Observation of Photon Polarization in the $b \to s\gamma$ Transition

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This Letter presents a study of the flavor-changing neutral current radiative $B^\pm \to K^\pm \pi^\mp \pi^\pm \gamma$ decays performed using data collected in proton-proton collisions with the LHCb detector at 7 and 8 TeV center-of-mass energies. In this sample, corresponding to an integrated luminosity of $1.1\,\text{fb}^{-1}$, near 14,000 signal events are reconstructed and selected, containing all possible intermediate resonances with a $K^\pm \pi^\mp \pi^\pm$ final state in the $[1.1, 1.9]\,\text{GeV}/c^2$ mass range. The distribution of the angle of the photon direction with respect to the plane defined by the final-state hadrons in their rest frame is studied in intervals of $K^\pm \pi^\mp \pi^\pm$ mass and the asymmetry between the number of signal events found on each side of the plane is obtained. The first direct observation of the photon polarization in the $b \to s\gamma$ transition is reported with a significance of $5.2\sigma$.

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The standard model (SM) predicts that the photon emitted from the electroweak penguin loop in $b \to s\gamma$ transitions is predominantly left-handed, since the recoiling $s$ quark that couples to a $W$ boson is left-handed. This implies maximal parity violation up to small corrections of the order $m_s/m_b$. While the measured inclusive $b \to s\gamma$ rate [1] agrees with the SM calculations, no direct evidence exists for a nonzero photon polarization in this type of decay. Several extensions of the SM [2], compatible with all current measurements, predict that the photon acquires a significant right-handed component, in particular, due to the exchange of a heavy fermion in the penguin loop [3].

This Letter presents a study of the radiative decay $B^+ \to K^+ \pi^- \pi^+ \gamma$, previously observed at the $B$ factories with a measured branching fraction of $(27.6 \pm 2.2) \times 10^{-6}$ [1,4,5]. The inclusion of charge-conjugate processes is implied throughout. Information about the photon polarization is obtained from the angular distribution of the photon direction with respect to the normal to the plane defined by the momenta of the three final-state hadrons in their center-of-mass frame. The shape of this distribution, including the up-down asymmetry between the number of events with the photon on either side of the plane, is determined. This investigation is conceptually similar to the historical experiment that discovered parity violation by measuring the up-down asymmetry of the direction of a particle emitted in a weak decay with respect to an axial vector [6]. In $B^+ \to K^+ \pi^- \pi^+ \gamma$ decays, the up-down asymmetry is proportional to the photon polarization $\lambda_\gamma$ [7,8] and therefore measuring a value different from zero corresponds to demonstrating that the photon is polarized. The currently limited knowledge of the structure of the $K^+ \pi^- \pi^+$ mass spectrum, which includes interfering kaon resonances, prevents the translation of a measured asymmetry into an actual value for $\lambda_\gamma$.

The differential $B^+ \to K^+ \pi^- \pi^+ \gamma$ decay rate can be described in terms of $\theta$, defined in the rest frame of the final state hadrons as the angle between the direction opposite to the photon momentum $\vec{p}_\gamma$ and the normal $\vec{p}_{\pi,\text{slow}} \times \vec{p}_{\pi,\text{fast}}$ to the $K^+ \pi^- \pi^+$ plane, where $\vec{p}_{\pi,\text{slow}}$ and $\vec{p}_{\pi,\text{fast}}$ correspond to the momenta of the lower and higher momentum pions, respectively. Following the notation and developments of Ref. [7], the differential decay rate of $B^+ \to K^+ \pi^- \pi^+ \gamma$ can be written as a fourth-order polynomial in $\cos \theta$

$$
\frac{d\Gamma}{dsds_{13}ds_{23}d\cos \theta} \propto \sum_{i=0,2,4} a_i(s, s_{13}, s_{23}) \cos^i \theta + \lambda_\gamma \sum_{j=1,3} a_j(s, s_{13}, s_{23}) \cos^j \theta, \quad (1)
$$

where $s_{ij} = (p_i + p_j)^2$ and $s = (p_1 + p_2 + p_3)^2$, and $p_1$, $p_2$, and $p_3$ are the four-momenta of the $\pi^-$, $\pi^+$, and $K^+$ mesons, respectively. The functions $a_i$ depend on the resonances present in the $K^+ \pi^- \pi^+$ mass range of interest and their interferences. The up-down asymmetry is defined as

$$
A_{\text{ud}} \equiv \frac{\int_{-1}^{0} d\cos \theta \frac{d\Gamma}{d\cos \theta} - \int_{0}^{1} d\cos \theta \frac{d\Gamma}{d\cos \theta}}{\int_{-1}^{1} d\cos \theta \frac{d\Gamma}{d\cos \theta}}, \quad (2)
$$

which is proportional to $\lambda_\gamma$.

The LHCb detector [9] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the
pp interaction region, a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes placed downstream. The combined tracking system provides a momentum measurement with relative uncertainty that varies from 0.4% at 5 GeV/c to 0.6% at 100 GeV/c, and impact parameter resolution of 20 μm for tracks with a few GeV/c of transverse momentum (p_T). Different types of charged hadrons are distinguished by information from two ring-imaging Cherenkov detectors. Photon, electron, and hadron candidates are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter, and a hadronic calorimeter. The trigger consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction.

Samples of simulated events are used to understand signal and backgrounds. In the simulation, pp collisions are generated using Pythia [10] with a specific LHCb configuration [11]. Decays of hadronic particles are described by EVTGEN [12], in which final state radiation is generated using PHOTOS [13]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [14] as described in Ref. [15].

This analysis is based on the LHCb data sample collected from pp collisions at 7 and 8 TeV center-of-mass energies in 2011 and 2012, respectively, corresponding to 3 fb⁻¹ of integrated luminosity. The B⁺ → K⁺π⁻π⁺γ candidates are built from a photon candidate and a hadronic system reconstructed from a kaon and two oppositely charged pions satisfying particle identification requirements. Each of the hadrons is required to have a minimum p_T of 0.5 GeV/c and at least one of them needs to have a p_T larger than 1.2 GeV/c. The isolation of the K⁺π⁻π⁺ vertex from other tracks in the event is ensured by requiring that the χ^2 of the three-track vertex fit and the χ^2 of all possible vertices that can be obtained by adding an extra track differ by more than 2. The K⁺π⁻π⁺ mass is required to be in the [1.1, 1.9] GeV/c^2 range. High transverse energy (> 3.0 GeV) photon candidates are constructed from energy depositions in the electromagnetic calorimeter. The absence of tracks pointing to the calorimeter is used to distinguish neutral from charged electromagnetic particles. A multivariate algorithm based on the energy cluster shape parameters is used to reject approximately 65% of the π^0 → γγ background in which the two photons are reconstructed as a single cluster, while keeping about 95% of the signal photons. The B⁺ candidate mass is required to be in the [4.3, 6.9] GeV/c^2 range. Backgrounds that are expected to peak in this mass range are suppressed by removing all candidates consistent with a D^0 → K⁺π⁻π^0 or ρ⁺ → π⁺π^0 decay when the photon candidate is assumed to be a π^0.

A boosted decision tree [16,17] is used to further improve the separation between signal and background. Its training is based on the following variables: the impact parameter χ^2 of the B⁺ meson and of each of the final state hadrons, defined as the difference between the χ^2 of a primary vertex (PV) reconstructed with and without the considered particle; the cosine of the angle between the reconstructed B⁺ momentum and the vector pointing from the PV to the B⁺ decay vertex; the flight distance of the B⁺ meson, and the K⁺π⁻π⁺ vertex χ^2.

The mass distribution of the selected B⁺ → K⁺π⁻π⁺γ signal is modeled with a double-tailed Crystal Ball [18] probability density function (PDF), with power-law tails above and below the B mass. The four tail parameters are fixed from simulation; the width of the signal peak is fit separately for the 2011 and 2012 data to account for differences in calorimeter calibration. Combinatorial and partially reconstructed backgrounds are considered in the fit, the former modeled with an exponential PDF, the latter described using an ARGUS PDF [19] convolved with a Gaussian function with the same width as the signal to account for the photon energy resolution. The contribution to the partially reconstructed background from events with only one missing pion is considered separately.

The fit of the mass distribution of the selected B⁺ → K⁺π⁻π⁺γ candidates (Fig. 1) returns a total signal yield of 13876 ± 153 events, the largest sample recorded for this channel to date. Figure 2 shows the background-subtracted K⁺π⁻π⁺ mass spectrum determined using the technique of Ref. [20], after constraining the B mass to its nominal value. No peak other than that of the K₁(1270)⁺ resonance can be clearly identified. Many kaon resonances, with various masses, spins, and angular momenta, are expected to contribute and interfere in the considered mass range [1].

The contributions from single resonances cannot be isolated because of the complicated structure of the

![FIG. 1 (color online). Mass distribution of the selected B⁺ → K⁺π⁻π⁺γ candidates. The blue solid curve shows the fit results as the sum of the following components: signal (red solid), combinatorial background (green dotted), missing pion background (black dashed), and other partially reconstructed backgrounds (purple dash-dotted).](image-url)
$K^+\pi^-\pi^+$ mass spectrum. The up-down asymmetry is thus studied inclusively in four intervals of $K^+\pi^-\pi^+$ mass. The [1.4, 1.6] GeV/$c^2$ interval, studied in Ref. [7], includes the $K_1(1400)^+$, $K_2(1430)^+$ and $K_3(1410)^+$ resonances with small contributions from the upper tail of the $K_1(1270)^+$. The hypothesis is defined as

$\chi^2$ test where the null hypothesis is defined as $\lambda_R = 0$, implying that the up-down asymmetry is expected to be zero in each mass interval. The corresponding $\chi^2$ distribution has 4 degrees of freedom, and the observed value corresponds to a $p$ value of $1.7 \times 10^{-7}$. This translates into a $5.2\sigma$ significance for nonzero up-down asymmetry. Up-down asymmetries can be computed also for an alternative definition of the photon

$$A_{ud} = c_1 - \frac{c_3}{4},$$

(4)

As a cross-check, the up-down asymmetry in each mass interval is also determined with a counting method, rather than an angular fit, as well as considering separately the $B^+$ and $B^-$ candidates. All these checks yield compatible results.

The results obtained from a $\chi^2$ fit of the normalized binned angular distribution, performed taking into account the full covariance matrix of the bin contents and all of the systematic uncertainties, are summarized in Table I. These systematic uncertainties account for the effect of choosing a different fit model, the impact of the limited size of the simulated samples on the fixed parameters, and the possibility of some events migrating from a bin to its neighbor because of the detector resolution, which gives the dominant contribution. The systematic uncertainty associated with the fit model is determined by performing the mass fit using several alternative PDFs, while the other two are estimated with simulated pseudoexperiments. Such uncertainties, despite being of the same size as the statistical uncertainty, do not substantially affect the fit results since they are strongly correlated across all angular bins.

The fitted distributions in the four $K^+\pi^-\pi^+$ mass intervals of interest are shown in Fig. 3. In order to illustrate the effect of the up-down asymmetry, the results of another fit imposing $c_4 = c_3 = 0$, hence forbidding the terms that carry the $\lambda_R$ dependence, are overlaid for comparison.

The combined significance of the observed up-down asymmetries is determined from a $\chi^2$ test where the null hypothesis is defined as $\lambda_R = 0$, implying that the up-down asymmetry is expected to be zero in each mass interval. The corresponding $\chi^2$ distribution has 4 degrees of freedom, and the observed value corresponds to a $p$ value of $1.7 \times 10^{-7}$. This translates into a $5.2\sigma$ significance for nonzero up-down asymmetry. Up-down asymmetries can be computed also for an alternative definition of the photon

TABLE I. Legendre coefficients obtained from fits to the normalized background-subtracted cos $\theta$ distribution in the four $K^+\pi^-\pi^+$ mass intervals of interest. The up-down asymmetries are obtained from Eq. (4). The quoted uncertainties contain statistical and systematic contributions. The $K^+\pi^-\pi^+$ mass ranges are indicated in GeV/$c^2$ and all the parameters are expressed in units of $10^{-2}$. The covariance matrices are given in Ref. [22].

<table>
<thead>
<tr>
<th>Mass Range</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$A_{ud}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1.1, 1.3]</td>
<td>6.3 ± 1.7</td>
<td>31.6 ± 2.2</td>
<td>−2.1 ± 2.6</td>
<td>3.0 ± 3.0</td>
<td>6.9 ± 1.7</td>
</tr>
<tr>
<td>[1.3, 1.4]</td>
<td>7.9 ± 1.7</td>
<td>27.0 ± 2.6</td>
<td>2.0 ± 3.1</td>
<td>6.8 ± 3.6</td>
<td>4.9 ± 2.0</td>
</tr>
<tr>
<td>[1.4, 1.6]</td>
<td>4.3 ± 1.9</td>
<td>43.1 ± 2.3</td>
<td>−5.2 ± 2.8</td>
<td>8.1 ± 3.1</td>
<td>5.6 ± 1.8</td>
</tr>
<tr>
<td>[1.6, 1.9]</td>
<td>−4.6 ± 1.8</td>
<td>28.0 ± 2.3</td>
<td>−0.6 ± 2.7</td>
<td>−6.2 ± 3.2</td>
<td>−4.5 ± 1.9</td>
</tr>
</tbody>
</table>
The shape of the angular distribution in each bin, as well as the values for the up-down asymmetry, may be used, if theoretical predictions become available, to determine for the first time a value for the photon polarization, and thus constrain the effects of physics beyond the SM in the $b \to s \gamma$ sector.

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