Chapter 6

Stochastic Model for Slugging Fluidization

6.1 Slugging Fluidized Beds

A stochastic model for particle transport in fluidized beds is proposed in this study. The mixing and segregation of binary particle mixtures in slugging fluidized beds is used as a case study. This model is based on a description of the particle displacement in the system resulting from the formation and rise of a single slug. It disregards any interaction between successive slugs in the bed. Using this approach, the entire process can be broken up into two distinct phases, slug formation and slug rise. The cumulative effect of consecutive slugs is found by superposition. The model is validated with experimental data for the particle mixing pattern in time. The predicted axial particle concentration profiles from the stochastic model are compared graphically with the experimental results. This simple and elegant modeling approach is shown to be successful even for a complex process like the one considered.

Slugging fluidization normally occurs in beds of high aspect ratio (>>1, Noordergraaf et al., 1987), often with a small diameter with respect to the particle size. A slug is simply a bubble whose diameter is nearly equal to that of the bed itself. Even so, slugs influence the bed material in a different way than bubbles do.

Some research literature is dedicated to slug type fluidization, though it is still scarce. Noordergraaf et al. (1987) studied slugging fluidized beds containing large particles in order to begin to explain the fluidization behavior of fluidized bed combustors. Abanades et al. (1998) also examined slugging fluidization in a British Coal pressurized fluidized bed gasifier.

Slugs can be generally categorized into two types: axisymmetric slugs and wall slugs as shown in Fig 6.1. The first category can again be split into two kinds: round-nosed and square-nosed slugs. Axisymmetric slugs usually occur in beds that contain large particles [Geldart B, D]. The square-nosed slugs are often found in the high velocity beds. Square-nosed slugs are the focus of our study.
6.2 Mixing and Segregation in Slugging Fluidized Beds

Although fluidization is known for its good mixing characteristics, the solid mixing is often incomplete when particles of different properties are present in the bed. Binary mixtures have often been used for studies of mixing in fluidized beds. Binary mixtures are usually referred to as ‘equal-density mixtures’ if the two fractions differ in size only, and ‘different-density mixtures’ if the density or both properties are different (Hoffmann, 1991; 1993). However, the difference in density tends to be the dominant driving force for segregation, and is our interest here. The development of a model for mixing and segregation in bubbling fluidized beds was pioneered by Gibilaro and Rowe (1974), who considered that solids mixing and segregation are both related to the motion of fluidization bubbles. These were the concepts used by Dehling et al. (1999) and Hoffmann et al. (1998) for their stochastic model for particle transport in continuous beds.

This study is focused on the mixing and segregation caused by slugs. The mechanistic concepts behind the model were suggested by Abanades et al. (1994) and are described in a following section.

The objective of this work is to formulate a stochastic model for mixing and segregation of a binary mixture of different density particles in slugging beds in order to predict the solid concentration profiles directly from the operating conditions and particle properties.
6.3 Modeling

6.3.1 Description of the Model

In this section we describe the particle transport processes involved in the model. We consider a bed of unit cross-sectional area. So that we can equate superficial velocities and volumetric flows. The inspiration for the model came from earlier research in segregation of limestone-coal mixtures in slugging fluidized beds (Abanades et al., 1994). The effect of formation and rise of a single slug are modeled first. The displacement of particles due to several consecutive slugs, even if more than one is present in the bed at any one time, is assumed to be a superposition of the effects of the individual slugs. On this basis, a model for the displacement of the bulk material in time is formulated, whereafter the segregation of the particle fraction tending to sink is considered in a subsequent section.

A fluidized bed with height at rest \( h_b \), is considered and can be seen schematically in Fig.6.2. Slugs are formed at some height \( h_o \) in the bed by the flow of fluidization bubbles in which the flow regime below \( h_o \) is bubbling fluidization. They grow in size until they have reached a certain height \( h_s \) at which time they detach and start rising as a whole towards the top of the bed while a new slug forms. When the nose of the slug has reached the top of the bed, the slug collapses. In accordance with our approach outlined above, the effect of a single slug is modeled, disregarding any effect of other slugs in the bed.

We can thus distinguish two distinct phases for the process, as follows:

**Slug formation:** The slug forms at a height \( h_o \) above the bottom of the bed and grows in vertical size at a speed \( v \) from 0 to \( h_s \). In this phase, all particles above the slug are pushed upwards, and as a result the bed height also rises above its original level \( h_b \). Assuming that the slugs nearly fill up the entire diameter of the reactor, the bed also rises at the same speed \( v \) to its maximum height \( h_b+h_s \) (see Figure 6.2 iii).

**Slug rise:** When the slug has reached its final height, it starts rising towards the top of the bed. In this phase the height of the slug, and consequently also the height of the bed, remain constant. The speed of rise is assumed to be \( v \) again, although in principle the slug formation speed and the slug rise speed can be different. At the end of the slug rise phase, the nose of the slug has reached the top of the bed. At this point the slug ceases to exist, and the bed collapses back to its original height.

As a result of slug formation and rise particles in the bed are displaced. To simplify the model, the horizontal component of particle displacement is neglected and only
vertical displacement is considered. This model incorporates two different kinds of displacement (see figure 6.2):

Particles above the slug are pushed upwards. In the slug formation phase, this happens at speed \( v \). In the slug rise phase, the speed is smaller than \( v \), as a result of material transport from above to below the slug. In this phase, the speed of rise becomes equal to \( v \) at the apex of the slug and zero at the top of the bed. In the slug rise phase, particles above the slug can fall through the annulus around the slug and then be deposited directly underneath the slug. The probability of this event is likely to depend on the vertical distance of the particle from the nose of the slug. In this initial formulation, it is assumed to be an increasing function of the distance from the nose. More experimental work will shed more light on this aspect.

The aim is to model the overall effect of the formation and rise of one slug. In order to achieve this, the process is studied at a shorter time scale where physical intuition can guide modeling. The total effect of one slug can then be found by aggregation over the entire time period.

During the slug formation phase all the bed particles are as mentioned, assumed to rise at a velocity \( v \) in front of the slug. We now proceed to specify the probabilities and directions of possible displacements of particles above the slug nose in the slug rise phase. Denote by \( h = h(t) \) the vertical position of the slug nose.

During the slug rise phase the particles in front of the slug, i.e. with \( x \) in the range \( h \leq x \leq h_s + h_t \) either rise further up in the bed or move into the annular region around the slug and are then deposited directly underneath the slug. The rate at which the latter happens is a function \( \lambda_h(x) \) of the vertical location of the particle.
in the bed. We assume that $\lambda_h(x)$ is an increasing function of $x$, i.e., more of the particles that deposit underneath the slug come from the top of the bed than from closer to the slug nose. We envisage the flow pattern sketched in Fig. 6.3, i.e., intensity of transition (probability per unit of time).

![Figure 6.3 A sketch indicating the movement of particles in front of slug nose, more particles that deposit underneath the slug come from the top of the bed than from closer to the slug nose. (Thickness of arrows indicates greater particle flux)](image)

The total flow around the slug must equal the speed at which the slug rises; therefore we obtain the continuity condition

$$\int_{h_0}^{h_0+h} \lambda_h(x)dx = v, \quad (6.1)$$

for any $h$ in the interval $h_0 \leq x \leq h_0 + h$. In the following, it is assumed that $\lambda_h$ follows a power law, that is $\lambda_h(x) = c(x-h)^r$ for some $r \geq 0$, where the constant $c$ is determined by the continuity condition (8.1). Thus this yields:

$$\lambda_h(x) = \frac{v(r+1)}{(h_0+h - h)^{r+1}}(x-h)^r, \quad h_0 \leq x \leq h_0 + h.$$  

The exponent $r$ still remains to be determined, either from experimental observations or from considerations concerning the flow of particles from the region above the slug via the annulus to the bottom part of the bed (see figure 6.3).

Particles above the slug that do not flow into the annulus rise at a speed $v_h(x)$, which depends on the vertical location. It is computed as the difference
between the speed of the slug nose and the flow of particles into the annulus below height $x$, that is $v_h(x) = v - \int_x^\infty \lambda_h(u)du$.

![Graph showing rate of flow into the annulus region as a function of height above the nose of the slug-plotted for 3 different values of the index $r$.]

Using (6.1), this can also be written as, $v_h(x) = \int_x^\infty \lambda_h(u)du$, which can be interpreted as an equality between the upward flow of the rising particles and the downward flow in the annulus. The position of the slug is defined by the position of its nose. Based on the above assumptions, this is a linear function of time, defined for $0 \leq t \leq T$ where $T$ is the maximum time equal to $\frac{h_b + h_s - h_0}{v}$ and given by

$$h(t) = h_0 + vt$$  \hspace{1cm} (6.2)

(see figure 6.5).
6.3.2 Discrete Markov Chain Model

The model outlined in the previous section defines a continuous-time stochastic process, more specifically a convection process with jumps. The concept of Markov chains is the main vehicle to formulate the model mathematically. The essence of the Markov chain model is the fact that the probability distribution of the future position of a particle only depends on the present position and not on the past. Thus the model is completely determined by the transition probabilities \( P(X_{n+1} = j \mid X_n = i) \), for any elements \( i, j \) of the state space.

In this section, we describe a discrete Markov chain approximation to the continuous process described in the previous section, which can then be solved numerically. To do so, we divide the bed into horizontal slices of width \( \Delta \) and discretize time into steps \( \varepsilon \) in such a way that \( \Delta / \varepsilon = v \). In this way, one step per unit of time corresponds to the speed \( v \). To simplify the model, we assume moreover that slug formation starts right above the distributor plate, i.e., that \( h_0 = 0 \).

We define \( H \) as \( h_b + h_s \), the maximum height of the bed and assume that both \( h_b \) and \( h_s \) are integer multiples of \( \Delta \), that is \( h_s = i_s \Delta \) and \( h_b = i_b \Delta \). The horizontal slices (cells) are numbered by \( i = 1, \ldots, I = i_s + i_b \). The discrete Markov chain model will have both a state space \( \{1, \ldots, I\} \) and transition probabilities \( p_j(n) \), which depend on time and are defined below. \( p_j(n) \) gives the probability that a particle is in state \( j \) at time \( n \), given that it was in state \( i \) at time \( n-1 \).
At time $n = 0$, slug formation starts with the slug occupying one extra cell after each transition. Hence at time steps $n = 1$ to $i_s$, the slug occupies the cells $i = 1, \ldots, n$. In this period, the transition probabilities are:

$$
 p_y(n) = \begin{cases} 
 1 & \text{if } j = i + 1 \\
 0 & \text{otherwise}
\end{cases}
$$

for $n \leq i \leq i_b + n - 1$. For $i \leq n - 1$ and $i \geq i_b + n$, we define $p_u(n) = 1$ and $p_y(n) = 0$ for $i \neq j$.

At time $n = i_s$, the slug has reached its maximum height and then enters the rise phase. When $n = i_s + 1$ to $I$ the slug just before the $n$-th transition occupies the cells $i = n-i_s, \ldots, n-1$. For particles below the slug, i.e., those in cells with index $i < n-i_s$, the transition probabilities are trivial in the sense that $p_u(n) = 1$ and $p_y(n) = 0$ if $i \neq j$. For particles in cells above the slug, that is for $i \geq n$, there are three possible transitions (see Figure 6.6):

- move to what will be the first cell below the slug after the time step, i.e., to $j = n-i_s$,
- move one cell up, i.e., to $j = i + 1$,
- stay in the same cell.
The probabilities for these three transitions are

\[
p_{i,i-1}(n) = \epsilon \frac{1}{\Delta} \int_{(i-1)\Delta}^{i\Delta} \lambda_{(n-1)\Delta}(x)dx = \frac{(i\Delta - h)^{r+1} - ((i-1)\Delta - h)^{r+1}}{(H - h)^{r+1}} \quad (6.3)
\]

\[
p_{i,i+1}(n) = \epsilon \frac{1}{\Delta} \int_{i\Delta}^{(i+1)\Delta} \lambda_{(n-1)\Delta}(x)dx = \frac{(H - h)^{r+1} - (i\Delta - h)^{r+1}}{(H - h)^{r+1}} \quad (6.4)
\]

\[
p_{i,j}(n) = 1 - p_{i,i+1} - p_{i,i-1} = \epsilon \frac{1}{\Delta} \int_{(i-1)\Delta}^{(i-1)\Delta} \lambda_{(n-1)\Delta}(x)dx = \frac{((i-1)\Delta - h)^{r+1}}{(H - h)^{r+1}} \quad (6.5)
\]

Transition probabilities from the cells that are occupied by the slug are irrelevant because these cannot contain particles. For the sake of completeness, we define in this case \( p_{i,i}(n) = 1 \) and for \( p_{i,j}(n) = 0 \) if \( i \neq j \).
In this way we have defined the transition probabilities \( p_{ij}(n) \), \( 1 \leq i, j \leq I \) for \( n = 1, \ldots, I \). Thus, the transition matrix \( P(n) = (p_{ij}(n))_{1\leq i,j \leq I} \) is defined. If the initial distribution is given by the vector \( p = (p_1, \ldots, p_I) \) the distribution at time \( n \) equals \( p(n) = p \cdot P(1) \cdots P(n) \).

After a full period of one slug formation and rise, the total effect is captured by the transition matrix \( Q \), defined as \( Q = P(1) \cdots P(I) \), and the probability distribution for the particle, which is also the distribution of a pulse of an infinite number of marked particles after one slug is given by \( p' = pQ \).

### 6.3.3 Modeling of Segregation Effects

What we have modeled so far is the particle behavior in a bed containing particles of uniform properties. A simple segregation process is introduced based on the idea of “jetsam” particles (the heavier and/or larger particles that tend to sink in the mixture) sinking when slugs disturb the system. Segregation of jetsam and flotsam particles in a slugging fluidized bed manifests itself as a difference in the downward drifts. The mass balance equation only holds on average, with

\[
\int_{h_r}^{h_s} \lambda_s(x)dx > v \text{ for jetsam particles and } < v \text{ for flotsam particles.}
\]

The segregation effect is simulated by increasing the probability of a down-transition for the jetsam particles, giving an extra downward drift. When modeling the dynamics of the jetsam particles simple way to account for their larger downward flow is to modify the transition probabilities (6.3), (6.4), (6.5) as follows:

\[
\bar{p}_{i,n-i} = \alpha + (1-\alpha)p_{i,n-i} \quad (6.6)
\]

\[
\bar{p}_{i,i+1} = (1-\alpha)p_{i,i+1} \quad (6.7)
\]

\[
\bar{p}_{i,i} = (1-\alpha)p_{i,i} \quad (6.8)
\]

where \( 0 \leq \alpha \leq 1 \) is a constant defining the strength of the segregation effects.
6.4 Numerical Simulation and Comparison with Experiments

Details of the experimental technique for the main batch of data with which we are comparing the model are given in Abanades and Atarés (1998). The technique used for determining the concentration of jetsam particles is analysis of images on videotape recorded during mixing experiments in deep fluidized beds of coarse particles. Properties of the solids used, which were pigment agglomerates, are shown in Table 6.1. The experiments were carried out in a column of 15 cm diameter and were performed at 3 superficial gas velocities, 1.57, 1.69 and 1.83 m/s, and 2 initial bed heights, 0.53 and 0.73 m ($h_b/D_{bed} = 3.53$ and $4.87$, respectively). The tracer volume fraction was varied between 0.2 and 0.35. In a typical experiment, a layer of white jetsam particles was initially arranged in the bottom of a bed of red particles.

Table 6.1 Properties of the solids used in the experiments (Abanades et al., 1998).

<table>
<thead>
<tr>
<th></th>
<th>$d_p$(mm)</th>
<th>$\rho_s$(kg/m$^3$)</th>
<th>$U_{mf}$(m/s)</th>
<th>$\phi$</th>
<th>$\varepsilon_{mf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>white</td>
<td>3.2</td>
<td>1554</td>
<td>1.150</td>
<td>0.98</td>
<td>0.391</td>
</tr>
<tr>
<td>red</td>
<td>3.2</td>
<td>1105</td>
<td>0.913</td>
<td>1.00</td>
<td>0.331</td>
</tr>
</tbody>
</table>

For the modeling of these experiments it was, as mentioned, assumed that slugging begins on the distributor plate. The slug frequency, $f$, was calculated from the correlation proposed by Noordergraaf et al. (1987):

$$f = 0.32 \frac{U^{-0.15}}{h_{mf}}$$

(6.9)

The physical properties of the slug are based on the axisymmetrical slug type. The height of a stable slug, $h_s$, is calculated by (Davidson et al., 1985):

$$\frac{h_s}{D_{bed}} = 0.495 \left( \frac{h_s}{D_{bed}} \right)^{1.2} \left( 1 - \frac{u-u_{mf}}{0.35(gD_{bed})^{0.5}} \right) + 0.061 - \left( \frac{1.939(u-u_{mf})}{0.35(gD_{bed})^{0.5}} \right) = 0$$

(6.10)

The experimental results and the results from our model are not in the same form. The experimental results are reported as volume of tracer in cell $i$ as the fraction of the total volume of material in the cell, we call this $V_{fr,i}$. The model calculations, on
the other hand, give $p_i$: the probability that the particle is in cell $i$, which can also be seen as the volume of tracer in cell $i$ as a fraction of the total volume of tracer in the whole bed. To compare model calculations with experimental results, the results from the model were converted to the same form as the experimental results. If the bed is discretized in $N$ cells of equal volume, the volume fraction of each cell is $1/N$. If the total volume of tracer in the experimental bed is $V_{tr}$, then $p_i$ can be converted to $V_{fr,i}$ as follows:

$$V_{fr,i} = \frac{V_{fr} p_i}{\left(\frac{1}{N}\right)}$$

In this study, we do not yet account for the effect of the jetsam particles filling up the cell. This would impose a maximum value for $p_i$ of

$$p_i(V_{fr,i} = 1) = p_{i,max} = \frac{1}{NV_{tr}}.$$

However, to take full account of these effects in the model, the change of the segregation behavior of the jetsam particles in response to the change in the local jetsam concentration would also have to be accounted for.

The two parameters $r$ and $\alpha$ in the model are functions of the experimental conditions and particle properties. When sufficient information is available, these can be quantified directly. In this study we use them as adjustable parameters.

In Figure 6.7, the plots of distribution of jetsam volume fraction with time step predicted by the model are compared with experimental observations. The profiles shown are averages during the period where the profiles can be said to have reached a steady state.

Figure 6.7 contains plots of distribution of jetsam volume fraction as a function of height in the bed for varying initial bed height, $h_b$, and superficial gas velocity, $U$. A, B and C show results for the same $h_b$ with increasing $U$, 1.57, 1.69 and 1.83 m/s, respectively. B and D have the same $u$ (1.69 m/s) with different $h_b$, 0.53 and 0.73 m, respectively. In our model, we keep both the segregation parameter, $\alpha$ and the rate of removal parameter, $r$ constant in all cases. We found that agreement is acceptable in all cases with values of $\alpha$ and $r$ equal to 0.17 and 1, respectively.
This means that both experimental results and model thus confirm that the intuitive idea of the degree of mixing and segregation caused by slugs is insensitive to the bed height and the superficial gas velocity.

Figure 6.7 shows good agreement between the predicted and experimental data. There are two very weak effects in the experimental data, which are not reflected in the model: 1) the particle mixing increases slightly with an increase in $u$ and 2) the particle mixing increases with a decrease in $h_b$. In our model at present, we have no effect of $U$. As $U$ increases, the slug size also increases. One possibility to improve the agreement between model and experiment further in this respect is to introduce a diffusion, which increases slightly with slug size. To make the model more consistent with the physics, we will also have to include the effect of the locally changing environment (change in jetsam concentration) on local segregation distance. When comparing the profiles in detail, it can be seen that the tail of the experimental profiles tends to be constant indicating a region of constant concentration like in bubbling beds while our model is still decreasing with a very slow rate.

In the experiment corresponding to $h_b = 0.53$ m and $U = 1.83$ m/s (Fig. 6.7C), the experimental profile is quite different from the other experiments. The first cell is not the one that has the most volume fraction of tracer. Whether this is a real effect of high fluidization velocity remains to be seen. The model does not account for such a feature at present.

To validate our stochastic model further, it was compared with extra series of experimental data from a completely different system. The details of the experimental set-up for this work can be found in Abanades et al. (1994). The system is a slugging bed containing coal and limestone. Coal acts as a flotsam ($\rho_f = 1200$ kg/m$^3$, $d_f = +1.0$-$1.4$ mm) and limestone as jetsam ($\rho_j = 2600$ kg/m$^3$, $d_j = +1.7$-$2.0$ mm) with bed aspect ratio of 5.77. Figure 8.8 shows the comparisons of the experimental data with model prediction. The superficial velocity for experiment A and B was, $U = 0.8$ m/s and for C, $U = 0.95$ m/s. The experiments were performed at different jetsam concentrations, $x_0 = 0.05$, 0.1 and 0.2, for A, B and C, respectively. Good agreement is seen also here.

Even though the experimental results and model prediction agree well, discrepancies can be seen in the bottom of the bed in both systems. An increasing fluidization velocity affects the concentration of particles at the bottom of the bed the most and only slightly in the higher part of the bed as seen in both systems. Our model does not reflect this. This can be explained since our model has not taken the bubble-forming-a-slug region into account. We assume that the slug forms right above the distributor while in the actual system, as mentioned before, it is likely...
that a region of bubbling fluidization exists below the slug forming region. The slugs are formed from coalescence of small bubbles and this process is affected by a change in the fluidization velocity.

Regarding the segregation effect in Figure 6.8, the coal-limestone system, we used $\alpha = 0.11$ instead of 0.17. Since this system is completely different from the other one, a difference in segregation can be expected. In our model a lower the value of $\alpha$ reflects a stronger segregation. This agrees qualitatively with experiment, as we show by means of the ‘segregation distance’ for bubbling beds.

From the equation for the “segregation distance” (a dimensionless measure of the distance jetsam particles segregate due to the passage of one fluidization bubble) in bubbling fluidized beds proposed by Taminoto et al. (1981) and modified by Hoffmann et al. (1991) to:

$$ Y = 0.8 \cdot \left( \frac{d_j}{d_{\text{bulk}}} \right)^{0.33} \frac{\rho_j}{\rho_{\text{bulk}}} - 0.8 \tag{6.11} $$

![Figure 6.7 Plots of distribution of jetsam volume fraction as a function of height of the bed for varying initial bed height, $h_b$, and superficial gas velocity, $u$ with $\alpha = 0.17$ and $r = 1$. Black dots are plots of experimental data. White dots are plots of results from our model.](image)
We find that at the same jetsam concentration, \( x_0 = 0.2 \), for the pigment agglomerate system \( Y = 0.24 \) while the coal limestone system has a segregation distance, \( Y = 0.74 \). Even though our system is not a bubbling bed this confirms that the pigment agglomerate system should exhibit less segregation than the coal-limestone system.

![Figure 6.8 Plots of comparison between experimental and predicted segregation profiles at different limestone concentrations, \( x_0 \), with \( \alpha = 0.11 \) and \( r = 1 \). Black dots are plots of experimental data. White dots are plots of results from our model. \( h_{mf} = 0.75, d_j = +1.7-2.0 \text{ mm}, d_l = +1.0-1.4 \text{ mm}, u = 0.8 \text{ m/s} \) (\( u = 0.95 \text{ m/s} \) in key \( x_0 = 20\% \)).](image)

Figures 6.7 and 6.8 represent different types of particles and different bed geometries. Both agree very well with our model prediction apart from the velocity effect in the lowest part of the bed mentioned above. This confirms that our stochastic model is suitable for slugging fluidized beds in general.

### 6.5 Conclusions

The agreement seen in Figure 6.7 and 6.8 show that our stochastic model is generally capable of predicting the profile of segregation in slugging fluidized bed. Further study is required to develop this model by relating the segregation to the physical properties of the particles and improve the agreement with experiment further as discussed in the previous section. One assumption of this current model is that the jetsam is infinitely dilute so that:
all the jetsam particles can collect in one cell (i.e. no interaction between particles in the sense that they can fill a cell).

- the segregation rate is not influenced by the presence of other jetsam particles (the local environment is not influenced by the concentration of jetsam particles).

The stochastic approach has successfully been used to formulate a model for a system so complex that modeling with the deterministic approach would have been difficult, if not impossible. This stochastic model for segregation in slugging fluidized bed has proved to be a simple and intuitive model. It has a further important attraction in significantly reducing calculation time when compared to the alternatives.

### 6.6 Notation

\[\begin{align*}
d_f &= \text{diameter of flotsam particle} \\
d_j &= \text{diameter of jetsam particle} \\
D &= \text{diffusion coefficient} \\
D_B &= \text{diameter of bubble} \\
D_{\text{bed}} &= \text{diameter of bed} \\
f &= \text{slug frequency} \\
g &= \text{gravitational acceleration} \\
h_o &= \text{height of bed at slug formation} \\
h_b &= \text{height of initial bed} \\
h_{\text{mf}} &= \text{height of bed at minimum fluidization conditions} \\
h_s &= \text{height of stable slug} \\
H &= \text{maximum height of bed} \\
i,j &= \text{indices denoting the number of cell} \\
I &= \text{number of cells internal to the discretized bed} \\
n &= \text{index denoting number of time step} \\
r &= \text{rate of } \lambda_d(x) \text{ function} \\
p &= \text{probability} \\
p &= \text{probability vector} \\
P &= \text{transition probability matrix} \\
Q &= \text{transition probability matrix constant in time} \\
t &= \text{time} \\
T &= \text{Maximum time from the formation to the collapse of a slug} \\
U &= \text{superficial gas velocity} \\
U_{\text{mf}} &= \text{minimum fluidization velocity} \\
v &= \text{slug velocity} \end{align*}\]
$V_{fr,i}$ = volume of tracer in cell $I$

$V_T$ = total volume of tracer

$x, z$ = position of the particle

$X$ = random variable for the particle’s position

$x_0$ = average weight fraction of jetsam

$Y$ = segregation distance

**Greek symbols:**

$\alpha$ = a constant defining the strength of the segregation effects

$\varepsilon$ = length of time interval

$\lambda_i$ = probability that particle move one cell upwards

$\mu_i$ = probability that particle move one cell downwards

$\pi$ = probability density

$\Delta$ = cell width

$\rho_f$ = density of flotsam particle

$\rho_j$ = density of jetsam particle

$\rho_s$ = density of solid

$\phi$ = sphericity factor

$\varepsilon_{mf}$ = voidage at minimum fluidization condition

### 6.7 References


