varepsilon)(\rho_s - \rho_l)$$  \hspace{1cm} (8)

Besides this dissipation, energy is also dissipated due to friction with the wall; this amount is equal to:

$$E_{\text{diss,w}} = \frac{\pi}{4} D_h^2 v_{sf} \Delta P_w$$ \hspace{1cm} \text{with} \hspace{0.5cm} \Delta P = 4f \frac{1}{2} \rho v_{sf}^2 \frac{H}{D_h}$$  \hspace{1cm} (9)

The friction factor $f$ can be calculated with the equation of Blasius (Bird 1960), assuming that the tube has a smooth surface. The friction loss at the wall is normally small compared to the friction in the bed of particles. With the above equation, the energy dissipation in empty tube cross flow filtration can also be calculated.

**Fluidisation velocity**

The terminal falling velocity, i.e. the falling velocity of a single sphere in an unbounded liquid can be obtained from the force balance:

$$\frac{\pi}{6} d_p^3 (\rho_s - \rho_l)g = C_D \frac{\pi}{4} d_p^2 \frac{1}{2} \rho_l v_{t,\infty}^2$$  \hspace{1cm} (10)

The friction factor $C_D$ is a function of the particle Reynolds number at the terminal falling velocity, $Re_t$. $C_D$ is equal to 24/$Re_t$ for $Re_t < 1$ and equal to 0.43 for $Re_t > 1000$. For intermediate Reynolds numbers Lali et al. (Lali 1989) give the following relation:

$$C_D = \frac{24}{Re_{t,\infty}} \left(1 + 0.15 Re_{t,\infty}^{0.687}\right)$$  \hspace{1cm} (11)
If the sphere falls down a tube, the falling velocity is reduced by the presence of the tube wall. The correction of Richardson and Zaki can be applied (Richardson 1954):

\[ v_t = v_{t,\infty} 10^{-d_p/D_t} \]  \hspace{1cm} (12)

The dependency of bed porosity on superficial velocity can be described by the relation:

\[ \varepsilon^n = \frac{v_{sf}}{v_t} \]  \hspace{1cm} (13)

For the exponent, \( n \), the relation of Garside and Al Dibouni (Garside 1977) is used:

\[ \frac{5.1 - n}{n - 2.7} = 0.1Re_t^{0.9} \]  \hspace{1cm} (14)

**Experimental**

**Materials**

Polyethylene glycol (Sigma) was used with an average mass of 10000 Da (PEG10000). Solutions of 2% were made in demineralized water. Diffusivity and viscosity data were obtained from literature. The diffusion coefficient is \( 7.9 \times 10^{-11} \text{ m}^2/\text{s} \) (Pradanos 1992). The viscosity was estimated from the following relation:

\[ \frac{\eta_{sp}}{C} = [\eta] + 0.33[\eta]^2 C, \quad \eta_{sp} = \frac{\eta - \eta_{water}}{\eta_{water}} \]  \hspace{1cm} (15)

In above equation \( \eta_{sp} \) is called the specific viscosity. The intrinsic viscosity \([\eta]\) was estimated from (Amu 1983):
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\[ [\eta] = 6.04 \cdot 10^{-5} M_n^{0.9} \] (16)

Albumin solutions were prepared by dissolving 10 g/L BSA Fraction V (Boehringer Mannheim) in 0.15 M NaCl (Merck) solution. The pH was adjusted to pH 7.0 with NaOH (Merck). The viscosity of BSA solutions was determined with the following relation, given by Kozinski and Lightfoot (Kozinsky 1972):

\[ \eta = \frac{10^{-3}}{1.11 - 0.00542C + 6.71 \cdot 10^{-6} C^2} \] (17)

The diffusivity of BSA at infinite dilution was given by Fair et al. (Fair 1978) to be \( 6.1 \times 10^{-11} \) m²/s at 25 °C. The coefficients for Na⁺ and Cl⁻ are equal to \( 1.33 \times 10^{-9} \) and \( 2.03 \times 10^{-9} \) m²/s respectively (Chapman 1967).

Four different kinds of particles were used for fluidisation. Glass beads used were obtained from P.M. Tamson B.V. (Zoetermeer, the Netherlands) with mean particle diameters of 0.46 and 1.04 mm (\( \rho_s = 2900 \) kg/m³). Stainless steel beads with diameters of 1.0 and 2.0 mm (\( \rho_s = 7800 \) kg/m³) were purchased from INA Naaldlager Mij B.V. (Barneveld, Netherlands).

**Equipment and measurements**

Experiments were carried out in a configuration as shown in figure 1. Tubular polysulfone membranes were used of type Stork Wafilin WFBX0121, with an inner diameter of 14.4 mm and 1.75 m length and a NMWC of 10 kD. Solutions were fed to the membrane module using a Jabsco S2 gear Lobe pump and a Verder 2035 gear pump. In experiments with a fluidised bed, the particles were added as soon as the permeate flow was stabilised. The feed velocity was adjusted such that the particles were visible just above and below the membrane module to ensure that the membrane tube is completely filled with a fluidised bed. Pressures on the feed and concentrate side were measured using Econosto pressure gauges. The pressure at the permeate side was assumed to be
During experiments permeate fluxes were measured gravimetrically. The permeate solution was fed back to the feed solution periodically to keep feed concentrations constant. Concentrations of PEG10000
or BSA in both concentrate and permeate streams were measured. Concentrations of PEG10000 were determined by measuring the refractive index of the solution, those of BSA by measuring UV adsorption at 280 nm. The rejection of BSA turned out to be always larger than 98% in all experiments, so 100% rejection was assumed. All experiments were carried out at 25ºC.

**Results**

*Velocity voidage relationship*

The relation between bed porosity and superficial velocity was measured for the different particles used in water at 25 ºC. The results are presented in fig. 2. It is seen that the experimental data can be well described by equation (13), with $n$ determined from eqn. (14) and $v_t$ from eqns. (10)-(12).

![Fig. 2.](image)

**Fig. 2.** Comparison between experimental data and theoretical relation between bed porosity and superficial velocity. (▲), glass 0.46 mm; (●), glass 1.0 mm; (▼), steel 1.0 mm; (□), steel 2.0 mm.

*Solutions of PEG10,000*

In figs. 3a-c results are presented of experiments without fluidised particles. At two cross flow velocities (0.29 and 1.0 m/s) the transmembrane pressure drop
(TMP) was varied. In fig. 3a the fluxes are plotted against the transmembrane pressure drop, in fig. 3b the observed rejection against the filtrate flux. In fig 3c the results are plotted according to eqn. (4), to obtain the true rejection of the membrane and the mass transfer coefficient. The flux was divided by $v^{0.8}$ to compare experiments at two different cross flow velocities.

![Graphs showing flux and rejection](image)

**Fig. 3.** Flux (a) and rejection (b) of PEG 10,000 in empty tube experiments at varying transmembrane pressure drop and cross flow velocities. Velocities are 1 m/s (▲) and 0.29 m/s (□).

**Fig. 3c.** $\ln\left(\frac{1-R_{obs}}{R_{obs}}\right)$ vs. the permeate flux, corrected for cross flow velocity. Velocities are 1 m/s (▲) and 0.29 m/s (□).

The graphs clearly show the effect of concentration polarisation; at increasing filtrate flux the rejection is decreased. At lower cross flow velocity, this effect
is more pronounced. Also, the flux is influenced by the cross flow velocity, another effect of concentration polarisation. From fig. 3c the value of $R_{true}$ was determined to be 0.84 by linear regression. The slope of the curve was found to be $2.24 \times 10^5$, which is slightly higher than predicted from the Dittus Boelter equation (5), $1.93 \times 10^5$.

![Fig. 4. Flux (a) and rejection (b) of PEG10,000 in the presence of fluidised steel particles of 2 mm compared to the empty tube experiment with a cross flow velocity of 0.29 m/s. (▼), $\varepsilon = 0.89$ ; (□), $\varepsilon = 0.95$ ; (●), no particles.](image)

![Fig. 5. Effect of particle size on flux (a) and rejection (b) of PEG10,000. Bed porosity is 0.89. (□), 1 mm steel particles ; (●), 2 mm steel particles.](image)
Experiments with fluidised particles are presented in figs. 4 and 5. In fig. 4 experiments with fluidised beds of 5% and 11% steel particles of 2 mm diameter are shown. The results are compared to an empty tube experiment with a cross flow velocity comparable to the superficial velocities in the fluidised bed experiments.

It is clearly shown that both flux and rejections are increased by the particles. Both effects are a result of better mixing of the feed liquid. This leads to lower concentrations of PEG at the membrane / liquid interface. Hereby the osmotic pressure loss is reduced, as is seen by the higher fluxes and the rejections are closer to the intrinsic values for the membrane. We see a small effect of particle concentration; the rejections with 11% particles are a little higher than with 5%, as are the fluxes.

The effect of particle size is shown in fig. 5. Neither fluxes nor rejections are influenced by particle size. The same degree of mixing is obtained. Plotting eqn. (4) yields empirical mass transfer coefficients for the different fluidised beds. This is depicted in fig. 6. The plots can all be drawn to the same intercept (i.e. $R_{true} = 0.84$), although the experiments with beds at higher concentration (11%) begin to deviate at low fluxes.

![Fig. 6. Ln((1-R_{obs})/R_{obs}) vs. the permeate flux, without and with fluidised steel particles. (□), $\varepsilon = 0.89$, $d_p = 1$ mm; (▲), $\varepsilon = 0.89$, $d_p = 2$ mm; (▼), $\varepsilon = 0.95$, $d_p = 2$ mm; (●), no particles.](image-url)
In Table 1 all the results obtained with PEG10000 are summarised. The experimentally determined mass transfer coefficients are compared with those predicted from relations (6) and (7). Also the rate of energy dissipation in one membrane tube is calculated. Application of fluidised particles results in higher mass transfer coefficients. If we compare mass transfer coefficients at equal feed velocity, we obtain improvements up to 5 times. They can also be compared at equal energy consumption rate. The results obtained with a bed of 11 % 1 mm steel particles can best be compared with the empty tube experiment at a cross flow of 1 m/s. At the same energy input, the mass transfer coefficient is increased by a factor 2. The mass transfer coefficients measured are in good agreement with the empirical relations of Smith and Yasunishi.

<table>
<thead>
<tr>
<th>$d_p$ (mm)</th>
<th>$\varepsilon$</th>
<th>$v_{cross}$ (m/s)</th>
<th>$E_{diss}$ (J/s)</th>
<th>$k_{exp}$ ($\mu$m/s)</th>
<th>$k_{calc}$ ($\mu$m/s)</th>
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<td>0.36</td>
<td>8.8</td>
<td>8.3 *; 6.9#</td>
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* calculated with the relation of Smith (8); # calculated with the relation of Yasunishi (9).

Experiments with protein solutions

In Fig. 7 the effect of fluid beds on fluxes is presented in concentration experiments. If we apply eqn. (1), with $C_p$ equal to zero (100% rejection of BSA) and assume the concentration at the wall to be fixed, the flux plotted against the concentration on a logarithmic scale should give a straight line. The point at which the flux is zero is around 240 g/L. According to the gel layer model this is the concentration at the membrane interface at which the protein starts to precipitate. This concentration is close to the value of 250 g/L as found by Van Oers (Van Oers 1994). However, van Oers also reported that it is possible to dissolve more than 400 g/L at pH 7.4. This has been confirmed in our laboratory.
The Maxwell-Stefan based model is an osmotic pressure model that allows the wall concentration to be varied. This wall concentration is calculated to be near to 440 g/L at fluxes close to zero in all cases, which fits well with the high solubility observed for BSA. Fig. 7 shows that the semi-logarithmic Maxwell-Stefan plots are roughly linear over the main part of the concentration range and resemble the classical concentration polarisation plot. At fluxes close to zero the plots bend, however. It is difficult to obtain the exact intercept of the X-axis (or the point of maximum concentration) from numerical calculations, but it is seen to be close to the value of the wall concentration as calculated.

We see that fluxes are increased by the presence of the fluid bed and that fluxes are higher at higher particle concentrations. Mass transfer coefficients were determined by comparing the Maxwell-Stefan model with the experimental results. They are presented in table 2. The value of the experimentally determined coefficients are somewhat, but not too much, lower than those predicted by eqn.(5) for the empty tube experiment and relations (6) and (7) for the fluid bed experiments. The observed flux increases can thus largely be explained by an increase in mass transfer coefficient. The main mechanism of flux enhancement is by augmentation of back diffusion.
In figs. 8a-d, the effect of the particles is shown at constant protein concentration. The small periodic flux decays that are shown in some plots are due to a slight increase in concentration of the feed solution in time (from 1 to 1.5%). Fluxes were restored after the permeate solution was fed back to the feed solution periodically. In table 2 the mass transfer coefficients are given as determined by applying the Maxwell-Stefan description. After a stationary flux was reached in an empty tube, fluidised particles were added and the flux increased within only a few minutes. This time scale indicates that flux increase is merely due to the effect of mixing up the boundary layer, and scouring effects do not play an important role. The absolute fluxes in the presence of
fluidised particles increase only slightly with increasing particle size and particle density. This is in agreement with the results obtained with PEG10000 (fig. 5), which showed that mass transfer is hardly affected by particle size. These results have important consequences, as the risk of membrane damage increases dramatically with increasing particle size or density. Besides that, the energy consumption in the system is also strongly increased. Therefore, the best particles are as small and as light as possible, as they can provide quite acceptable fluxes with a low energy consumption and low risk. With the use of the heaviest particles these risks come clear to the eye. The concentration polarisation layer of protein is unable to protect the membrane against the impacts of the 2 mm steel particles and permeate flux decreases in time.

Table 2. Mass transfer coefficients in experiments with BSA solutions

<table>
<thead>
<tr>
<th>$\rho_s$</th>
<th>$d_p$ (mm)</th>
<th>$\varepsilon$</th>
<th>$v_{cross}$ (m/s)</th>
<th>$E_{diss}$ (J/s)</th>
<th>$k_{exp}$ ($\mu$m/s)</th>
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<td>8.5* ; 8.1#</td>
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</table>

* calculated with the relation of Smith (8); # calculated with the relation of Yasunishi (9)

In the results presented, it was shown that by adding fluidised particles the fluxes are increased enormously compared to empty tube experiments at the same feed velocity, by up to a factor of 13. For practical considerations, it is interesting to compare the mass transfer coefficients obtained in fluid bed and empty tube systems at equal energy consumption rates. This is done in figs. 9. The experimental data are compared to predictions with the equation of Yasunishi (7) and Dittus and Boelter (5). The effect of particle size is
considered at constant bed porosity. In the calculations the particle size was varied from 0.5 to 2 mm.

From the mass transfer relations it is predicted that at the same energy consumption a fluid bed system is favoured over empty tube systems. This trend is also visible in the experimental results obtained with PEG solutions.
The experimental results of BSA are more difficult to compare, as the predicted values for the mass transfer tend to be higher than the experimentally determined values, both for the fluid bed and the empty tube experiments.

The advantage of fluidised particles is that mass transfer rates are obtained at lower energy consumption, as compared to conventional cross flow. Application of fluidised beds also has disadvantages; one is the risk of membrane damage (to be discussed in the next chapter), another one is the lower flexibility of the system. Therefore its use should be restricted only to systems in which energy dissipation is a problem. When highly viscous liquids are filtered, energy consumption can be quite high and then application of fluidised particles is favoured over high cross flow velocity.

![Figure 10](image)

**Fig. 10.** Calculation of the rate of energy consumption, needed to maintain $k$ at 3 µm/s in fluid bed and empty tube systems at increasing relative viscosity. Calculations were done for fluid beds of glass and steel particles. Porosities varied from 0.95 down to 0.6 at increasing relative viscosity. (○), glass, 0.46 mm ; (●), glass 1.0 mm ; (□), steel 1 mm ; (■), steel 2mm.

We have carried out some calculations on the effect of viscosity on mass transfer coefficients in both fluidised bed and cross flow systems. These results are presented in fig 10, in which the energy consumption is plotted against the viscosity of the solution, relative to water. The energy consumption is
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calculated to maintain a mass transfer coefficient of 3 µm/s \( (D = 7 \times 10^{-11} \text{ m}^2/\text{s}) \), based upon the relation of Yasunishi (7) and Dittus-Boelter (5). Calculations were carried out for the four different particles that were used before in this work. Bed porosities were varied from 0.95 to 0.6.

Fluidised beds seem to be especially attractive for liquids with higher viscosity. Then energy consumption becomes an important factor in the process. At a relative viscosity of 10 the energy consumption in a fluid bed system is predicted to be 5 times lower than in an empty tube system. So in processing viscous liquids application of fluidised particles as turbulence promoters is attractive, especially because the risk of membrane damage is reduced with increasing viscosity.

Conclusions

Both rejection and fluxes can be improved considerably by the use of fluidised particles. The fluxes and rejections obtained were at most a weak function of particle properties such as size and density. The best choice is to use particles as light and small as possible, as this minimises the energy consumption in the process and also the risk of membrane damage. The application of fluidised beds is especially attractive for viscous liquids. The main mechanism of flux improvement is the better mixing of the liquid in front of the membrane.

Symbols

\[ C \quad : \quad \text{concentration, kg/m}^3 \]

\[ CD \quad : \quad \text{drag coefficient} \]

\[ dp \quad : \quad \text{particle diameter, m} \]

\[ D \quad : \quad \text{diffusivity, m}^2/\text{s} \]

\[ Dt \quad : \quad \text{tube diameter, m} \]

\[ E \quad : \quad \text{Rate of energy consumption, J/s} \]

\[ f \quad : \quad \text{friction factor} \]

\[ g \quad : \quad \text{gravity constant, m/s}^2 \]

\[ H \quad : \quad \text{height of the fluidised bed} \]
Chapter 5

\( k \) : mass transfer coefficient, m/s  
\( J \) : volume flux through the membrane, m/s  
\( m \) : mass, kg  
\( M_n \) : number average molecular weight  
\( P \) : pressure, bar  
\( R \) : rejection, fraction of solute that is retained  
\( \text{TMP} \) : transmembrane pressure drop, bar  
\( v \) : velocity m/s  

\( \delta \) : membrane thickness, m  
\( \epsilon \) : porosity of the fluidised bed  
\( \eta \) : viscosity, Pa s  
\( \rho \) : density, kg/m³  

\( Re \) : Reynolds number in empty tube flow, \( \rho \cdot v_{\text{cross}} \cdot D_t / \eta \)  
\( Re_h \) : Reynolds number used in relation of Smith, eqn. (6), \( \rho \cdot (v_{sf} / \epsilon) \cdot d_h / \eta \)  
\( Re_t \) : Reynolds number at terminal velocity of a falling particle, \( \rho \cdot v_t \cdot d_p / \eta \)  
\( Sc \) : Schmidt number, \( \eta / \rho \cdot D \)  
\( Sh \) : Sherwood number, \( k \cdot D_t / D \)  

subscripts

\( b \) : bulk  
\( \text{calc} \) : calculated  
\( \text{cross} \) : cross flow  
\( \text{exp} \) : experimental  
\( h \) : hydraulic  
\( \text{diss} \) : dissipated  
\( \text{fb} \) : fluid bed  
\( l \) : liquid  
\( \text{obs} \) : observed  
\( p \) : permeate  
\( \text{true} \) : true rejection, intrinsic property of the membrane  
\( s \) : solid  
\( sf \) : superficial  
\( t \) : terminal
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\( w \): wall
\( \infty \): in unbound liquid

References


Meijer, J.A.M., 1984 Inhibition of calcium sulfate scale by a fluidized bed. PhD thesis University of Delft, the Netherlands.


Appendix

Appendix: $R_{true}$ as function of the Flux

In this appendix justification is given for the assumption of constant value for $R_{true}$ at the volume fluxes in the experimental work on PEG10000 presented here.

According to the hydrodynamic theory of membrane filtration, (Deen 1987, Opong 1991) the actual rejection can be related to both the volume flux and the ratio of solute diameter to pore diameter, $\lambda$:

$$ R_{true} = 1 - \frac{(1 - \lambda)^2 K_c \exp(Pe_m)}{(1 - \lambda)^2 K_c - 1 + \exp(Pe_m)} $$

The membrane Peclet number, $Pe_m$, is the ratio between diffusive and convective transport through the membrane and is given by:

$$ Pe_m = \left( \frac{K_c}{K_d} \right) \left( \frac{j_v}{\varepsilon} \frac{\delta}{D_\infty} \right) $$

In this equation $\varepsilon$ is the porosity of the membrane. The constants $K_c$ and $K_d$ are the convective and diffusive hindrance factors respectively and are related to the ratio of solute to pore diameter, $\lambda$. This relation is based upon simulations done by Bungay and Brenner (Bungay 1973) and translated into these hindrance factors by Deen (Deen 1987):

$$ K_c = \frac{(2 - (1 - \lambda)^2)K_s}{2K_t} $$

$$ K_d = \frac{6\pi}{K_t} $$
The coefficients, $K_s$ and $K_t$ are themselves a function of $\lambda$:

\[
\begin{pmatrix}
K_s \\
K_t
\end{pmatrix} = \frac{9}{4} \pi^2 \sqrt{2} (1-\lambda)^{-5/2} \left[ 1 + \sum_{n=1}^{2} \left( \begin{array}{c}
a_n \\
b_n
\end{array} \right) (1-\lambda)^n \right] + \sum_{n=0}^{4} \left( \begin{array}{c}
a_{n+3} \\
b_{n+3}
\end{array} \right) \lambda^n
\]

The coefficients $a_n$ and $b_n$ are given in table A1.

Table A1. Values of the coefficients needed in the approach of Bungay and Brenner

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n$</td>
<td>-1.2167</td>
<td>1.5336</td>
<td>-22.5083</td>
<td>-5.6117</td>
<td>-0.3363</td>
<td>-1.216</td>
<td>1.647</td>
</tr>
<tr>
<td>$b_n$</td>
<td>0.1167</td>
<td>-0.00442</td>
<td>4.0180</td>
<td>-3.9788</td>
<td>-1.9215</td>
<td>4.392</td>
<td>5.006</td>
</tr>
</tbody>
</table>

The true rejection at infinite Peclet number, i.e. independent volume flux, is given by:

\[ R_{true,\infty} = 1 - (1-\lambda)^2 K_c \]

Now the relation of $R_{true}$ to the volume flux can be determined for the present work with PEG10000. The solute diameter of PEG10000, as calculated with the Stokes-Einstein relation for diffusion coefficients, is equal to 6.2 nm. From the experimental data it can be estimated that the true rejection of the membrane at high fluxes should be around 0.8 - 0.9, which means that $\lambda$ is about 0.6-0.75 (the $\lambda$ corresponding to a $R_{true}$ value 0.84 is equal to 0.66). From these values a pore diameter can be calculated. In addition, the value of the ratio $\varepsilon/\delta_m$ needs to be determined to calculate the Peclet number as function of the volume flux, $J_v$. This is done from data on the pure water flux of the membrane. The latter was determined to be 60 L m$^{-2}$ hr$^{-1}$ bar$^{-1}$. If Poiseuille flow is assumed, the pure water flux is given by:

\[ J_c = \frac{\varepsilon}{\delta} \frac{d_p^2}{32} \frac{\Delta P}{\mu} \]
From above equation, the ratio $\frac{\varepsilon}{\delta_m}$ can be calculated, if $d_p$ and $J_c$ are known. Next, the $Pe$ number as a function of the volume flux and thus of the actual rejection as a function of the volume flux. This plot is shown for three different values of $\lambda$ in fig. A1. It can be seen that above a flux of 3 L m$^{-2}$ hr$^{-1}$ the $R_{true}$ is independent of the volume flux. In the present work on PEG10,000 the volume flux varies from 5 to 80 L m$^{-2}$ hr$^{-1}$. A constant $R_{true}$ can therefore safely be assumed.

**Fig. A1.** True rejection as a function of flux for different ratios of solute to pore diameter, $\lambda$. Above roughly 3 l m$^{-2}$ hr$^{-1}$ the rejection attains a constant value.