Mixed Integer Programming models for planning maintenance at offshore wind farms under uncertainty

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Abstract

We introduce the Stochastic Maintenance Fleet Transportation Problem for Offshore wind farms (SMFTPO), in which a maintenance provider determines an optimal, medium-term planning for maintaining multiple wind farms while controlling for uncertainty in the maintenance tasks and weather conditions. Since the maintenance provider is typically not the owner of a wind farm, it needs to adhere minimum service requirements that specify the required service. We consider three of such settings: 1) perform all maintenance tasks, 2) allow for a fraction of unscheduled tasks, and 3) incentivize to perform maintenance rather quickly. We provide a two-stage stochastic mixed integer programming model for the three SMFTPO settings, and solve it by means of Sample Average Approximation. In addition, we provide an overview of the, what we discovered, non-aligned modeling assumptions in the literature regarding operational decisions. By providing a series of special cases of the second-stage problem resembling the different modeling assumptions, we aim to establish a common consensus regarding the key modeling decisions to be taken in maintenance planning problems for offshore wind farms. We provide newly constructed, and publicly available, benchmark sets. We extensively compare the different SMFTPO settings and its special cases on those benchmark sets, and we show that the special case reformulations are very effective for solving the second-stage problems. In addition, we find that for particular cases, established modeling techniques result in overestimations and increased running times.

Keywords: Mixed Integer Programming, Stochastic Programming, Offshore Wind, Transportation, Service Logistics, Maintenance

1. Introduction

Successful offshore wind maintenance service logistics requires a thought-out maintenance strategy describing service-vessel\(^1\) utilization strategies for the medium- and long-term. Such strategies need to consider a highly stochastic, operational environment, in which the turbines’ failure behavior is difficult to predict and in which weather conditions determine the wind farm’s daily accessibility (Shafiee 2015, Shafiee and Sørensen 2017). In addition, misaligned objectives between the wind farm owner (i.e., profit maximization) and the maintenance ser-
vice provider (i.e., cost minimization) has led to current practices where operations are streamlined by imposing minimum service requirements (Ferreira et al. 2014). These are contractually binding requirements specifying the required performance of the maintenance provider. In this context, we study a stochastic maintenance planning problem for offshore wind farms, controlling for uncertain maintenance tasks and weather conditions, in which we explicitly take the viewpoint of a maintenance service provider that is subject to such minimum service requirements. We refer to this problem as the Stochastic Maintenance Fleet Transportation Problem at Offshore wind farms (SMFTPO).

When considering decisions on a tactical level as in the SMFTPO, simplifications on the underlying operational planning problem are insurmountable as the operational planning is already shown to be computationally challenging in a deterministic setting (see, e.g., Irawan et al. 2017, Schrotenboer et al. 2018, 2019). However, the extent of such simplifications affects the medium-term vessel utilization and thereby long-term vessel charter strategies. A wide variety of approximations (and underlying assumptions) have been made in the literature, and we show, by means of a thorough review, that there is no common consensus on those assumptions. We present four categories of modeling decisions that are crucial for the resulting complexity and practicality of the optimization problems. We aim to provide more insight into the impact of those assumptions on the tractability of the resulting optimization problems, and consequently, how this affects the computational efficiency. This structural overview of the impact of such assumptions on the underlying optimization problem, both computationally and mathematically, will contribute to the development of a common consensus on key modeling decisions in offshore wind maintenance logistics.

Inspired by offshore wind practices observed in the Netherlands, and contrary to previous work in offshore wind maintenance service logistics (see e.g., Stålhane et al. 2016b), we take the perspective of a single, large maintenance service provider that is responsible for the maintenance of one or multiple offshore wind farms. This maintenance service provider is not the owner of the wind farms and, therefore, does not bear the risk of uncertain production revenues due to the highly volatile energy prices and the risk of production losses due to downtime of the turbines. The maintenance service provider’s sole priority is adhering the minimum service requirements specified in a service contract between the wind farm owner and the maintenance service provider, typically resulting in a misalignment of objectives between wind farm owner (production maximization) and the maintenance service provider (maintenance cost minimization). This is in contrast with the current literature, in which it is assumed that objectives are aligned and the sum of total costs will be minimized, without imposing hard constraints on the service requirements.

In this paper, we introduce the Stochastic Maintenance Fleet Transportation Problem for Offshore wind farms (SMFTPO). Its goal is to develop a cost-minimizing, medium-term maintenance planning by assigning vessels to depots (O&M bases) in the first stage, and after the uncertain maintenance tasks and weather conditions are revealed, to assign maintenance tasks to the vessels in the second stage. This is, to the best of the authors’ knowledge, the first study on a tactical level of decision making in offshore wind maintenance service logistics. We consider three settings of the SMFTPO, each describing a different variant of minimum service requirements. These set-
tings are 1) to perform all maintenance tasks, 2) to allow for a fraction of unscheduled tasks, and 3) to incentivize performing maintenance rather quickly. We provide a general two-stage stochastic mixed integer programming formulation and its scenario-based large scale representation which we solve by Sample Average Approximation (see, e.g., Kleywegt et al. 2002, Santoso et al. 2005). The second-stage is modelled by using decomposed and time-expanded networks, which are commonly used in network design applications (see, e.g., Crainic 2000, Andersen et al. 2011) and for which sophisticated solution approaches have been developed (see, e.g., Boland et al. 2017). We provide insights in the value of the stochastic solution and the effect of the different service requirements, both from a computational and a managerial point of view.

In addition, we exemplify the impact of different decisions regarding the identified modeling categories by studying five special cases of the second-stage problem of the SMFTPO, resulting in a series of reformulations for each SMFTPO setting. The special cases are inspired on practical observations in offshore wind, on the need for further mathematical insights as identified in our literature review, or on both. The special cases elaborate on differences between single and multiple wind farm settings, maintenance task pre-processing techniques, and the level of detail of the operational problem. A numerical investigation on the computational tractability of the special cases is performed to assess the trade-off between the level of modeling detail and the computational performance. The formulations of the special cases are shown to be very efficient in terms of computational efficiency. Moreover, it is numerically shown that pre-processing maintenance tasks into bundles (see, e.g., Gundegjerde et al. 2015) will result in an overestimation of the total incurred maintenance costs, although it is computationally attractive to do so. To foster future research in this area, we made our set of benchmark instances publicly available².

1.1. Literature Review

The SMFTPO relates to so-called fleet size and mix problems in offshore wind, which focus on strategic decision making (buying or chartering a vessel) as opposed to the tactical decision making (assigning vessels to wind farms and maintenance tasks) in the SMFTPO. Halvorsen-Weare et al. (2013) presents the first application of fleet size and mix problems in the context of offshore wind. They propose a MIP formulation, which formed the basis of the 3-stage stochastic programming approach by Gundegjerde et al. (2015). It includes stochastic vessel spot rates, weather conditions, electricity prices, and turbine failures.

Motivated by this seminal work, a number of papers have studied slight variants of this optimization problem or introduced new solution approaches. The work by Stålhane et al. (2016a) considers a similar setting as Gundegjerde et al. (2015). By advanced preprocessing of maintenance tasks, they provide the first sophisticated solution approach in this context. Its mathematical model forms the basis of three other works. First, Stålhane et al. (2016b) use the same concepts to study optimal fleet size and mix over the complete lifetime of a wind farm. Second, Gutierrez-Alcoba et al. (2017) provide additional insights by means of a sophisticated case-study. Third, Halvorsen-Weare et al. (2017) developed a metaheuristic approach in which uncertainty is assessed by means of

²https://sites.google.com/rug.nl/albertschrotenboer
simulation. Slightly different works are those by Stålhane et al. (2017) studying optimal jack-up vessel chartering strategies and the work of Sperstad et al. (2017) on the robustness of different decision support tools.

Since the accurateness of the modeling of operational activities such as vessel routing and short-term maintenance planning is crucial for the difficulty of fleet size and mix problems, we shortly review the most important contributions in that area. The short-term planning of maintenance tasks in offshore wind farms has become increasingly popular among researchers. A series of research articles (Dai et al. 2015, Stålhane et al. 2015, Irawan et al. 2017, Raknes et al. 2017, Schrottenboer et al. 2019, 2018) discuss this problem, leading to a branch-and-price-and-cut method for the single wind farm case (Schrottenboer et al. 2019), and a metaheuristic approach for the multiple wind farm case (Schrottenboer et al. 2018).

In addition, a number of articles are written about decisions support tools for offshore wind maintenance logistics (see, e.g., Stålhane et al. 2015, 2016a). The interested reader is referred to the work of Hofmann (2011) for a review of papers on decisions support tools for offshore wind (maintenance) logistics. For interested readers on the installation phase of offshore wind farms, see the papers by Vis and Ursavas (2016) and Ursavas (2017). For all contributions on fleet size and mix problems in onshore logistics, we refer the interested reader to the recent review by Braekers et al. (2016). Finally, for all other aspects related to offshore wind maintenance logistics that are not stringently relevant to the SMFTPO, we refer the reader to the excellent review by Shafiee and Sørensen (2017).

The notion of minimum service requirements is, to the best of the authors’ knowledge, not considered before in maintenance planning optimization problems focusing on offshore wind. However, it is well-known that maintenance performance should be measured and the right incentives should be given (Parida et al. 2015). Especially in more mature industries that include applications in rail (see, e.g., Lidén 2015) and power systems (see, e.g., Froger et al. 2016) the use of minimum service requirements is common. However, the impact of stochastic weather conditions that limit the maximum working hours in a period is typical for offshore wind, and can not directly be related to other applications. We, therefore, introduce three basic minimum service requirements and investigate their impact on the maintenance planning problem as discussed in this paper.

1.1.1. Modeling decisions and assumptions.

From the literature review it is clear that a series of research articles has been devoted to offshore wind maintenance planning problems. However, those studies are typically not aligned with regards to their modeling decisions. We discuss, what we call, four modeling categories, and we detail how the existing studies have taken decisions within.

First, there is a difference between maintenance tasks that take more than a single period, and less than a single day. The papers by Halvorsen-Weare et al. (2013) and Gundegjerde et al. (2015) partition the set of maintenance tasks based on their duration (whether they take a single period or more). However, the impact of this on the computational tractability is not discussed by the authors. In Stålhane et al. (2016b) and Stålhane et al. (2016a), task durations are assumed to be at most a single period, i.e., tasks are not scheduled among multiple periods.
Second, taking the perspective of a single, large maintenance provider in offshore wind (as in the SMFTPO), the question how to jointly use multiple depots (ports or harbours) is of utmost importance. In Gundegjerde et al. (2015), they assume vessels can switch depots, however, they consider only a single wind farm in their experiments. Hence it is not clear what the impact of this assumption is. In Stålhane et al. (2016b), vessels may switch depots, however as the period length resembles several months, this does not have a great influence. In Stålhane et al. (2016a) and Halvorsen-Weare et al. (2017) it is assumed that vessels are associated with a single base.

The third modeling category is the modeling and categorization of maintenance tasks. Halvorsen-Weare et al. (2013) and Gundegjerde et al. (2015) consider every task on its own, but belonging to different categories that define whether or not a vessel is eligible to perform the maintenance task. Moreover, they assume that a task can only be completed by a single vessel. In Stålhane et al. (2016b), the considered time horizon length equals the wind farm lifetime and the length of single period is in the order of several months. Tasks of the same category are assumed to demand a number of technicians, and a piecewise linear relation between vessel fleet capacity and incurred downtime costs is proposed. The same holds for, Stålhane et al. (2016a) and Halvorsen-Weare et al. (2017) though they have a similar focus as Ahmadi-Javid and Seddighi (2012), i.e., they consider a time horizon of a year and periods reflect days. In this paper, we are especially interested in the impact of assuming that a single job is performed by one vessel only.

The final modeling category is the extent to which minimum service requirements are imposed, as we detailed for the SMFTPO in the former. Halvorsen-Weare et al. (2013) and Gundegjerde et al. (2015) consider a penalty cost for not scheduling maintenance tasks. Before solving actual instances, penalty costs need to be set so that enough tasks are completed in order to reflect practical situations. This is structurally different from the SMFTPO, as we model minimum service requirements with hard constraints. The papers by Stålhane et al. (2016b), Stålhane et al. (2016a), and Halvorsen-Weare et al. (2017) model the assigned vessel’s capacity to wind farms and penalize under- and over-coverage of the expected number of needed technicians.

1.2. Contributions and outlook

Summarizing, four key differences of this paper with the current literature are observed. First, all the above-mentioned papers considered the wind farm as a single entity of which the total costs are to be minimized. We, however, take the viewpoint of a maintenance service provider subject to minimum service requirements. Such a maintenance service provider is not concerned with production losses but minimizes their own costs so that the maintenance service requirements are met. Second, all the above-mentioned papers on optimizing maintenance planning at offshore wind farms did not provide mathematical insights into the underlying MIP formulations and its relation to the observed computational performance. In this paper, we provide such insights in Section 4, by discussing a series of reformulations for special cases in offshore wind. Third, we study tactical decision making, i.e., we allocate an already existing vessel fleet to wind farms and depots instead of focusing on strategic or operational decisions, allowing us to elaborate further on those mathematical insights. Fourth, by taking decisions on a tactical level, the SMFTPO is the first model that allows vessels to be utilized from multiple depots.
The remainder of this paper is structured as follows. In Section 2, we provide, next to a formal problem statement, the two-stage stochastic mixed integer programming formulation and its scenario-based large-scale monolithic formulation. In Section 3, we present five special cases of the second-stage problem of the SMFTPO that focus on the modeling categories and assumptions discussed in Section 1.1.1. A numerical analysis of all the special cases and the SMFTPO is provided in Section 4. We conclude our work in Section 5, where we provide numerous avenues for further research as well.

2. Problem Formulation

In this section, we present the Mixed Integer Programming (MIP) formulation for the Stochastic Maintenance Fleet Transportation Problem for Offshore wind farms (SMFTPO). We first describe the system upon which the SMFTPO is based, and discuss the first and second stage decisions to take. After that, we provide a time expanded and decomposed network formulation upon which we model the second-stage decisions. We discuss three distinct second-stage optimization models each of which is tailored towards a particular setting of minimum service requirements. In the first setting, all maintenance tasks need to be scheduled. In the second setting, a fraction \( \alpha \) of so-called technician hours can be left unscheduled. The third setting restricts the maximum fraction \( \beta \) of periods on which turbines are left unrepaired. Those three settings, although stylized, will provide realistic cost estimations of the medium-term maintenance planning accounting for different incentives; the setting with \( \alpha \) unscheduled technician hours is commonly encountered and results in delaying tasks that are most unprofitable, and the setting with at most \( \beta \) downtime periods resembles a maintenance contract giving incentives to perform maintenance tasks rather quickly.

We end this section by providing the two-stage stochastic programming model and its scenario-based monolithic formulation. In the remainder, we denote uncertainty by the set \( \Xi \). Dependency on this set is denoted by the general descriptor \( \cdot(\xi) \), where \( \xi \in \Xi \).

2.1. System description

An overview of the notation used for describing the system is given in Table 1. We consider a time horizon \( T = \{1, \ldots, T\} \) that is partitioned in \( L \) lease terms of equal size. We refer to each \( t \in T \) as a period. Each period has a maximum number of working hours. The corresponding periods of each lease term \( \ell \in L = \{1, 2, \ldots, L\} \) are given by \( T^\ell = \{(T/L)(\ell - 1) + 1, \ldots, (T/L)\ell\} \). In Figure 1, an example planning horizon with \( T = 12 \) periods and \( L = 4 \) leaseterms is presented. Throughout the paper we will assume that \( T/L \in \mathbb{N} \).
Table 1: Overview of the main sets, parameters, and decision variables

<table>
<thead>
<tr>
<th>Deterministic sets and parameters</th>
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<tbody>
<tr>
<td>( T = {1, \ldots, T} )</td>
<td>The complete time horizon</td>
</tr>
<tr>
<td>( \mathcal{W} = {1, \ldots, W} )</td>
<td>Set of wind farms</td>
</tr>
<tr>
<td>( \mathcal{D} = {1, \ldots, D} )</td>
<td>Set of depots</td>
</tr>
<tr>
<td>( L )</td>
<td>Number of leaseterms with equal length ( T/L \in \mathbb{N} )</td>
</tr>
<tr>
<td>( \mathcal{V} = {1, \ldots, V} )</td>
<td>Set of vessels</td>
</tr>
<tr>
<td>( S_1^v )</td>
<td>Number of technicians on board vessel ( v \in \mathcal{V} )</td>
</tr>
<tr>
<td>( S_2^v )</td>
<td>Number of hours that technicians can perform maintenance per period on vessel ( v \in \mathcal{V} )</td>
</tr>
<tr>
<td>( \hat{C}_{dv} )</td>
<td>Costs of assigning vessel ( v ) to depot ( d ) in leaseterm ( \ell ).</td>
</tr>
<tr>
<td>( \hat{p} )</td>
<td>Exogenous costs of changing the assigned depot of a vessel in the second stage</td>
</tr>
<tr>
<td>( \mathcal{L}^m )</td>
<td>Set of maintenance categories</td>
</tr>
<tr>
<td>( \mathcal{L}^v )</td>
<td>Set of vessel types</td>
</tr>
<tr>
<td>( \Theta(\ell^v) )</td>
<td>Set of maintenance categories that can be performed by vessel type ( \ell^v )</td>
</tr>
<tr>
<td>( \Theta(\ell^m) )</td>
<td>Set of vessel types that can perform maintenance category ( \ell^m )</td>
</tr>
<tr>
<td>( \theta(\ell^m, \ell^v) )</td>
<td>Equals 1 if vessel type ( \ell^v ) can perform maintenance category ( \ell^m )</td>
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<table>
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<tr>
<th>Stochastic sets and parameters</th>
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<tbody>
<tr>
<td>( \mathcal{M}(\xi) = {1, \ldots, M(\xi)} )</td>
<td>Set of maintenance tasks</td>
</tr>
<tr>
<td>( M^w(\xi) )</td>
<td>Number of maintenance tasks at wind farm ( w )</td>
</tr>
<tr>
<td>( M^m(\xi) )</td>
<td>Number of maintenance tasks at wind farm ( w ) in period ( t )</td>
</tr>
<tr>
<td>( S_m(\xi) )</td>
<td>First period in which maintenance task ( m ) can be scheduled</td>
</tr>
<tr>
<td>( E_m(\xi) )</td>
<td>The latest period in which maintenance task ( m ) can be scheduled</td>
</tr>
<tr>
<td>( T^m(\xi) )</td>
<td>The set of periods in which maintenance task ( m ) can be scheduled</td>
</tr>
<tr>
<td>( F^m(\xi) )</td>
<td>Wind farm in which maintenance task ( m ) is located</td>
</tr>
<tr>
<td>( D^v_m(\xi) )</td>
<td>Number of technicians demanded for maintenance task ( m )</td>
</tr>
<tr>
<td>( D^v(\xi) )</td>
<td>Number of hours of work (for each of the demanded technicians) ( v \in \mathcal{V} )</td>
</tr>
<tr>
<td>( H^v_{tw}(\xi) )</td>
<td>Number of hours vessel type ( \ell^v ) can work offshore in period ( t ) at wind farm ( w )</td>
</tr>
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<tr>
<th>Decision variables</th>
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<tbody>
<tr>
<td>( y^v_d(\xi) )</td>
<td>First stage binary variable equalling 1 if vessel ( v ) is assigned to depot ( d ) in period ( \ell ).</td>
</tr>
<tr>
<td>( x^v_{ij}(\xi) )</td>
<td>Second Stage binary variable whether arc ((i, j)) is traversed by vessel ( v )</td>
</tr>
<tr>
<td>( z^v_{ij}(\xi) )</td>
<td>number of technician hours send along arc ((i, j)) by vessel ( v )</td>
</tr>
</tbody>
</table>

The set of wind farms is denoted by \( \mathcal{W} \). Each wind farm \( w \in \mathcal{W} \) consists of \( N^w \) turbines and is geographically represented by a single set of coordinates \((X^w, Y^w)\). All distances within a wind farm will be ignored. Let \( \mathcal{D} \) be the set of depots (O&M bases). Each depot \( d \in \mathcal{D} \) is geographically placed at \((X^d, Y^d)\). Distances between depots and wind farms are assumed to be Euclidean.

The set of stochastic maintenance tasks is denoted by \( \mathcal{M}(\xi) = \{1, \ldots, M(\xi)\} \). A maintenance task \( m \in \mathcal{M}(\xi) \) is part of a particular wind farm \( F^m \). The set of maintenance tasks at wind farm \( w \in \mathcal{W} \) is defined as \( \mathcal{M}^w(\xi) := \{m \in \mathcal{M}(\xi) : F^m = w\} \). Every maintenance task \( m \) has a number of consecutive periods \( T^m(\xi) \) in which it can be scheduled, the first period being denoted by \( S_m \), and the latest period is denoted by \( E_m \). We call the periods
between \( S_m(\xi) \) and \( E_m(\xi) \) the maintenance window. Then, the set of maintenance tasks of windfarm \( w \) at period \( t \) is defined as \( M^w(\xi) := \{ m \in M^w(\xi) : S_m(\xi) \leq t \leq E_m(\xi) \} \).

We consider a given fleet of vessels \( \mathcal{V} \) that are deployed for performing maintenance tasks. Each vessel \( v \in \mathcal{V} \) transports \( S^v_1 \) technicians that can work \( S^v_2 \) hours in each period. Each maintenance task requires \( D^v \) technicians to work for \( D^v_1 \) hours to be completed. We will model the demand (resp. supply) of technicians in terms of technician hours: The number of technicians multiplied with the number of hours they are required (resp. available) to work. The supply of technician hours can be restricted to reflect travel time, unloading time, or severe weather conditions.

Each maintenance task \( m \) can be categorized to a maintenance category \( \ell^M \in \mathcal{L}^M \), where the set \( \mathcal{L}^M = \{1^M, \ldots, L^M\} \) contains all maintenance categories. Similarly, each vessel \( v \) can be categorized to a vessel type \( \ell^V \in \mathcal{L}^V \), where the set \( \mathcal{L}^V = \{1^V, \ldots, L^V\} \) is the set of vessel types. A vessel cannot perform all maintenance categories. We use \( \ell^M(m) \) and \( \ell^V(v) \) to denote the category of a maintenance task \( m \) and the type of a vessel \( v \), respectively. We let \( \theta(\ell^M, \ell^V) = 1 \) if maintenance category \( \ell^M \) can be performed by vessel type \( \ell^V \). For notational convenience, we let \( \Theta(\ell^M) := \{ \ell^V \in \mathcal{L}^V : \theta(\ell^M, \ell^V) = 1 \} \), and \( \Theta(\ell^V) := \{ \ell^M \in \mathcal{M} : \theta(\ell^M, \ell^V) = 1 \} \). In other words, \( \Theta(\ell^M) \subseteq \mathcal{L}^V \) are all the vessel types that can perform maintenance category \( \ell^M \), and \( \Theta(\ell^V) \subseteq \mathcal{L}^M \) are all the maintenance categories that can be performed by vessel type \( \ell^V \).

We define \( H^w_{\ell^M}(\xi) \) as the number of hours that a vessel of type \( \ell^V \) can perform maintenance tasks in period \( t \) at wind farm \( w \). This reflects the impact of weather conditions on the daily operations. We, hereby, imply that vessels cannot change wind farms within a period. However, wind farms being close are likely to be exposed to similar weather conditions, and such wind farms will be modeled as a single wind farm.

Then, the SMFTPO is a two-stage stochastic optimization problem. In the first stage, we need to assign vessels to depots for each lease period \( \ell \in \mathcal{L} \). Then, in the second stage, after the set of maintenance tasks and the weather conditions are revealed, we need to assign the vessels to the maintenance tasks. We allow a vessel to change depot in the second stage with a penalty cost \( \hat{P} \). The first-stage decisions are modelled by binary decision variables \( y^v_{\ell,d} \), equalling 1 if vessel \( v \in \mathcal{V} \) is assigned to depot \( d \in \mathcal{D} \) in leaseterm \( \ell \in \mathcal{L} \), and 0 otherwise.

2.2. The second stage problem

We model the second-stage problem, i.e., given a first-stage decision and after observing uncertain parameters \( \xi \in \Xi \), as a network design problem on a decomposed and time-expanded network. An overview of the notation used is provided in Table 2. The time expansion is made on the period level, i.e., nodes encode a maintenance task in a particular period. The decomposition is made in the vessel dimension, as we assume their movements are independent. To enhance readability, we omit the dependency on \( \cdot(\xi) \) in this subsection.

We first consider a flat network (i.e., no time expansion or decomposition) that forms the basis of the decomposed and time-expanded formulation. Let \( G=(\mathcal{N}, \mathcal{A}) \) be this network, where the node set \( \mathcal{N} \) consists of a node for each wind farm \( w \in \mathcal{W} \), depot \( d \in \mathcal{D} \), and maintenance task \( m \in \mathcal{M} \). The arc set \( \mathcal{A} \) consists of two types of arcs:

1. We create arcs \((i, j)\) for each \( i, j \in (\mathcal{D} \cup \mathcal{W}) \), i.e., arcs between depots and reachable wind farms.
We create arcs \((i, w)\) and \((w, i)\) for each \(i \in M^w\) and for all \(w \in W\), i.e., arcs between maintenance tasks and their corresponding wind farms.

2.2.1. Decomposed and time-expanded network.

Let \(T\) and \(v \in V\) be given. The corresponding, decomposed and time expanded graph is then defined as \(G^T_v = (N^T_v, A^T_v)\) for each vessel \(v \in V\). The node set is defined as \(N^T_v := N^T_D \cup N^T_W \cup N^T_M \cup N^T_{\text{art}}\). Here \(N^T_D := \{(d, t) \mid d \in D, t \in T\}\), \(N^T_W := \{(w, t) \mid w \in W, t \in T\}\), \(N^T_M := \{(m, t) \mid m \in M : \ell^V(v) \in \Theta(t^M(m)), t \in (T^m \cap T)\}\), and \(N^T_{\text{art}} := \{\ell \mid \ell \in \mathcal{L}\}\). In other words, those sets contain node copies for each \(t \in T\) representing the depots, wind farms, the eligible maintenance tasks for vessel \(v\), and artificial nodes modeling the availability of vessels.

The arc set \(A^T_v\) is partitioned into the sets \(A^T_{1,v}, A^T_{2,v}, A^T_{3,v}, A^T_{4,v},\) and \(A^T_{5,v}\), which are constructed as follows:

1. Arcs \((d, t), (w, t)\) \(\in A^T_{1,v}\) for each \(d \in D, j \in W, t \in T\). Note that we only consider arcs from a depot to a windfarm if it is reachable from that depot. The costs of these arcs represent the daily traveling costs. The capacity of this arc represents the realization of \(H^w_t\).

2. Arcs \((w, t), (d, t + 1)\) \(\in A^T_{2,v}\) for each \(w \in W, d \in D, t \in T\setminus T\). Here \(t + 1\) refers to the first period that follows \(t\) in \(T\). Again, we only consider wind farm to depot arcs if the wind farm can be reached from the depot. The costs represent the traveling costs.

3. Arcs \((d, t), (d', t + 1)\) \(\in A^T_{3,v}\) for each \(d, d' \in D, t \in T\setminus T\). These arcs between depots represent either no maintenance (if they connect the same depot) or the recourse action that can be taken to change the allocation of the vessel to a different depot. In case of the latter, travel costs and the penalty \(P\) are incurred.
(4) Arcs \( ((i, t), (w, t)) \in \mathcal{A}_4^{T, v} \) and \( ((w, t), (i, t)) \in \mathcal{A}_4^{T} \) for each \( i \in M^v, t \in T, w \in W \). These arcs represent performing a particular maintenance task in a wind farm. The costs represent maintenance specific costs. The capacity of these arcs resemble the demand for technician hours by the maintenance task.

(5) Arcs \( (\ell, (d, t')) \) and \( ((d, t'), \ell + 1) \in \mathcal{A}_5^{T, v} \), with \( t' \) being the earliest period \( t \) in lease period \( \ell \) and \( t'' \) the latest period \( t \) in lease period \( \ell \), for all \( \ell \in \mathcal{L}\setminus L \). These arcs model the inflow of vessels in a lease term, as depicted by the first-stage solution. The costs are already included in the first-stage decision. The capacity of this arc equals the supply of technician hours of vessel type \( \ell \).

Example 1. An illustrative example of a time-expanded network \( \mathcal{G}_v^T \) for an arbitrary vessel \( v \in V \) is presented in Figure 2. Using this graph, we can model the second-stage problem of the SMFTPO as a network design problem. In the example, we included two wind farms ‘wf1’ and ‘wf2’, two depots ‘D1’ and ‘D2’. In this particular example, wf1 is only reachable from D1, and wf2 is only reachable from D2. On the left, the node ‘ART’ is a artificial node acting as source (and sink) of the vehicle flow. In the example, six jobs ‘j1’-’j6’ are depicted. It is seen that jobs ‘j1’-’j3’ are located in the first wind farm, and the remaining jobs in the second wind farm. Each job is only present in its maintenance window, e.g., job six is not present in period 2. Finally, note the (red) arcs between the depots, which model either a period in which no maintenance is performed (an arc between the same depot) or a period where the vessel changes depot and incurs the penalty \( \hat{P} \).

2.2.2. Second-stage mixed integer programming formulation.

We can model the second-stage problem of the SMFTPO on the decomposed and time-expanded networks \( \mathcal{G}^T \). Let \( x^v_{ij} \) be a binary decision variable equaling 1 if arc \( (i, j) \in \mathcal{A}_3^{T, v} \) is traversed by vessel \( v \in V \). Let \( z^v_{ij} \) be a continuous decision variable indicating the number of technician hours sent along arc \( (i, j) \in \mathcal{A}_4^{T} \). For each arc


\((i, j) \in \mathcal{A}^T_v\), we let \(U^v_{ij}\) be the total number of technician hours (or capacity) that can be sent along \((i, j)\) with vessel \(v\), as described in the former.

We define \(C^v_{ij}\) as the costs of traversing arc \((i, j)\) and we let \(F^v_{ij}\) be the task-specific maintenance cost per supplied technician hour. Note that these costs are exogenously given and might be used to make a distinction between corrective or preventive maintenance costs. We elaborate more on the actual construction of the arc capacities and costs in the numerical results in Section 4.

Some additional notation is required in order to obtain a concise formulation. Let \(\delta^+(S) := \{(i, j) \in \mathcal{A}^T_v \mid i \in S, j \notin S\}\) and \(\delta^-(S) := \{(i, j) \in \mathcal{A}^T_v \mid i \notin S, j \in S\}\) for any \(S \subseteq \mathcal{N}^T_v\). In addition, we denote with \(\delta^n\) the difference in incoming and outgoing technician hours. This equals 0 for each node except the artificial nodes \(\mathcal{N}^T_{\text{art}}\), in which \(\delta^n\) models the availability of vessels and their corresponding supply of technician hours (i.e., the first-stage decision).

Finally, we refer to the complete vectors of decision variables by denoting them in bold, e.g., with \(y^v\) we denote \(y^v_{dt}\) for all \(d \in D, \ell \in \mathcal{L}, \text{ and } v \in \mathcal{V}\). Then, the second-stage problem of the SMFTPO asks for solving

\[
Q(x, z \mid \xi) := \min \sum_{v \in \mathcal{V}} \sum_{i,j \in \mathcal{A}^T_v} C^v_{ij}x^v_{ij} + \sum_{v \in \mathcal{V}} \sum_{m \in \mathcal{M}} \sum_{(i,j) \in \mathcal{A}^T_v(m)} F^v_{ij}z^v_{ij} \tag{1}
\]

\[
\text{s.t. } \sum_{(i,j) \in \delta^+(n)} x^v_{ij} \leq 1 \quad \forall n \in \mathcal{N}^T_{\text{art}}, v \in \mathcal{V} \tag{2}
\]

\[
\sum_{(i,j) \in \delta^-(n)} x^v_{ij} \leq 1 \quad \forall n \in \mathcal{N}^T_{\text{art}}, v \in \mathcal{V} \tag{3}
\]

\[
\sum_{(i,j) \in \delta^+(n)} z^v_{ij} - \sum_{(i,j) \in \delta^-(n)} z^v_{ij} = \delta^n \quad \forall n \in \mathcal{N}^T_v, v \in \mathcal{V} \tag{4}
\]

\[
z^v_{ij} \leq U^v_{ij}x^v_{ij} \quad \forall (i, j) \in \mathcal{A}^T_v \tag{5}
\]

\[
\sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}^T_v(m)} z_{ij} \geq D^m_T D^m_S \quad \forall m \in \mathcal{M} \tag{6}
\]

\[
x^v_{ij} \in \{0, 1\}, \quad z^v_{ij} \in \mathbb{R}_+ \quad \forall (i, j) \in \mathcal{A}^T_v, v \in \mathcal{V} \tag{7}
\]

The Objective (1) minimizes the costs over the arcs in the network. Constraints (2) and (3) ensure that a vessel cannot be split to multiple wind farms in the same period and that it returns to a single depot, respectively. Constraints (4) ensure the flow conservation of technicians, and Constraints (5) ensure that only a positive flow of technician hours can be sent along an arc if it is traversed. Constraints (6) ensure that every task is being performed and the variable domains are denoted by (7).

**Example 2.** In Figure 3, we provide two feasible example flows of technician hours (of vessel \(v\)) through the same graph as in Example 1. In black, we depict a solution that visits wind farm 1 and works on job 1 and job 3 in period 1, and switch depots in period 2. In blue, we work in wind farm 2 on job 6 in period 1, and switch depots in period 2.
2.2.3. The three SMFTPO settings.

The minimum service requirements, as modeled by Constraints (6), impose that all the tasks should be scheduled within their imposed time windows. This is the first setting of the SMFTPO. However, this might not reflect the practical incentives of a maintenance service provider that does not bear any risk of incurred downtime costs.

In the second setting, we allow that a fraction $\alpha^w$ of demanded technician hours in wind farm $w \in W$ is left unassigned. The values of $\alpha^w$ are typically small (e.g., 0.10 or 0.05). We model this as follows: The total supply of technician hours to maintenance tasks should be at least $(1 - \alpha^w)$ times the total demand of technician hours of all the maintenance tasks. The second setting asks for solving

$$Q^\alpha(x, z | \xi) := \min \sum_{v \in V} \sum_{(i, j) \in A^v} C^v_{ij} x_{ij}^v + \sum_{v \in V} \sum_{m \in \mathcal{M}} \sum_{(i, j) \in A^v_{(m)}} F^v_{ij} z_{ij}^v$$

subject to

$$\sum_{v \in V} \sum_{(i, j) \in A^v} z_{ij}^v \geq (1 - \alpha^w) \sum_{w \in W} D_w^1 D_w^2 \quad \forall w \in W$$

Constraints (2) - (5), (7)

Constraints (9) ensure that, for each wind farm $w$, the fraction of technician hours supplied by the vessels is at least $(1 - \alpha^w)$ of the total number of required technician hours. Constraints (2) - (5), and (7) are unchanged, as no other modeling restrictions need to be taken into account.

In the third setting, we consider a maximum fraction $\beta^w$ of so-called downtime periods. These are defined as the difference between the first possible period of scheduling a task and the latest period in which a task is actually performed. We introduce the variable $\eta_m$, being equal to the latest period in which task $m$ is scheduled. This is required since tasks could be split up among different periods and vessels. The third setting of the SMFTPO asks
for solving

\[
Q^\beta(x, z, \eta | \xi) := \min \sum_{v \in V} \sum_{(i,j) \in A_T^v} C_{ij}^v x_{ij}^v + \sum_{v \in V} \sum_{m \in M} \sum_{(i,j) \in A_T^m} F_{ij}^v x_{ij}^v
\]

s.t. \[
\sum_{m \in M} \eta_m - S_m \leq \beta^w N^w T \quad \forall w \in W
\]

\[
\eta_m \geq t x_{ij}^v \quad \forall (i, j) \in A_T^v (m), m \in M, t \in T, v \in V
\]

\[
S_m \leq \eta_m \leq E_m \quad \forall m \in M
\]

\[
\eta_m \in \mathbb{R}_+ \quad \forall m \in M
\]

Constraints (2) - (7)

Constraints (11) ensure that the number of downtime periods is, for each windfarm \( w \), at most \( \beta^w \) times the maximum number of production periods. Constraints (12) ensure that the variables \( \eta_m \) are equal to the last period in which maintenance task \( m \) is scheduled, and Constraints (13) impose a trivial upper and lower bound on the variables \( \eta_m \). Finally, Constraints (14) indicate that \( \eta_m \) is a continuous variable.

2.3. Two-stage stochastic programming formulation

In order to provide a concise two-stage mixed integer stochastic programming formulation for each of the three SMFTPO variants, we introduce the following notation. We gather feasibility of the second stage variables \( x, z, \eta \) in the feasibility sets \( X, Z, \eta \) so that we can denote \( x \in X \) to ensure feasibility of the second stage decision. Similar notation is used for \( z \) and \( \eta \). Let \( \phi_{ij}^{dv \ell} \) be equal to 1 if arc \( (i, j) \in A_T^v \) connects the artificial source node at least term \( \ell \) to depot \( d \). Then, the SMFTPO asks for solving

\[
z := \min \sum_{d \in D} \sum_{v \in V} \sum_{x_{dv}^v} e_{dv} x_{dv}^v + E_\xi \left[ Q(x, z, \xi) \right]
\]

s.t. \[
\sum_{d \in D} y_{dv} \leq 1 \quad \forall \ell \in L, v \in V
\]

\[
\sum_{(i,j) \in A_T^v} y_{ij}^v \phi_{ij}^{dv \ell} - x_{dv}^v \leq 0 \quad \forall d \in D, v \in V, \ell \in L
\]

\[
x \in Y, z \in Z
\]

Here, Objective (15) minimizes the sum of first-stage vessel assignments and the expected costs of the second-stage decisions. Constraints (16) ensure that a vessel is assigned at most once every period. Constraints (17) are the non-anticipativity constraints linking the first and second stage decisions. Finally, with Constraints (18) we indicate second-stage feasibility of \( x \) and \( z \).

The above formulation is for the first setting of the SMFTPO, i.e., when all maintenance tasks need to be scheduled. Replacing \( Q(x, z, \xi) \) with \( Q^z(x, z, \xi) \) or \( Q^\beta(x, z, \eta, \xi) \) results in the second SMFTPO setting in which we...
can leave a fraction \( \alpha \) of demanded technician hours unscheduled (denoted by \( z^\alpha \)) and the third SMFTPO setting where the fraction of downtime periods is at most \( \beta \) (denoted by \( z^\beta \)), respectively.

### 2.3.1. Monolithic formulation by scenario generation.

Instead of directly working with the two-stage stochastic mixed integer programming formulation above, we will consider set of generated scenarios \( \Xi \) of size \( N \) and its accompanying monolithic formulation. This formulation can directly be solved with commercial MIP solvers and is provided in Appendix A.

We will employ a Sample Average Approximation (SAA) approach to solve this monolithic formulation (see, e.g., Kleywegt et al. 2002, Santoso et al. 2005). In order to detail this procedure, we write solving the scenario-based formulation of the SFTMPO as

\[
\varepsilon^{\text{SAA}} := \min_{y \in Y} \left[ \hat{C}y + \sum_{\xi \in \Xi} \frac{1}{N} \left[ Q(x^\xi, z^\xi, \xi \mid x^\xi \in X^\xi, z^\xi \in Z^\xi) \right] \right]
\]  

(19)

Here, the superscript \( \xi \) in the second-stage decision variables \( x^\xi \) and \( z^\xi \) denotes that we explicitly define the variables for each scenario \( \xi \in \Xi \) (see also appendix A).

Then, SAA consists of the following two steps:

1. We take \( M \) samples of \( N \) scenarios. Let \( \varepsilon_i^{\text{SAA}} \) denote the solution of the monolithic MIP imposed by the \( N \) scenarios in sample \( i \leq M \). Then \( \hat{\text{LB}} := \frac{1}{M} \sum_{i=1}^{M} \varepsilon_i^{\text{SAA}} \) is an estimation of a lower bound on an optimal solution of the two-stage stochastic mixed integer program.

2. Let \( j := \arg \min_i \varepsilon_i^{\text{SAA}} \) the sample with the lowest objective value. Let \( \hat{y} \) be the corresponding first-stage solution. To estimate an upper bound, we take this first-stage solution and evaluate it on \( M' \) scenarios. That is, for each scenario \( \xi^i \), \( 1 \leq i \leq M' \), we calculate \( \varepsilon_i^{\text{SAA}} = \hat{C}\hat{y} + Q(x^\xi, z^\xi, \xi \mid \hat{y}) \). Then, we estimate an upper bound for the two-stage stochastic mixed integer program by \( \hat{\text{UB}} := \frac{1}{M'} \sum_{i=1}^{M'} \varepsilon_i^{\text{SAA}} \).

Once again, we can replace \( Q(\cdot) \) with \( Q^\alpha(\cdot) \) or \( Q^\beta(\cdot) \) to solve the monolithic formulations of the second and third setting of the SMFTPO, respectively.

### 3. Modeling decisions for the second-stage problem

In this section, we present a series of reformulation for the second-stage problem of the SMFTPO (for each setting) by imposing additional assumptions. We refer to these formulations as special cases for the second-stage problem of the SMFTPO, or in short special cases. As stated in Section 1.1.1, we identified four modeling categories that are both relevant from an optimization perspective and a practical point of view. These modeling categories are (1) the duration of maintenance tasks, (2) the allocation of vessels to depots, (3) the modeling and categorization of maintenance tasks, and (4) the induced minimum service requirements.

Table 3 provides an overview of the special cases and the corresponding modeling decisions. With regards to the task duration, a distinction is made based on whether they take more than 1 period, or not. A preprocessing step
Table 3: Overview of special cases and relation to modeling categories. Note that Special Case IV equals the SMFTPO for a single scenario $\xi$, $L = 1$ and $\hat{P} = 0$

<table>
<thead>
<tr>
<th>Special Case</th>
<th>1) task duration</th>
<th>2) vessel allocation</th>
<th>3) maintenance modeling</th>
<th>4) service requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Special Case I</td>
<td>$\geq 1$ period</td>
<td>1 depot</td>
<td>free</td>
<td>all, $\alpha, \beta$</td>
</tr>
<tr>
<td>Special Case II</td>
<td>$\geq 1$ period</td>
<td>1 depot</td>
<td>no split jobs</td>
<td>all, $\alpha, \beta$</td>
</tr>
<tr>
<td>Special Case III</td>
<td>$\leq 1$ period</td>
<td>1 depot</td>
<td>no split jobs &amp; bundles</td>
<td>all, $\alpha, \beta$</td>
</tr>
<tr>
<td>Special Case IV</td>
<td>$\geq 1$ period</td>
<td>$\geq 2$ depots</td>
<td>free</td>
<td>all, $\alpha, \beta$</td>
</tr>
<tr>
<td>Special Case V</td>
<td>$\leq 1$ period</td>
<td>$\geq 2$ depots</td>
<td>no split jobs &amp; bundles</td>
<td>all, $\alpha, \beta$</td>
</tr>
</tbody>
</table>

is required that splits the tasks that take more than 1 period into tasks that take at most a single day. The vessel allocation indicates if chartered vessels are allowed to change depots in their lease term. Special Cases I-III are single windfarm, single depot cases, and inherently assume it is not possible to change depots. Special Cases IV and V consider multiple depots, and we allow vessels to change depots during the lease term. The maintenance modeling has two distinct assumptions that we investigate. First, whether jobs can be completed by more than a single vessel (referred to as ‘free’), and second, whether the preprocessing step is taken (referred to as ‘bundles’). Finally, we specify the special cases for each of the service requirements as discussed in Section 2, which we refer to as ‘all’, ‘$\alpha$’, and ‘$\beta$’. Note that this leads us to fifteen distinct models (fifteen special cases with each three different service requirements).

In the following, we describe these special cases, show how they reduce to classic problems in the field of Operations Research, and provide an outlook of their computational efficiency. Note that, without additional assumptions, maintenance tasks might take any amount of time, vessels are allowed to use all the depots available, and all the maintenance tasks are treated individually.

Finally, to not distract the reader from our goal to highlight the impact of different modeling assumptions and to keep the exposition concise, we will assume that there are no costs attached for assigning vessels to depots for the special cases. Therefore, we will assume that the complete planning horizon is comprised of a single leaseterm for the special cases and that $\hat{P} = 0$.

3.1. Special Case I: Single wind farm

In the case of a single depot, there is no need to track the vessels’ depot and the vessels’ wind farm location over time. Hence, constraints ensuring that vessels can only depart from, and work in, a single wind farm or depot are abundant. The analysis in the following will also hold for multiple wind farms when vessels cannot change depot, since, then, the problem can be decomposed in a straightforward way.

Assuming a single wind farm allows the reformulation of the second-stage problem of the SMFTPO into (variants of) the capacitated facility location problem. In facility-location terms, we need to open capacitated facilities (a vessel traveling to the wind farm in a particular period), and assign customers’ demand (technician hours) to the opened facility. Let $y_{tm}^v$ be the fraction of the demanded technician hours of task $m$ fulfilled by vessel $v$ in period $t$. Let $u_{vt}^v$ be a binary decision variable whether vessel $v$ is deployed in period $t$. Let $U_{vt}^v$ be the total
technician hours that can be supplied from vessel \( v \) in period \( t \). Let \( C_{tm} \) be the costs of maintaining task \( m \) in period \( t \), and let \( G_v^t \) be the fixed vessel deployment costs.

We need to impose upper bounds \( Y_v^t \) on the variable \( y_{v tm}^t \) in the following way. First, if task \( m \) does not exist in period \( t \), we set \( Y_v^t = 0 \) for all \( v \in V \). Second, the fraction \( (D_m^{1m}S_m^1)/(D_m^{1m}D_m^{2m}) \) indicates the maximum fraction of technician hours that can be sent from vessel \( v \) to task \( m \) in period \( t \). This ensures, for example, that if a task demands 3 technicians for 16 hours, no supply of 48 (3 \( \times \) 16) technician hours in a single period can be sent to satisfy the tasks’ demand. Hence we set \( Y_v^t = \min\{1, (D_m^{1m}S_m^1)/(D_m^{1m}D_m^{2m})\} \). Notice that this still allows for multiple vessels supplying a particular task with more technicians than possible in a single period. However, this is typically not observed in the resulting optimal solutions due to the set-up costs of visiting a wind farm, and we, therefore, do not include additional constraints to cover those scenarios. Then, this special case is solved by finding

\[
Q_{\alpha f 1}(\hat{y}, u \mid \xi) := \min \sum_{t \in T} \sum_{m \in M} \sum_{v \in V} C_{tm} \hat{y}_{v tm}^t + \sum_{v \in V} \sum_{t \in T} G_v^t u_v^t
\]

s.t.
\[
\sum_{m \in M} D_m^{1m} D_m^{2m} \hat{y}_{v tm}^t \leq U_v^t u_v^t \quad \forall t \in T, v \in V (21)
\]

\[
\sum_{v \in V} \sum_{t \in T} \hat{y}_{v tm}^t \geq 1 \quad \forall m \in M (22)
\]

\[
0 \leq \hat{y}_{v tm}^t \leq Y_v^t u_v^t \in \{0, 1\} \quad \forall t \in T, m \in M, v \in V (23)
\]

Objective (20) minimizes the fixed costs of deploying a vessel and the task specific costs. Constraints (21) ensure that no more technicians are deployed than possible in each period. Constraints (22) ensure that all tasks are performed, and Constraints (23) indicate the domain of the decision variables. The formulations \( Q_{\alpha f 1}(\hat{y}, u \mid \xi) \) and \( Q_{\beta f 1}(\hat{y}, u, \eta_1, \eta_2 \mid \xi) \) are similar as discussed in Section 2, i.e., they model the minimum service requirements in which at most \( \alpha \) unscheduled technician hours are allowed and in which the fraction of downtime periods is at most \( \beta \), respectively. To keep our exposition concise, we provide these formulations in Appendix Appendix B.

3.2. Special Case II: Single wind farm and dedicated vessels

An often encountered constraint is that a single maintenance task should completely be performed by a single vessel. We refer to this assumption as ‘dedicated vessels’. This implies that the we need additional binary variables \( \lambda_m^v \) equaling 1 if vessel \( v \in V \) is assigned to task \( m \). This special case then reduces to solving
$$Q_{f_{1-v1}}(\hat{y}, u, \lambda | \xi) := \min \sum_{t \in T} \sum_{m \in M} \sum_{v \in V} \hat{y}_{vm} C_{vm} + G_{tv} u_{tv}$$

\begin{align*}
&\text{s.t.} \quad \sum_{m \in M} \sum_{v \in V} \sum_{t \in T} \hat{y}_{vm} C_{vm} + G_{tv} u_{tv} \\
&\quad \sum_{v \in V} \sum_{t \in T} \hat{y}_{vm} \geq 1 \quad \forall m \in M \\
&\quad \sum_{t \in T} \hat{y}_{vm} \leq \lambda_{vm} \quad \forall m \in M, v \in V \\
&\quad \sum_{v \in V} \lambda_{vm} \leq 1 \quad \forall m \in M \\
&\quad 0 \leq \hat{y}_{vm} \leq \lambda_{vm}, u_{tv} \in \{0, 1\} \quad \forall t \in T, m \in M, v \in V
\end{align*}

Constraints (27) and (28) ensure that each maintenance task is only performed by a single vessel. Hence, finding $Q_{f_{1-v1}}(\hat{y}, u, \lambda | \xi)$ is equal to solving a facility location problem in which demand of a ‘customer’ can only be assigned to a, to be determined, subset of opened facilities. Similar to the previous exposition, the models for this special case with the generalized minimum service requirements ($Q_{f_{1-v1}}(\hat{y}, u, \lambda | \xi)$ and $Q_{f_{1-v1}}(\hat{y}, u, \lambda, \eta^1, \eta^2 | \xi)$) are provided in Appendix Appendix B.

3.3. Special Case III: Single wind, dedicated vessels, and bundle preprocessing

We can preprocess the maintenance tasks such that each vessel is assigned to a so-called bundle of tasks in every period (Gundegjerde et al. 2015). These bundles are sets of maintenance tasks that will be performed on a single day. Inherently, it is assumed that tasks will take less than a period. One could argue that tasks can be split up into smaller pieces that fit into a period, a strategy we will follow in our numerical analysis in Section 5. The major concern is the number of bundles being generated. For the $z_{\text{basic}}$ variants one can assume that each period contains at most $\phi$ tasks, the total number of bundles will be approximately $V \sum_{i=1}^{\phi} \binom{M_v}{i}$. The number of bundles might be reduced in two ways. First one could provide sophisticated enumeration algorithms in which particular bundle compositions are suboptimal. Second, one could use a dynamic discretization discovery algorithm (Boland et al. 2017) which iteratively enlarges the set $T$.

For the single wind farm case, with bundle preprocessing, we need to solve a set covering problem, i.e., one needs to assign preprocessed task bundles to a vessel in a particular period. We only need a single set of variables, as technician hours cannot be split between tasks as in the previous models. We let $B$ the set of all possible bundles of maintenance tasks. The binary parameter $\phi_b^m$ equals 1 if maintenance task $m$ is contained in bundle $b \in B$. Other notation for bundles $b \in B$ is straightforwardly obtained from the notation on maintenance tasks: $D_1^b$ and $D_2^b$ denote the demanded technicians and the demand working hours of those technicians for bundle $b \in B$, respectively. Binary variables $\tilde{y}_{tb}$ indicate whether vessel $v$ in period $t$ is assigned to bundle $b$. The upper bounds
on the $\hat{y}_{tb}$ variables are similarly obtained as their task counterpart. Then, this variant asks for solving:

$$Q_{r1-a}(\hat{y} | \xi) := \min \sum_{\tau \in T} \sum_{b \in B} \sum_{v \in V} \hat{y}_{tb} C_{tb}^v$$  \hspace{1cm} (30)

subject to:

$$\sum_{v \in V} \sum_{t \in T} \hat{y}_{tb} \phi^m_{tb} \geq 1 \hspace{1cm} \forall b \in B$$  \hspace{1cm} (31)

$$\sum_{b \in B} \hat{y}_{tb} \leq 1 \hspace{1cm} \forall t \in T, v \in V$$  \hspace{1cm} (32)

$$0 \leq \hat{y}_{tb} \leq Y_{tb} \hspace{1cm} \forall t \in T, v \in V, b \in B$$  \hspace{1cm} (33)

The Objective (30) minimizes the costs of assigning bundles to vessels, and Constraints (31) ensure that each maintenance task is being completed. The generalizations to the other minimum service requirements ($Q_{r1-a}^\beta(\hat{y} | \xi)$ and $Q_{r2-a}^\beta(\hat{y} | \xi)$) are provided in Appendix Appendix B. The above formulation classifies as a traditional set-covering formulation.

3.4. Special Cases IV and V: Multiple farms and bundle preprocessing

We refer to the second-stage problem of the SMFTPO (see (1) - (14)) with $\hat{P} = 0$ and $L = 1$ as Special Case IV. We like to impose the idea of bundle preprocessing on the general second-stage problem of the SFTMPO, and refer to that as Special Case V.

To consider job bundles instead of individual tasks, we need to merge the task nodes, as illustrated in Figure 2, into sets of tasks. We thereby implicitly assume that only complete tasks are part of a bundle, otherwise the number of generated bundles becomes too large, or one should allow a separate flow for each task in the bundle thereby undoing the whole benefit of introducing bundles. We do so by preprocessing the maintenance tasks in tasks of at most a single day, see Section 4.

Let us revise some of the notation. First, we disregard the flow variables $z_{ij}$, since selecting an arc entering a bundle immediately implies performing all the tasks within the bundle. This implies that the flow conservation (i.e., Constraints (4)) are modelled in terms of the $x_{ij}$ variables. Moreover, we assume that the task-specific maintenance costs $F_{ij}^v$ are incorporated in the $C_{ij}^v$, as we only have binary decision variables. Let $A^T_{ij}(b)$ be the set of incoming arcs of bundle $b \in B$. Then the basic formulation with bundle preprocessing reduces to finding

$$Q_4(x | \xi) := \min \sum_{v \in V} \sum_{(i,j) \in A^T_{ij}} C_{ij}^v x_{ij}^v$$  \hspace{1cm} (34)

subject to:

$$\sum_{(i,j) \in A^T_{ij}(n)} x_{ij}^v \leq 1 \hspace{1cm} \forall n \in N^T_{ij}, v \in V$$  \hspace{1cm} (35)

$$\sum_{(i,j) \in A^T_{ij}(n)} x_{ij}^v \leq 1 \hspace{1cm} \forall n \in N^T_{ij}, v \in V$$  \hspace{1cm} (36)

$$\sum_{(i,j) \in A^T_{ij}(n)} x_{ij}^v - \sum_{(i,j) \in A^T_{ij}(n)} x_{ij}^v = 0 \hspace{1cm} \forall n \in N^T_{ij}, v \in V$$  \hspace{1cm} (37)

$$\sum_{v \in V} \sum_{(i,j) \in A^T_{ij}} x_{ij}^v \phi^m_{ij} \geq 1 \hspace{1cm} \forall m \in M$$  \hspace{1cm} (38)

$$x_{ij}^v \in \{0, 1\} \hspace{1cm} \forall (i,j) \in A^T_{ij}, v \in V$$  \hspace{1cm} (39)
Objective (34) minimizes the costs of traveling along edges in the network. Constraints (35) and (36) ensure that vessels cannot be split between different depots and wind farms. Constraints (37) ensure that flow is conserved in all nodes but the source and sink nodes. Constraints (38) ensure that all tasks are being performed, and Constraints (39) indicate the domain of the \( x \) variables.

As in the previous expositions, the formulations for the other minimum service requirements \( Q^a(x | \xi) \) and \( Q^b(x | \xi) \) are provided in Appendix Appendix B.

4. Numerical Results

The goal of this section is twofold. First, we provide a numerical analysis of the second-stage special cases of the SMFTPO as discussed in Section 3. We assess their computational tractability on a set of newly created benchmark instances. Second, we provide an analysis of the three different settings of the SMFTPO by solving the two-stage stochastic optimization models, using the the monolithic formulations and an SAA approach.

In the following, we first describe how we constructed the benchmark instances for the second-stage models. Afterward, we show the performance of the different special cases on the benchmark instances. We analyze the proposed solutions by the different formulations and assess which formulations are the most suitable for particular offshore wind scenarios. Then, we discuss the implications for managers and scientists in the field of offshore wind maintenance service logistics, in which we focus on how the identified modeling categories (Sections 1.1.1 and 3) relate to the presented results. Then, we discuss the benchmark instances used for testing the two-stage stochastic models, and we finally present results on that.

4.1. Benchmark instances for the second-stage models

The newly constructed benchmark instances are inspired on the works of Gundegjerde et al. (2015) and Stålshane et al. (2016a). In addition, information and knowledge gathered from industry partners participating in our research project on “Sustainable service logistics for offshore wind farms”3 is used as well. The analysis consists of two parts, one for comparing the different models and assumptions for the single wind farm case (Benchmark Set A - Special Cases I-III), and one for comparing it in the context of multiple wind farms (Benchmark Set B - Special Cases IV-V).

Within the benchmark sets, the instances differ in the number of turbines in the wind farm(s) and in the total number of periods. Three instances are randomly constructed, as described below, for each combination of the number of wind farms and the number of periods. This results in 30 instances for Benchmark Set A and 24 instances for Benchmark Set B. See Table 5 and 6 for an overview of the instances and the corresponding solutions.

The maintenance tasks are generated as follows. We consider five different maintenance categories: (i) small preventive maintenance, requiring 2 - 4 technicians for 2 - 12 hours; (ii) large preventive maintenance, requiring 2 - 6 technicians for 12 - 24 hours; (iii) small corrective maintenance, requiring 2 - 3 technicians for 2 - 6 hours;

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3https://www.rug.nl/cope/projecten/servicelogistiek-windmolens
(iv) severe corrective maintenance, requiring 3 - 5 technicians for 12 - 36 hours; (v) Lifting operations, requiring 3 technicians for 2 hours. Each turbine has a tuple $p = < p_1, p_2, p_3, p_4, p_5 >$ denoting the probability of a maintenance event of each type. Technician demand and required hours are then drawn uniformly from the intervals as specified in the former. We ensure that the number of lifting operations is smaller or equal to the number maintenance tasks performed. We ignore any precedence relations between maintenance tasks (see, e.g., Gundegjerde et al. 2015).

An overview of the considered transportation modes is provided in Table 4. We assume all the transportation modes are available in each period. Although varying fleets over time can be handled by the special cases, we choose to keep the fleet composition constant to not overly complicate the analysis.

Weather conditions limit the vessel’s daily availability. For this, historical data (see, e.g., Uit het Broek et al. 2019) is used similar to Gundegjerde et al. (2015). A Weibull distribution with shape 2.17 and scale $1.128 \bar{\varphi}$, where $\bar{\varphi}$ is the mean wind speed of an arbitrarily drawn month from the historical data, is used to generate wind speeds for all the periods. We assume all the drawings are independent, similar to Gundegjerde et al. (2015).

The vessels’ daily working hours are affected by the observed wind speed $\varphi_t$ in period $t$ in the following way: If $\varphi_t$ is larger than the maximum allowed wind speed $\varphi_{safe}^v$ than no operation are allowed. If $\varphi_{safe}^v > \varphi_t$, the working hours of vessel $v$ in period $t$ are reduced to $\min\{S_v^2, \varphi_{safe}^v - \varphi_t\}$ hours. For the single wind farm case, we directly incorporated further travel and transfer times between depot and wind farm into the vessels’ daily working hours.

<table>
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<th>work. hours ($S_2^v$)</th>
<th>Travel speed</th>
<th>Fuel cost</th>
<th>max wind speed</th>
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<td>50</td>
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<td>20 m/s</td>
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<td>Large CTV</td>
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<td>12 h</td>
<td>35</td>
<td>25</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Helicopter</td>
<td>4</td>
<td>10 h</td>
<td>200</td>
<td>30</td>
<td>20 m/s</td>
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<tr>
<td>Supply Vessel (with lifting)</td>
<td>20</td>
<td>12 h</td>
<td>35</td>
<td>20</td>
<td>35 m/s</td>
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</table>

4.2. A comparison of second-stage special cases

We provide an overview of the computational performance of the different formulations in two parts. First, we consider Benchmark Set A and provide an overview of the three variants of minimum service requirements models for the single wind farm case, with and without dedicated vessels, with and without bundle preprocessing. In other words, we obtain the Special Case I-III solutions for each of the three settings regarding minimum service requirements. The results are presented in Table 5. Next, we solve the instances of Benchmark Set B as Special Cases IV and V, i.e., with and without bundle preprocessing. These results are provided in Table 6. We will omit the arguments of the $Q(\cdot)$ functions to enhance readability.

All the instances are solved by means of CPLEX 12.8.0 via its callable library in C++. The runtime is limited to 10800 seconds or when 24gb of RAM is used. At most four parallel threads are exploited by CPLEX. In the following, we will discuss the results presented in Tables 5 and 6. Initial experiments have shown that the results are robust with respect to the values of $\alpha$ and $\beta$, and we, therefore, fixed those on 0.0175 and 0.02, respectively.
4.2.1. The single wind farm case.

We compare solving the Special Case models assuming a single wind farm \((Q_1, Q_1^a, \text{ and } Q_1^b)\), assuming a single wind farm with dedicated vessels \((Q_{1-v}, Q_{1-v}^a, \text{ and } Q_{1-v}^b)\), and assuming a single wind farm with bundle-preprocessing \((Q_{1-b}, Q_{1-b}^a, \text{ and } Q_{1-b}^b)\). Recall that, the Special Cases I and IIs are (variants of) the capacitated facility location problem, whereas the Special Case III models are (variants of) a set-covering formulation (see Section 3 for further details).

The models with bundled maintenance tasks assume that all tasks can be performed within a single period, hence a preprocessing step is done to convert the instances so that the models \((Q_{1-b}, Q_{1-b}^a, \text{ and } Q_{1-b}^b)\) can be solved: Every task lasting more than 8 hours is partitioned into tasks of 8 hours and a task that takes the remaining hours. In this way, every task can be performed within a period. Preliminary experiments have shown that this provides us with the most convenient and comparable instances, i.e., a significant amount of instances becomes infeasible if we increase the maximum task duration to more than 8 hours. In addition, we ensure that there is no bundle containing multiple tasks that correspond to the same original task.

In Table 5, the results of solving all the models is provided. Some instances of the Special Case III models \((Q_{1-a})\) were not solved to optimality (indicated with an asterisk), but their final optimality gaps were so small (0.20% or smaller) that reporting it is not relevant. It is observed that the runtime of all the models become larger if the instance sizes grow, which is expected. Significant differences are observed between instances of the same size, which is typically caused by differing distances between wind farm and depot. Larger distances inherently complicate the planning process, which is expressed by the runtime for the particular instances.

A few observations stand out. First, averaging over all the results, the special cases with the \(\beta\) downtime setting take on average 969 seconds compared to on average 25 and 140 for the second setting where \(\alpha\) technician hours are unmet and the first setting where all jobs need to be scheduled, respectively. Second, although \(\alpha\) is only small (0.02), the decrease in cost-estimation between the first and second setting is on average 10.13%. Third, the differences between whether or not vessels are dedicated (Special Case II) or not have their influence on both the running time and on the objective values. The objective values raise on average with 5.10%, 4.34%, and 5.43% for the three settings of the SMFTPO (i.e, \(Q_{1-v}, Q_{1-v}^a, \text{ and } Q_{1-v}^b\)), respectively. The runtime remarkably decreases when vessels are dedicated, which might be explained by a more restricted solution space. In other words, branch and bound is more efficient which decreases the runtime of the instances that are difficult to solve, e.g., the runtime of instance 25 reduces from 6623 seconds to 2757 seconds.

Fourth, the effect of \(\beta\) on the resulting objective values is not very large, i.e., the objective value increases with 1.27%, 1.59%, and 0.80% for the single wind farm formulation, the single wind farm with dedicated vessels formulation, and the bundled formulation, respectively. Finally, the average increase in objective value that is observed when using the bundled formulations (Special Cases III) instead of the non-bundled formulations (Special Cases I and II) is remarkable. This increase equals 9.91%, 9.03%, and 9.40% for the three SMFTPO settings, respectively. Comparing the runtime, those differences are 10779%, 7444% and -98% for the three settings, respectively.
The last result has severe practical implications for operations managers in offshore wind maintenance service logistics. Although the bundled formulations have their popularity (see, e.g., Gundegjerde et al. 2015), and are quite intuitive to incorporate and model, they come at an overestimation of the actual maintenance planning costs. The major difficulty is how the bundles are generated, and how tasks that take more than a single period are split up over multiple tasks. Summarizing, taking into account the cost estimation increase and the differences in runtime, only for the third SMFTPO setting (with at most \( \beta \) downtime periods) one might prefer the use of a bundled formulation. For the other minimum service requirements (all the tasks or the at least \( \alpha \) technician hours variant), we advise to using the non-bundled formulations.

4.2.2. The multiple wind farm case.

The results of solving the Special Case IV models \( Q(\cdot) \), \( Q'(\cdot) \), and \( Q^b(\cdot) \) and their bundle-preprocessed counterparts (the Special Case V models) \( Q_\alpha(\cdot) \), \( Q'_\alpha(\cdot) \), and \( Q^b_\alpha(\cdot) \) are given in Table 6. The columns headed with “UB”, “LB”, and “Sec” denote the best upper bound, the best lower bound, and the runtime of the solver, respectively.

What stands out is the difference in computational efficiency between the Special Case IV and V models. The average optimality gap for the Special Case IV models equals 10.00% against 0.07% for their bundled counterparts formulations. Since the lower bounds of the basic formulations are significantly lower than the optimal solutions found by the bundled formulations, we infer that the bundled formulations are overestimating the costs similarly as in the single wind farm case, but no hard conclusions can be drawn from this.

Regarding the difference between the cost estimations for different minimum service requirement policies, we focus on the models with bundled tasks (Special Case V). Average cost differences of -10.18% and 6.10% are observed for the \( Q'(\cdot) \) and \( Q^b(\cdot) \) settings over the \( Q(\cdot) \) setting, respectively. The 6.10% increase of incorporating the at most \( \beta \) downtime periods constraints stands out when compared with the increases around 2% in case of a single wind farm. Finally, similar to the single wind farm case, the second setting of the SMFTPO (\( \alpha \) technician hours unscheduled) is relatively difficult to solve compared to the other two minimum service requirement settings.

4.2.3. Implications.

The analysis of the results in Tables 5 and 6 made clear that it is important to have a good understanding of the different models and the underlying assumptions since it severely impacts the accuracy of the cost-estimations for tactical maintenance planning at offshore wind farms. What stands out is that the effect of bundling maintenance tasks. It essentially assumes that the tasks will be performed in a single period, by a single vessel, and this simplifies the underlying optimization problem at the expense of a quite expensive preprocessing step (of which the computation times are not taken into account in the results). This relates to the first three Modeling Categories, as identified in Sections 1.1.1 and 3. Especially when multiple wind farms are included and when the necessary skills to develop sophisticated solution approaches are lacking, the bundled tasks are efficient and effective for obtaining quick cost-estimations for medium-term maintenance planning problems.

The effect of the different minimum service requirement settings (Modeling Category 4) is also noteworthy. Operations managers should be aware of the fact that, when a fraction of jobs can be left unscheduled in the
Table 5: Computation performance of the different models and formulations to the instances of Benchmark Set A.

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The entries marked with '-' are infeasible due to bundle preprocessing. The \(\varepsilon_{\text{basic}}\) formulations of instance 9 are not solved to optimality (but with optimality gaps around 1 \%).
Table 6: Computation performance of the different models and formulations to the instances of Benchmark Set B.

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Average: 57270  52895  8234  54721  46625  8129  57249  52904  8349  57467  57467  4  51618  51499  1415  60968  60967  7

The entries marked with '-' are infeasible due to bundle preprocessing.
mathematical model, a multiplying effect is observed with regards to the cost estimations, i.e., not satisfying 2% of the demanded technician hours will result in cost-decreases of more than 2%. This is explained by noticing that in such cases the most expensive tasks will be left (partly) unscheduled. However, it might result in a more realistic description of the actual behavior of the maintenance service provider, as such minimum service requirements are commonly encountered. Hence, if the models as presented in this paper will be used for predicting the behavior of maintenance providers, it is advisable to include such minimum service requirements in the mathematical model formulation.

Furthermore, when a restriction in downtime periods is included in the minimum service requirements, it can be seen that the resulting estimated cost-increases will slightly increase. If such requirements are part of a service contract between wind farm owner and maintenance provider, it is likely that the maintenance provider will ask a higher price for the maintenance operations. The results have shown, however, that this increase should only be slight. This holds as well for the assumption that a maintenance task cannot be performed by multiple vessels, although this assumption reflects upon the operations of the maintenance provider and is less likely to be part of the service contract negotiations.

Finally, the implications are not only of interest for practitioners (e.g., operations managers at wind farms), but also provide guidance for scientists to build upon the models we propose for the SMFTPO settings and its special cases.

4.3. Computational results of the SMFTPO

In the following, we present experiments on the three settings of the SMFTPO in its two-stage stochastic optimization model. We briefly discuss how we generated a suitable set of benchmark instances and the configuration of the Sample Average Approximation algorithm. Consequently, we analyze the obtained solutions and provide insights in the computational difficulty.

As as indicated by the results of the Special Case IV and V models, the pre-processed maintenance tasks (i.e., considering bundles) is computationally the most efficient. We, therefore, apply this preprocessing and solve the instances as such in the following.

4.3.1. Benchmark instances for the SMFTPO.

To evaluate the three settings of the SMFTPO in the full two-stage stochastic optimization model by means of Sample Average Approximation (SAA), two main ingredients are required. First, we need to be able to draw scenarios in an independent and identical fashion, and second, we need a series of benchmark instances with varying characteristics of the base system. The scenarios are identically generated as described for the previous benchmark instances with the major distinction that the previous benchmark instances were in fact single scenario realizations, and we use them in a sampling context as part of the SAA approach.

We differed the base system by the number of considered lease terms and the number of periods per lease term, see Table 7 for detailed characteristics. We considered three windfarms that each can be served from a single depot. The wind farm sizes are set to 30 turbines per farm. We consider the same vessel types as denoted in Table 4. The
fleet is of a relatively large size: The number of small CTVs equals twice the number of depots, the number of medium sized CTV’s equals the number of depots, we considered two helicopters and a single supply vessel. Their charter prices per period equal 3500, 5000, 10000, and 25000 respectively. The penalty \( \hat{p} \) for changing depots within a lease-term equals 2500.

4.3.2. Sample Average Approximation, EVPI, and VSS.

The SAA algorithm, as explained in Section 2, is used to evaluate the three settings of the SMFTPO on the benchmark instances described in the former. The lower bound estimates are obtained by averaging the lower bounds of \( M = 96 \) samples of \( N = 25 \) scenarios. Preliminary experiments have shown that this ensures a relatively low variance of the obtained estimators. Then, an upper bound estimation is obtained by selecting the first-stage solution corresponding to the solution with lowest cost from the \( M = 96 \) samples. This first-stage solution is then evaluated on \( M' = 1000 \) scenarios. The average objective from these \( M = 1000 \) evaluations is an upper bound estimation of the SMFTPO.

Except for the lower and upper bound estimations, we are interested in two additional estimations. First, we are interested in the so-called Expected Value of Perfect Information (EVPI), which expresses the amount by which costs are reduced in a perfect information situation. We calculate it by solving the complete two-stage stochastic optimization model for each of the \( M' = 1000 \) scenario’s obtained for estimating the upper bound. The average is then an estimator for the perfect information solution. The difference between this perfect information solution and the estimated upper bound is an estimation of the EVPI.

In addition, we are interested in the so-called Value of the Stochastic Solution (VSS). It expresses the reduction in costs if one considers the two-stage optimization model instead of using simply the expected values of the uncertain parameters. This expected value solution is obtained by solving the two-stage optimization model with a single scenario where all parameters are equal to their expected value. We evaluate this solution on \( M' = 1000 \) individually drawn scenarios and average the obtained objective values. The VSS is then the difference between this average and the estimated upper bound.

Finally, for computational reasons we terminate the MIP solver if the optimality gap is smaller than 2%. The experiments are performed on two Intel Xeon E5 2680v3 CPUs, totalling 24 cores. Each MIP is solved by four dedicated threads. We allow for a maximum of three days to calculate lower bound, upper bound, EVPI and VSS. Finally, to ensure relatively complete recourse decisions we include artificial binary variables (to perform an individual maintenance task) that we penalize at 15000 each.

4.3.3. Results of the SAA.

The results of solving all the benchmark instances as each of the three SMFTPO settings are given in Table 7. For each setting, we provide the estimated lower bound (in \( \cdot 10^3 \)), upper bound (in \( \cdot 10^3 \)), and the EVPI (in \( \cdot 10^3 \)). We further provide the difference \( \Delta(z, z^\alpha) \) in percentages of using \( z^\alpha \) instead of \( z \), and similarly \( \Delta(z, z^\beta) \) denotes this difference for using \( z^\beta \) instead of \( z \). We do no provide the VSS in this table as its values are very large, i.e, the estimated expected value solution is 2-5 times as large as the estimated upper bound. The explanation is simple;
Table 7: Overview of the solutions of the two-stage optimization model

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$\tilde{L}$, $\tilde{U}$, and $\tilde{E}_{VPI}$ are given in $\cdot 10^3$. The variance of the estimators in the order of 500 - 2000, and is not presented to enhance readability reasons. Instances 9-11 and 16-18, setting $\varepsilon^\alpha$, are solved with 18 scenarios instead of 24 to estimated the lower bound.

The difficulty of the SFMTPO is caused by the high variability in weather conditions and in maintenance activities which is not reflected by the ‘nicely’ behaving expected value solution.

First it should be noted that the estimated lower and upper bounds for all the SFMTPO settings, and all the instances, are rather tight. As we terminated the MIP solver at 2%, this gap should is at least 2%, and the differences are only slightly larger. Second, we see that the estimated differences between scheduling all tasks and leaving $\alpha$ technician hours unscheduled (i.e., $\Delta(z, z^\alpha)$) is becoming larger for longer planning horizons, whereas for $\Delta(z, z^\beta)$ no such trend is visible. This is explained by the fact that for longer planning horizons more variability in the uncertainty realizations is observed, which increases the importance to anticipate upon the uncertain demand by not scheduling maintenance tasks.

The EVPI is, as expected, increasing for instances with longer planning horizons. This is trivial, as a longer planning horizon includes more uncertainty to deal with, and therefore a larger variability (in absolute value) in the uncertainty realizations. We furthermore observe that the EVPI is rather small for a tactical planning problem as the one we propose. Namely, although vessels will be allocated more in the solution corresponding to the estimated upper bound (i.e., the stochastic programming solution), the effect on individual perfect information solutions is not devestating. Still, the EVPI is around 10%, which might motivate further research in this area for more dynamic
planning algorithms.

5. Conclusion

In this paper, we introduced the Stochastic Maintenance Fleet Transportation Problem for Offshore wind farms (SMFTPO). In the SMFTPO, we aim to find a cost-minimizing assignment of maintenance tasks to vessels while controlling for uncertain maintenance tasks and weather conditions. We take the viewpoint of a single maintenance provider which is responsible for the maintenance at multiple wind farms, a situation often encountered in practice. This is in high contrast with existing research on tactical and strategic decisions in offshore wind maintenance service logistics; current research does not make the distinction between the maintenance provider and wind farm owner.

We introduced the notion of minimum service requirements, in the context of offshore wind, describing the contractual obligations of the logistics provider to the wind farm owner. Three settings of those minimum service requirements were considered. We then modeled those settings of the SMFTPO as a generic two-stage stochastic mixed integer programming model. The second-stage problem is modelled on a decomposed and time expanded-network, and Sample Average Approximation was used to solve a scenario-based large-scale mixed integer programming model.

In addition, by means of a thorough literature review, we provided an overview of the key-modeling decisions in related work by presenting four modeling categories in the context of offshore wind maintenance service logistics. To gain insights into the (computational) tractability of those key-modeling decisions, we presented five (re)formulations of the second-stage problem of each of the three SMFTPO settings.

We provided a new set of benchmark instances, which we made publicly available, and used those to assess the computational performance of all the models. First, focussed on the (re)formulations of the second-stage problems of the SMFTPO. It is shown, amongst others, that an established method for bundling maintenance tasks results in overestimating medium-term maintenance costs but is a computationally attractive. Moreover, it is shown that incorporating additional constraints incentivizing quickly scheduling maintenance tasks is especially costly in a multiple wind farm setting. Second, we studied the solutions of the two-stage stochastic optimization model for each of the three SMFTPO settings. It is clearly shown that considering uncertainty is a must, as the value of the stochastic solution is large, while on the other hand, the expected value of perfect information is rather small.

The directions for further research are numerous. A natural extension of this exploratory work is the incorporation of the notion of minimum service requirements in a multi-stage stochastic programming model, instead of a two-stage stochastic programming model. The development of such multi-stage models goes hand-in-hand with the need for advanced solution algorithms (e.g., integer L-shaped algorithms). Up until now, no such advanced algorithms exist in the context of strategic and tactical offshore wind maintenance service logistics. Such algorithms are required for solving practical cases in a more dynamic fashion since the increasing number of offshore wind farms and increased collaboration between the wind farms complicates the optimization problem significantly.
Furthermore, building upon the increasing popularity of approximate dynamic programming (Powell 2007), a fundamentally different approach might be interesting to pursue. Namely, to model the SMFTPO by means of a stochastic dynamic program, instead of a two- or multi-stage stochastic mixed integer program. A comparison of the required level of dynanism in such optimization problems is interesting for researchers and practitioners.

A different, but not less interesting extension could be the development of polynomial approximation algorithms to obtain cost-estimations for particular offshore wind scenarios. Then tactical planning models, as the ones developed in this paper, might be incorporated in real-time decision making which requires instant calculation of cost-estimations.

Acknowledgements

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References


Andersen, J., Christiansen, M., Crainic, T. G., Grønhaug, R., 2011. Branch and price for service network design with asset management constraints. Transportation Science 45 (1), 33–49.


Appendix A. Monolithic formulation for solving the SMFTPO

Here we detail the complete MIP for solving the scenario-based formulation of the SMFTPO. We only provide this for the first setting of the SMFTPO (i.e., where all the jobs need to be performed).

\[
\begin{align*}
\min & \quad \sum_{d \in \mathcal{D}} \sum_{\ell \in \mathcal{L}} \sum_{v \in \mathcal{V}} \hat{C}^{v}_{d} y^{v}_{d \ell} + \frac{1}{N} \sum_{\xi \in \Xi} \left[ \sum_{v \in \mathcal{V}} \sum_{(i, j) \in \mathcal{A}_{\ell}^{v}} \nu^{v}_{i j} x^{v}_{i j} + \sum_{v \in \mathcal{V}} \sum_{m \in \mathcal{M}} \sum_{(i, j) \in \mathcal{A}_{\ell}^{v}} F^{v}_{i j \xi} z^{v}_{i j} \right] \\
\text{s.t.} & \quad \sum_{d \in \mathcal{D}} y^{v}_{d \ell} \leq 1 \quad \forall \ell \in \mathcal{L}, v \in \mathcal{V} \quad (A.2) \\
& \quad \sum_{(i, j) \in \mathcal{A}_{\ell}^{v}} y^{v}_{i j} d^{d}_{i j} - x^{v}_{d \ell} \leq 0 \quad \forall d \in \mathcal{D}, v \in \mathcal{V}, \ell \in \mathcal{L}, \xi \in \Xi \quad (A.3) \\
& \quad \sum_{(i, j) \in \delta^{+}(n)} x^{v}_{i j} \leq 1 \quad \forall n \in \mathcal{N}_{\ell}^{T,v,\xi}, v \in \mathcal{V}, \xi \in \Xi \quad (A.4) \\
& \quad \sum_{(i, j) \in \delta^{-}(n)} x^{v}_{i j} \leq 1 \quad \forall n \in \mathcal{N}_{\ell}^{T,v,\xi}, v \in \mathcal{V}, \xi \in \Xi \quad (A.5) \\
& \quad \sum_{(i, j) \in \delta^{+}(n)} z^{v}_{i j} - \sum_{(i, j) \in \delta^{-}(n)} z^{v}_{i j} = \delta^{v}_{n} \quad \forall n \in \mathcal{N}^{T,v,\xi}, v \in \mathcal{V}, \xi \in \Xi \quad (A.6) \\
& \quad z^{v}_{i j} \leq U^{v}_{i j} x^{v}_{i j} \quad \forall (i, j) \in \mathcal{A}_{\ell}^{v}, \xi \in \Xi \quad (A.7) \\
& \quad \sum_{v \in \mathcal{V}} \sum_{(i, j) \in \mathcal{A}_{\ell}^{v}} z^{v}_{i j} \geq D^{n}_{\ell} \quad \forall m \in \mathcal{M}, \xi \in \Xi \quad (A.8) \\
& \quad x^{v}_{i j} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A}_{\ell}^{v}, \xi \in \Xi \quad (A.9) \\
& \quad z^{v}_{i j} \in \mathbb{R}_{+} \quad \forall (i, j) \in \mathcal{A}_{\ell}^{v}, \xi \in \Xi \quad (A.10) \\
& \quad y^{v}_{d \ell} \in \{0, 1\} \quad \forall d \in \mathcal{D}, \ell \in \mathcal{L}, v \in \mathcal{V} \quad (A.11)
\end{align*}
\]

The above formulation is similar to the stochastic mixed integer programming formulation presented in Section 2. Only difference is that all second-stage decision variables are indexed by \( \xi \), and in the objective we take the average costs of all the second-stage decisions.
Appendix B. Additional MIP formulations of special cases

This appendix provides the MIP formulations referred to in Section 3.

Appendix B.1. Special Case I: Single wind farm

No additional variables or notation is required for finding \( Q_{\alpha}^{f} (\hat{y}, u, \lambda | \xi) \), i.e, the generalization of the single wind farm case to include the \( \alpha \)-uptime minimum service requirements. Notice we drop the indices \( w \in W \), since we assume that there is only a single wind farm. This special case asks for solving

\[
Q_{\alpha}^{f} (\hat{y}, u, \lambda | \xi) := \min \sum_{m \in M} \sum_{v \in V} C_{im} \hat{y}_{im} + \sum_{v \in V} G_{i}^{u} u_{i}^{v} \tag{B.1}
\]

subject to

\[
\sum_{m \in M} D_{i}^{m} \hat{y}_{im} \leq U_{i}^{v} u_{i}^{v} \quad \forall t \in T, v \in V \tag{B.2}
\]

\[
\sum_{v \in V} \sum_{t \in T} \hat{y}_{im} \geq (1 - \alpha) \sum_{m \in M} D_{i}^{m} \tag{B.3}
\]

\[
\sum_{t \in T} \sum_{v \in V} \hat{y}_{im} \leq 1 \quad \forall m \in M \tag{B.4}
\]

\[
0 \leq \hat{y}_{im} \leq \bar{Y}_{im}, \quad u_{i}^{v} \in \{0, 1\} \quad \forall t \in T, m \in M, v \in V \tag{B.5}
\]

Objective (B.1) minimizes the fixed costs of deploying a vessel and the task specific costs. Constraints (B.2) ensure that the supplied technicians do not exceed the availability. Constraint (B.3) ensures that at least \( (1 - \alpha) \) of the total technician hours are supplied. Since a task cannot be performed more than once, Constraints (B.4) are required. Finally, Constraints (B.5) denote the domain of the decision variables.

To model \( Q_{\beta}^{f} (\hat{y}, u, \eta_{1}, \eta_{2} | \xi) \) (at most \( \beta \) downtime periods with a single wind farm), additional variables are needed to track the latest period in which flow is send to a maintenance task. We define the continues variables \( \eta_{m}^{1} \) and the binary variables \( \eta_{m}^{2} \) to indicate the latest period in which task \( m \) is maintained and whether or not task \( m \) is performed in period \( t \), respectively. Then, it boils down to finding

\[
Q_{\beta}^{f} (\hat{y}, u, \eta_{1}, \eta_{2} | \xi) := \min \sum_{v \in V} \sum_{t \in T} \sum_{m \in M} C_{im} \hat{y}_{im} + \sum_{v \in V} G_{i}^{u} u_{i}^{v} \tag{B.6}
\]

subject to

\[
\sum_{m \in M} D_{i}^{m} \hat{y}_{im} \leq U_{i}^{v} u_{i}^{v} \quad \forall t \in T, v \in V \tag{B.7}
\]

\[
\sum_{m \in M} \eta_{m}^{1} - S_{m} \leq \beta NT \tag{B.8}
\]

\[
\eta_{m}^{2} \geq \hat{y}_{im}^{v} \quad \forall m \in M, t \in T, v \in V \tag{B.9}
\]

\[
\eta_{m}^{1} \geq \eta_{m}^{2} \quad \forall m \in M, t \in T \tag{B.10}
\]

\[
0 \leq \hat{y}_{im} \leq \bar{Y}_{im}, S_{m} \leq \eta_{m}^{1} \leq E_{m}, \quad u_{i}^{v}, \eta_{m}^{2} \in \{0, 1\} \quad \forall t \in T, m \in M, v \in V \tag{B.11}
\]

Objective (B.6) minimizes the fixed costs of deploying a vessel and the task specific costs. Constraints (B.7) ensure that no more technicians are deployed than possible in each period. Constraints (B.8) ensure that at most \( \beta \) downtime periods are observed in any feasible solution. Constraints (B.9)-(13) ensure that the \( \eta_{m} \) variables are modeled correctly. Finally, the domain of the variables is indicated by Constraints (B.11).
Appendix B.2. Single wind farm case with dedicated vessels

We extend model $z_{v1}^{1-v1}$ to the case in which at least $(1-\alpha)$ technician hours are supplied of the total demanded technician hours in a straightforward way. It asks for solving

$$Q_{v1}^{f}(\hat{y}, u, \lambda | \xi) := \min \sum_{t \in T} \sum_{m \in M} \sum_{v \in V} C_{tm} \hat{y}_{tm} + \sum_{v \in V} \sum_{t \in T} G_i v_i \tag{B.12}$$

subject to

$$\sum_{m \in M} D_m^1 D_m^2 \hat{y}_{tm} \leq U_{v} u_{t} \quad \forall t \in T, v \in V \tag{B.13}$$

$$\sum_{m \in M} \sum_{v \in V} \sum_{t \in T} \hat{y}_{tm} D_m^1 D_m^2 \geq (1-\alpha) \sum_{m \in M} D_m^1 D_m^2 \tag{B.14}$$

$$\sum_{m \in M} \sum_{v \in V} \hat{y}_{tm} \leq 1 \quad \forall m \in M \tag{B.15}$$

$$\sum_{m \in M} \sum_{v \in V} \hat{y}_{tm} \leq \lambda_{m} v \quad \forall m \in M, v \in V \tag{B.16}$$

$$\sum_{v \in V} \lambda_{m} v \leq 1 \quad \forall m \in M \tag{B.17}$$

$$0 \leq \hat{y}_{tm} \leq \bar{y}_{tm} \text{ and } \lambda_{m}, u_{t} \in [0, 1] \quad \forall t \in T, m \in M, v \in V \tag{B.18}$$

Objective (B.12) minimizes the fixed costs of deploying a vessel and the task specific costs. Constraints (B.13) ensure that no more technicians are deployed than possible in each period. Constraints (B.14) make sure that the fraction of not supplied technician hour is at most $\alpha$, and Constraints (B.15) ensure that at task is not served more than once. Constraints (B.16) and (B.17) ensure that each task is at most assigned to a one vessel. Finally, the variables’ domains are modeled via Constraints (B.18).
The extension to at most $\beta$ downtime periods asks for solving

$$Q^{\beta}_{r_{1,v_{1}}}(\hat{y}, u, \lambda, \eta^2 | \xi) := \min \sum_{t \in T} \sum_{m \in M} \sum_{v \in V} C_{mv} \hat{y}_{tm} + G_{mv} \hat{u}_v$$

s.t. \[\sum_{m \in M} D_{tm}^1 D_{tm}^2 \hat{y}_{tm} \leq U_{v_t} \hat{u}_v \forall t \in T, v \in V\]  
(B.19)

s.t. \[\sum_{m \in M} \eta^1_m - S_m \leq \beta^w N^w T\]  
\forall w \in W

(B.20)

s.t. \[\eta^1_m \geq \eta^2_m \forall m \in M, t \in T\]  
(B.21)

s.t. \[\eta^2_m \geq \hat{y}_{tm} \forall m \in M, t \in T, v \in V\]  
(B.22)

s.t. \[\sum_{i \in T} \hat{y}_{tm} \leq \lambda_{tm} \forall m \in M, v \in V\]  
(B.23)

s.t. \[\sum_{v \in V} \lambda_{tm} \leq 1 \forall m \in M\]  
(B.24)

s.t. \[0 \leq \hat{y}_{tm} \leq \bar{y}_{tm}, S_m \leq \eta^1_m \leq E_m, \text{ and } \eta^2_m, \lambda_{tm}, \hat{u}_v \in \{0, 1\} \forall t \in T, m \in M, v \in V\]  
(B.25)

Objective (B.19) minimizes the fixed costs of deploying a vessel and the task specific costs. Constraints (B.20) ensure that no more technicians are deployed than possible in each period. Constraints (B.21) ensure that at most $\beta^w$ downtime periods are observed in any feasible solution. Constraints (B.22)-(B.25) ensure that the $\eta_m$, $\hat{y}_{tm}$, and $\lambda_{tm}$ variables are modeled correctly. Finally, Constraints (B.26) indicate the decision variables’ domains.

**Appendix B.3. Bundle Preprocessing**

The extension of $z_{\text{BASIC}}^{1,a_{\beta}}$ to the $\alpha$ and $\beta$ minimum service requirement variants is presented in the following. The variant in which at most a fraction $(1 - \alpha)$ of the total technician hours is left unassigned asks for solving

$$Q^{\alpha}_{r_{1,a_{\beta}}} (\tilde{y} | \xi) := \min \sum_{t \in T} \sum_{b \in B} \sum_{v \in V} \tilde{y}_{tb} c_{tv}$$

s.t. \[\sum_{m \in M} \sum_{v \in V} \sum_{t \in T} \hat{y}_{tb} D_{tb}^1 D_{tb}^2 \geq (1 - \alpha) \sum_{m \in M} D_{tm}^1 D_{tm}^2 \]  
(B.28)

s.t. \[\sum_{b \in B} \tilde{y}_{tb} \leq 1 \forall t \in T, v \in V\]  
(B.29)

s.t. \[\sum_{t \in T} \sum_{v \in V} \sum_{b \in B} \tilde{y}_{tb} \theta_{tb} \leq 1 \forall m \in M\]  
(B.30)

s.t. \[\tilde{y}_{tb} \in \{0, 1\} \forall t \in T, v \in V, b \in B\]  
(B.31)
Objective (B.27) minimizes the costs of assigning bundles to vessels. Constraints (B.28) ensures that at least \((1 - \alpha)\) technician hours are supplied. Moreover, Constraints (B.29) ensure that each bundle is assigned only once, and Constraints (B.30) ensure that a maintenance task is not scheduled more than once. The domains of the decision variables are given in Constraints (B.31).

To model the \(z_{\text{basic-2}}\) variant, we introduce the integer parameter \(\xi_b\) indicating the number of downtime periods incurred when assigning bundle \(b\). The formulation then becomes

\[
Q_b^{\beta} (\bar{y} \mid \xi) := \min \sum_{t \in T} \sum_{b \in B} \sum_{v \in V} \bar{y}_{tb} c_{tb} \tag{B.32}
\]

s.t. \[
\sum_{v \in V} \sum_{t \in T} \bar{y}_{tb} \phi_m^b \geq 1 \quad \forall m \in M \tag{B.33}
\]
\[
\sum_{b \in B} \bar{y}_{tb} \leq 1 \quad \forall t \in T, v \in V \tag{B.34}
\]
\[
\sum_{m \in M} \sum_{v \in V} \sum_{t \in T} \bar{y}_{tb} \xi_b \leq \beta NT \tag{B.35}
\]
\[
\bar{y}_{tb} \in \{0, 1\} \quad \forall t \in T, v \in V, b \in B \tag{B.36}
\]

Objective (B.32) minimizes the costs of assigning bundles to vessels. Constraints (B.33) ensures that each task is performed, while not assigning more bundles to vessels on a single period than possible (Constraints (B.34)). Constraints (B.35) ensures that the downtime constraints are respected. Finally, the domain of the decision variables is indicated via Constraints (B.36).

Appendix B.4. Bundle selection in basic formulation

Introducing bundles in the \(z_{\text{basic-1}}\) formulation leads to finding

\[
Q^b (x \mid \xi) := \min \sum_{v \in V} \sum_{(i,j) \in A_{T,v}} c_{ij} x_{ij} \tag{B.37}
\]

s.t. \[
\sum_{(i,j) \in \delta^+(n)} x_{ij} \leq 1 \quad \forall n \in N_{D,v}, v \in V \tag{B.38}
\]
\[
\sum_{(i,j) \in \delta^-(n)} x_{ij} \leq 1 \quad \forall n \in N_{D,v}, v \in V \tag{B.39}
\]
\[
\sum_{(i,j) \in \delta^+(n)} x_{ij} - \sum_{(i,j) \in \delta^-(n)} x_{ij} = \delta^v \quad \forall n \in N_{T,v}, v \in V \tag{B.40}
\]
\[
\sum_{v \in V} \sum_{(i,j) \in A_{T,v}^+ (b)} x_{ij} \phi_b^m \leq 1 \quad \forall m \in M \tag{B.41}
\]
\[
\sum_{v \in V} \sum_{(i,j) \in A_{T,v}^- (b,w)} x_{ij} D_b^1 D_b^2 \geq (1 - \alpha^w) \sum_{m \in M} D_m^1 D_m^2 \quad \forall w \in W \tag{B.42}
\]
\[
x_{ij} \in \{0, 1\} \quad \forall (i,j) \in A_{T,v}, v \in V \tag{B.43}
\]

Here, we let \(A_{T,v}^+ (b,w)\) be the set of arcs incoming at bundle \(b \in B\) at windfarm \(w \in W\). Objective (B.37) minimizes the transportation and maintenance costs. Constraints (B.38) - (B.40) ensure that vessels are not split
among wind farms and depots and ensure flow conservation of technician hours. Constraints (B.41) models that a task can only be performed once, and Constraints (B.42) model that the fraction of supplied technician hours is at least \((1 - \alpha^w)\). Finally, the domain of the decision variables is denoted by Constraints (B.43).

The variant in which the fraction of downtime periods is at most \(\beta^w\) asks for finding

\[
Q^{\beta}(x \mid \xi) := \min \sum_{v \in V} \sum_{(i,j) \in A_{T,v}} C_{ij}^v x_{ij}^v \quad (B.44)
\]

\[
\text{s.t.} \quad \sum_{(i,j) \in \delta^+(n)} x_{ij}^v \leq 1 \quad \forall n \in N_{D}, v \in \mathcal{V} \quad (B.45)
\]

\[
\sum_{(i,j) \in \delta^-(n)} x_{ij}^v \leq 1 \quad \forall n \in N_{D}, v \in \mathcal{V} \quad (B.46)
\]

\[
\sum_{(i,j) \in \delta^-(n)} x_{ij}^v - \sum_{(i,j) \in \delta^+(n)} x_{ij}^v = 0 \quad \forall n \in N_{D}, v \in \mathcal{V} \quad (B.47)
\]

\[
\sum_{v \in V} \sum_{(i,j) \in A_{T,v}} x_{ij}^v \phi_m^v \geq 1 \quad \forall m \in M \quad (B.48)
\]

\[
\sum_{v \in V} \sum_{(i,j) \in A_{T,v}} x_{ij}^v \xi_b \leq \beta N_w T \quad \forall w \in \mathcal{W} \quad (B.49)
\]

\[
x_{ij}^v \in \{0, 1\} \quad \forall (i, j) \in A_{T,v}, v \in \mathcal{V} \quad (B.50)
\]

Constraints (B.44) - (B.48) are similar to the \(z_{\text{base}}\) formulation provided in Section 4. We let \(\xi_b\) be the number of downtime periods when bundle \(b\) is chosen to be performed. Then constraints (B.49) model the minimum service requirements. Finally, the domain of the decision variables is denoted by Constraints (B.50).