Is $C_{50}$ a superaromat? Evidence from electronic structure and ring current calculations†

Ana Sanz Matias,* Remco W. A. Havenith, Manuel Alcamí & Arnout Ceulemans**

The fullerene-$50$ is a ‘magic number’ cage according to the $2(N + 1)^2$ rule. For the three lowest isomers of $C_{50}$ with trigonal and pentagonal symmetries, we calculate the sphericity index, the spherical parentage of the occupied $\pi$-orbitals, and the current density in an applied magnetic field. The minimal energy isomer, with $D_3$ symmetry, comes closest to a spherical aromat or a superaromat. In the $D_{5h}$ bond-stretch isomers the electronic structure shows larger deviations from the ideal spherical shells, with hybridisation or even reversal of spherical parentages. It is shown that relative stabilities of fullerene cages do not correlate well with aromaticity, unlike the magnetic properties which are very sensitive indicators of spherical aromacity. Superaromatic diamagnetism in the $D_3$ cage is characterized by global diatropic currents, which encircle the whole cage. The breakdown of sphericity in the $D_{5h}$ cages gives rise to local paratropic countercurrents.

1 Introduction

Orbital levels of globular molecules can be associated with scalar or tensor spherical harmonic functions on the basis of a common nodal structure.1–2 Spherical aromaticity arises whenever the occupied frontier orbitals correspond to a closed parent spherical shell. For scalar harmonics these magic electron counts fulfill the $2(N + 1)^2$ rule.3 In contrast to 2D Hückel aromats, 3D spherical aromats or superaromat s are particularly hard to find, since in fact they have to obey three requirements: (i) the overall shape of the molecule has to be close to a hollow sphere, (ii) the electron count has to fulfill the magic rule, and (iii) the projection of the orbitals on the spherical harmonics must yield complete spherical shells.4

Fullerenes are hollow carbon cages formed by hexagons and pentagons which, in spite of allowing closure, bring strain to the structure.5–7 Although a plethora of fullerene cages is mathematically possible, only a few have been detected experimentally.8 In view of their approximate globular shape fullerene cages could be expected to be promising targets for spherical aromaticity. However, the two most prominent peaks in the mass spectra of evaporated graphite correspond to $C_{60}$ and $C_{70}$,9 neither of which holds a magic electron count.10,11 The exceptional stability of icosahedral $C_{60}$ has been attributed to it being the first leapfrog cage with isolated pentagons.7 The frontier orbitals of leapfrog cages are based on entangled spherical shells,12,13 which is exactly the opposite of the spherical shell model. Likewise $C_{70}$ represents the first case of a cylindrical fullerene with isolated pentagons.14 It too corresponds to a severe symmetry breaking of the spherical model.

Another intense peak in the mass spectrum corresponds to $C_{50}$. This is a magic electron count which fulfills the $2(N + 1)^2$ rule with $N = 4$. Despite theoretical research has already been carried out on its isomers,15–18 little attention has been paid to what is mentioned as a cause of their different stability: spherical aromaticity.19 This paper will be devoted, through a distinct approach, to a detailed examination of the electronic structure of $50$-fullerenes, with special attention to the role of spherical aromaticity.

1.1 Isomers of $C_{50}$

Among the 271 possible isomers of $C_{50}$, high level calculations invariably point to the extra stability of only two $C_{50}$ isomers, with $D_3$ and $D_{5h}$ symmetry.15–18 Interestingly, these are the only isomers without triplets of directly fused pentagons. They are shown in Fig. 1, together with their Schlegel diagrams.

For the $D_3$ isomer a modified Schlegel diagram was adopted to obtain a better visualization of the symmetry. Since the main

---

* Department of Chemistry, University of Leuven, Celestijnenlaan 200F, B-3001 Leuven, Belgium. E-mail: ana.sanzmatias@chem.kuleuven.be, arnout.ceulemans@chem.kuleuven.be; Fax: +32-16-327992
† Theoretical Chemistry, Zernike Institute for Advanced Materials and Stratingh Institute for Chemistry, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands. E-mail: r.w.a.havenith@rug.nl; Tel: +31-50-3637754
‡ Ghent Quantum Chemistry Group, Department of Inorganic and Physical Chemistry, Ghent University, Krijgslaan 281 (S3), B-9000 Gent, Belgium
§ Departamento de Química, Módulo 13, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain. E-mail: manuel.alcamii@uam.es; Tel: +34-91-4973857
|| Instituto Madrileño de Estudios Avanzados en Nanociencias (IMDEA-Nanociencia), Cantoblanco 28049 Madrid, Spain. E-mail: manuel.alcamii@imdea.org
† Electronic supplementary information (ESI) available: CASSF(18,12) calculation results for $C_{50} D_3$ and $C_{50} D_{5h}$, detailed results of spherical parentages for $C_{50} D_3$ and $D_{5h}$, See DOI: 10.1039/c5cp04970a
So far experimental isolation of 50-fullerenes has been limited to the preparation of the chloro-derivative, decachloro-fullerene\([\text{C}_{50}\text{Cl}_{10}]^{24}\) It was demonstrated by a variety of techniques that this derivative most likely has a \(D_{5h}\) structure, with the ten chlorine atoms attached to the equatorial pentagon–pentagon fusions.\(^{24}\) This compound was nicknamed ‘saturnene’ in view of the chlorine ring around its equator. In contrast the ground state of unsubstituted \(\text{C}_{50}\) has not yet been characterized experimentally.

## 2 Methodology

\(D_3\), \(D_{5h}\) (A) and \(D_{3h}\) (B) symmetry structures were generated with \(\text{CaGe}^{25}\) software and optimized using density functional theory (DFT). In particular, the B3LYP functional\(^{26, 27}\) and the 6-31G\(^*\) basis set were chosen. These calculations were carried out using the GAUSSIAN program.\(^{28}\) Isomerization between the A and B states was further investigated with MOLPRO.\(^{29}\) A Hückel calculation was performed in order to obtain energy level diagrams and \(\pi\) bond orders.\(^{30}\) For a given fullerene, keeping to a minimum the sum of squares of the distance from each atom to a least squares sphere yields \(S\), the least squares sphericity parameter:\(^{31}\)

\[S = \sqrt{\frac{n \sum r_i^2}{n} - 1} \]  

where \(n\) is the total number of atoms (50 in our case) and \(r_i\) is the distance from the \(i\)th atom to the geometric center of the molecule. If the atoms of the molecule lie over a perfect sphere, \(S\) is equal to zero. Cage orbitals with a well defined spherical parentage are obtained by projecting the spherical harmonic functions onto the atomic basis set:

\[Y_{|m|}^l(\theta, \phi) = N \sum_{i=1}^{n} Y_{|m|}^l(\theta_i, \phi_i)|i\]  

here \(Y_{|m|}^l\) is the real spherical harmonic obtained from the combination of the spherical harmonics defined by an angular momentum quantum number \(l\) and magnetic quantum numbers \(+m_i\) and \(-m_i\). The coordinates \(\theta_i, \phi_i\) refer to the angular position of carbon atom \(i\), and \(|i\) is the atomic \(2p_z\) basis function on that atom. \(N\) is a normalizer. Strict orthogonality of orbitals with different spherical quantum numbers is not guaranteed, since they are based on the projection of continuous spherical functions onto a finite basis set. Nevertheless for the occupied combinations deviations from orthogonality turn out to be very small. The spherical parentage of the \(k\)th Hückel molecular orbital (MO) is determined by calculating the overlap between the spherical orbitals and the Hückel result:

\[\langle Y_{|m|}^l|\text{MO}^k\rangle = N \sum_{i=1}^{n} \langle Y_{|m|}^l(\theta_i\phi_i)|C_i^k\rangle \]  

here \(C_i^k\) is the coefficient of the \(i\)th atomic orbital in the (normalized) \(k\)th molecular orbital.
Nucleus independent chemical shifts (NICS, ppm)\textsuperscript{32} were computed at the centre of the cage using the GIAO method. Current density maps were calculated at the BLYP/6-31G* level using the isopercentric choice of origin as implemented in GAMESS-UK\textsuperscript{33} and SYMMSO.\textsuperscript{34} Diatropic and paratropic currents are represented, respectively, by anticlockwise and clockwise circulations.

## 3 Results

### 3.1 Ground state of C\textsubscript{50}

A summary of the optimization calculations results can be found in Table 1. In excellent agreement with the literature, \( D_3 \) is the most stable isomer followed by \( D_{5h} (B) \) and \( D_{5h} (A) \), 2.23 and 5.58 kcal mol\(^{-1}\) higher in energy. The \( D_3 \) HOMO–LUMO gap is also significantly higher, indicative of a larger aromaticity.\textsuperscript{4} The different sphericity is due to the distribution of adjacent pentagon pairs (6 and 5, respectively) on the structures and has been pointed previously as the reason for the energy difference between these isomers.\textsuperscript{14} However, the higher sphericity of the atomic framework is not a warranty of a neat closed spherical shell electronic structure.

The difference in geometry between \( D_{5h} \) A and B isomers is most pronounced on the equatorial belt. The calculated length of C–C bonds for the equatorial pentagon–pentagon fusions is 1.396 Å for the A isomer, versus 1.478 Å for the B isomer. As noted by Lu \textit{et al.}, in the A isomer the pentaphenyl belt has a quinoid-like valence-bond structure, while in the B isomer it is a sequence of aromatic benzenes,\textsuperscript{17} as shown in Fig. 2. Accordingly the two bonding schemes can be characterized as a Fries structure for A, and a Clar structure for B.

While the two bond-stretch isomers are close in energy, their HOMO and LUMO are interchanged: for A, the HOMO has \( a_1' \) orbital symmetry and the LUMO \( a_2' \), while for B it is the opposite (HOMO \( a_2' \) and LUMO \( a_1' \)). These MOs are mainly concentrated along the equator of the molecule. The \( a_1' \) orbital is bonding on the C–C bonds at the pentagon–pentagon fusions, and thus will be occupied in the Fries structure. In contrast the \( a_2' \) is antibonding and thus will favor single bonds on these sites, yielding a chain of isolated aromatic sextets which is typical for a Clar structure. Since the two structures are close both in energy and in geometry, we have investigated whether electron correlation effects could stabilize an intermediate resonant structure.

First it should be noted that both \( D_{5h} \) isomers A and B have, as shown by a CASSCF(18,12) 6-31G calculation, an undoubtedly monodeterminantal wavefunction. The obtained energy difference between them (3.06 kcal mol\(^{-1}\), Table S9, ESI\textsuperscript{†}) is in agreement with the DFT results, which yield a 3.35 kcal mol\(^{-1}\) gap (Table 1).

In order to explore the possibility of the above mentioned resonant isomer lying lower in energy than A and B, further calculations are performed on intermediate structures. The ground state singlet and a state average calculation including the GS and first excited state singlet are carried out using CASSCF(6,6) with a 6-31G basis set. The intermediate coordinates are obtained through linear combinations (interpolation) of A and B leading to the interconversion of A and B. While no energy well is observed, there seems to be an avoided crossing at 0.6 B + 0.4 A (Fig. S1, ESI\textsuperscript{†}), a coordinate in which both A and B electronic structures coexist in a resonant structure. The barrier between B and A amounts to 33 kcal mol\(^{-1}\).

The Hückel calculation for the \( D_{5h} \) graph, with all nearest neighbour interactions equal, invariably yields the ground state of the B isomer. However by changing the orbital occupation numbers of the frontier orbitals, one can study the bonding scheme in the A isomer as well. The Hückel π bond order of C–C bonds in equatorial pentagon–pentagon fusions clearly reproduces the DFT results: in the A isomer this bond order is 0.528 in agreement with the proposed Fries structure, while for B it is 0.385, showing a preponderant Clar structure.

### 3.2 Spherical parentages

Spherical parentages of occupied molecular π-orbitals have been calculated on the basis of the Hückel eigenvectors, which only take into account the connectivity of the cage. As an example in Table 2 we report the results of overlap calculations between the nine highest occupied Hückel MOs (HMOs 17–25) and the g-type spherical harmonics. As can be seen the g-parentage of these orbitals is never lower than 81%. A global survey of the overlap percentages between HMOs and spherical harmonics is provided in Table 3.

For the \( D_3 \) cage the valence orbital shell shows a close match with the particle on a sphere model: the 25 occupied orbitals correspond to a closed spherical shell up to \( l = 4 \). This is confirmed by the energy diagram in Fig. 3. Since the symmetry

---

**Table 1.** Adjacent pentagons, relative energy difference with respect to the \( D_3 \) singlet (kcal mol\(^{-1}\)), HOMO–LUMO gap (eV), sphericity (S) and NICS (ppm) for \( D_3 \), \( D_{5h} (\text{A}) \) and \( D_{5h} (\text{B}) \) \( C_{50} \) isomers.

<table>
<thead>
<tr>
<th>Isomer</th>
<th>AP</th>
<th>( E_{rel} )</th>
<th>Gap</th>
<th>S</th>
<th>NICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_3 )</td>
<td>6</td>
<td>0</td>
<td>2.27</td>
<td>0.024</td>
<td>-40.3</td>
</tr>
<tr>
<td>( D_{5h} (\text{A}) )</td>
<td>5</td>
<td>5.58</td>
<td>1.27</td>
<td>0.037</td>
<td>-32.5</td>
</tr>
<tr>
<td>( D_{5h} (\text{B}) )</td>
<td>5</td>
<td>2.23</td>
<td>1.37</td>
<td>0.051</td>
<td>-2.7</td>
</tr>
</tbody>
</table>
and shape of the Hückel and DFT MOs are similar, the spherical shell structure is also retrieved in the DFT orbital ordering.

For $D_{3h}$, on the other hand, the results show a large difference between the A and B isomers. The Hückel result for the $D_{3h}$ cage yields a HOMO with $a_{1g}$ symmetry, which corresponds to the HOMO of the B-isomer. For this orbital the sign of the eigen-

As a result the B isomer is found to be characterized by a pseudo-spherical shell, in which one of the g-orbitals is replaced by a h-orbital. The ground state of the B isomer thus is not a spherical aromat, and consequently, the filling rule for spherical shells does not apply. In contrast, in the A isomer the HOMO has $a_{1g}$ symmetry, and can be identified as the LUMO of the Hückel calculation. This orbital has a close resemblance to the $2^g$ harmonic function of the g-shell. By occupying this orbital instead of the $a_{1g}$ HOMO, the occupation of the g-shell is completed, and the sphericity of the electronic structure is nicely restored. The results of overlap calculations thus indicate that for the $D_3$ and $D_{3h}$ (A) $C_{50}$ isomers the projection of the occupied $\pi$-orbitals on the spherical harmonics yields complete spherical shells, while for the $D_{3h}$ (B) isomer it does not. However, as we pointed out in the introduction while the mapping on closed spherical shells is a criterion for spherical aromativity, it is not sufficient. The degree of distortion of sphericity is equally important. Ring current density plots provide a way to investigate this further.

### 3.3 Ring currents

Annulenes which satisfy Hückel’s rule display a diatropic ring current when subjected to a magnetic field which is oriented perpendicular to the plane of the molecule. The diatropic ring current is mainly due to the diamagnetic response of the $\pi$-electron cloud. Likewise in true spherical aromats one expects that application of a magnetic field would induce a global diatropic rotation of the electron cloud about the axial direction of the magnetic field. To study this effect we have applied a magnetic field along the principal rotational axis of the $C_{50^*}$ isomers, and made plots of the current densities in perpendicular planes at various latitudes. Current density plots showing the contribution from the $\pi$ orbitals for the $C_{50}$ isomers are presented in Fig. 4. When the projection plane is next to an atom, current density may be higher than average, hence the large arrows that appear occasionally in the plots. As shown in Fig. 4 the current plots for the $D_3$ isomer are the ones that are most consistent with a global diatropic circulation. This is especially clear for the plots in the planes at 5.0 and 2.5 a.u., which avoid direct atomic contact. Due to the large HOMO–LUMO gap, the coefficients of the transitions are small, i.e., the total currents seem to be composed of many allowed transitions, each with a small contribution.

The case for the $D_{3h}$ geometry is less clear cut and the two isomers behave differently. For the $D_{3h}$ (A) isomer currents are still diatropic on the poles and tropics, but near the equator
paratropic counter currents are observed. The B isomer shows analogous characteristics but the paratropic aspects are more pronounced in line with the incomplete filling of the spherical g-shell. In both cases, the centered paratropic current that appears at 4.42 a.u. (Fig. 4) corresponds to the bottom pentagon of the cage. Since the projection plane is parallel to it, the plot
obtained is comparable to those of annulenes. Thus it is not a
global current but a local one, not representative of the char-
acter of the molecule as a whole.

A more detailed understanding can be obtained by analyzing
the separate excitations to virtual orbitals that contribute to the
magnetic response. Both $D_{5h}$ isomers are characterized by a
small HOMO–LUMO gap. The corresponding excitations have
the symmetry of the $z$-component of the magnetic dipole operator:

$$a'_1 \times a'_2 = A' = \Gamma(R_z)$$

(5)

The matrix element in this operator appears to be strong
since both levels share an $l = 5$ parentage.

As we have already indicated the $a'_1$ orbital has no $g$-character,
but its Hückel projection overlaps to almost 99% with the $a'_2$
component of the $l = 5$ shell. The relevant $h$-orbital is expressed as:

$$\sin^5 \theta \sin 5\phi \sim 5x^4y - 10x^2y^3 + y^5$$

(6)

The remaining $a'_1$ orbital shows a strong overlap with the $g$-shell, 92.2% according to Table 3, nevertheless it also over-
laps to a significant extent of 26% with the $a'_1$ component of the
$l = 5$ shell. The corresponding function is given by:

$$\sin^5 \theta \cos 5\phi \sim 5xy^4 - 10x^3y^2 + x^5$$

(7)

The large simultaneous overlaps with equisymmetric $l = 4$
and $l = 5$ parent harmonics indicate that we have arrived at a
point in the spectrum where orthogonality of the projected
spherical harmonics breaks down as a result of the finite size of
the atomic basis set. The affiliation of the HOMO and the LUMO
to two complementary components of the $h$-shell explains the
large paratropic contribution of this excitation, since the mixing
of both by the magnetic dipole operator will create a large orbital
moment along the $z$-axis.

As for the $D_{5h}$ A isomer, it is clear that the weak surrounding
diatomic current is generated by small contributions of many
different excitations from the $\pi$ molecular orbitals below the
HOMO (from the MOs 142–149 to the LUMO and above). However,
besides the main HOMO–LUMO excitation, other transitions seem
to be also highly contributing to the global paratropic picture, such
as 146 $\rightarrow$ 151, 147 $\rightarrow$ 151 (mainly) and 150 $\rightarrow$ 156.

4 Discussion

Our results show that, as compared to 2D cylindrical aromats,
spherical analogues show more extensive hybridization between
spherical shells. The reason is that point group symmetries are
often too low to discriminate between spherical parentages. This
effect becomes more pronounced in the frontier orbital region.
As an example total symmetry in the $D_{5h}$ point group was unable
to distinguish $l = 4$ and $l = 5$ components. This would imply that
one should direct the search for superaromats to clusters with
high symmetries. The highest point groups are the tetrahedral,
octahedral and icosahedral groups, but these give rise to atomic
orbit counts that are divisors of the group orders 24, 48, and 120 respectively, and these counts are mostly not commen-
surate with magic numbers such as 32, 50 or 72.

Compatibility between magic counts and symmetry numbers
thus would require a different strategy, by adjusting electron
counts through doping with hetero-atoms. A successful example
is the endohedral U@C$_{28}$ metallo-fullerene, where the carbon
cage has tetrahedral symmetry and attains the magic electron
count of 32 by doping with four electrons from uranium.$^{37}$

The link between fullerene stability and magic electron counts
is not well established either. The most stable fullerenes C$_{60}$ and
C$_{70}$ are not aromats. Clearly the stability of the neutral fullerenes
is dictated more by the frequency of occurrence of stable motifs,
\emph{i.e.} the fact that pentagons are isolated as in C$_{60}$ and C$_{70}$ or the
absence of pentagon triplets as in C$_{50h}$ rather than by electron
count. Nevertheless the higher aromatic character of some iso-
mer can affect the expected stability by taking only into account
the distribution of motifs. For the three analysed structures of C$_{50h}$
the $D_3$ isomer complies best with a spherical aromat. It has a
higher degree of sphericity and a neat closed shell spherical
parentage. According to DFT calculations it is slightly more stable
than the $D_{3h}$ isomers, even though the PAPR rule would predict it
to be around 20 kcal mol$^{-1}$ less stable. Therefore a most favour-
able electronic structure (\emph{i.e.} enhanced aromaticity) can stabilize
some isomers of neutral fullerenes and favour structures with a
worst distribution of structural motifs.

These effects should be more evident for negatively charged
fullerenes, and in these systems electronic effects can determine
the stability instead of strain. For instance in the case of
endohedral metallo fullerenes, where stability is mainly dictated
by the charge transfer to the carbon cage and the most stable
isomers correspond to the most stable negatively charged carbon
cages, the structure observed experimentally does not fulfill
many cases the isolated pentagon rule.$^{38}$ The stability of these
isomers has been directly related to the electronic structure of the
$\pi$ shell$^{39}$ and with maximum aromaticity.$^{40}$

Of the two $D_{3h}$ isomers, the A isomer fulfils the criteria for
spherical aromaticity, while the B isomer is an anti-aromat as far
as spherical parentage is concerned. While the relative stabilities
do not typically correlate with aromaticity, the magnetic proper-
ties are very sensitive to the spherical shell nature of the cages.
The aromaticity of the $D_3$ isomer is confirmed by a substantial
diamagnetic NICS value, and a global diatomic ring current. The
NICS values of the $D_{3h}$ isomers also reflect their spherical shell
character (Table 1). The main difference between A and B isomers
lies in the currents surrounding the equator, since the cap
current density plots look quite similar (diatropic except for the
apical pentagons, which are paratropic). The current density
maps however indicate paratropic currents even for the A isomer,
which is more aromatic. The reason is the strong hybridization
between the spherical shells in the frontier orbital region, made
possible by the prismatic symmetry of these isomers.

5 Conclusions

Spherical aromaticity appears to remain an elusive concept. Our
investigation of fullerenes with a magic number of 50 electrons
has confirmed that although the distribution of the pentagons
dominates the fullerene energy landscape, enhanced spherical aromaticity of some isomers can compensate the destabilization due to the presence of an extra pair of adjacent pentagons. It can also lead to a breakdown of the spherical filling rule, as we have demonstrated for the case of the $D_{3h}$ (B) isomer, or to strong hybridization of spherical components, due to symmetry breaking, as was found in the strong hybridization of spherical components, due to symmetry to achieve a magic electron count. The quest for spherical aromats should continue, preferably by considering high-symmetry cages in combination with doping to achieve a magic electron count.

Acknowledgements

A.S.M. would like to thank Mutua Madrileña for a predoctoral scholarship, and Yang Wang for helpful discussions and providing useful programs. R.W.A.H. acknowledges the Zernike Institute for Advanced Materials for financial support. MA acknowledges support from MICINN project CTQ2013-43698-P and CAM project NANOFRONTMAG-CM ref. S2013/MIT-2850. A.C. is indebted to the FWO-Vlaanderen for financial support.

References

30. Programs to generate and analyse Hückel orbitals and energies in fullerenes have been provided by Y. Wang.
33. M. F. Guest, I. J. Bush, H. J. J. van Dam, P. Sherwood, J. M. H. Thomas, J. van Lenthe, R. W. A. Havenith and


