Precision Measurement of the $e^+ e^- \to \Lambda_c^+ \bar{\Lambda}_c^-$ Cross Section Near Threshold


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The electromagnetic structure of hadrons, parametrized in terms of electromagnetic form factors (EMFFs), provides a key to understanding quantum chromodynamics effects in bound states. The nucleon has been studied rigorously for more than sixty years, but new techniques and the availability of data with larger statistics from modern facilities have given rise to a renewed interest in the field, resulting in a nonzero cross section at threshold, and thereby enabling us to measure the nucleon’s electric over magnetic form-factor ratios \((|G_E/G_M|)\) are measured for the first time. They are found to be 1.14 ± 0.14 ± 0.07 and 1.23 ± 0.05 ± 0.03, respectively, where the first uncertainties are statistical and the second are systematic.

The cross section of the \(e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-\) process is measured with unprecedented precision using data collected with the BESIII detector at \(\sqrt{s} = 4574.5, 4580.0, 4590.0\) and 4599.5 MeV. The nonzero cross section near the \(\Lambda_c^+\Lambda_c^-\) production threshold is cleared. At center-of-mass energies \(\sqrt{s} = 4574.5\) and 4599.5 MeV, the higher statistics data enable us to measure the \(\Lambda_c\) polar angle distributions. From these, the \(\Lambda_c\) electric over magnetic form-factor ratios \((|G_E/G_M|)\) are measured for the first time. They are found to be 1.14 ± 0.14 ± 0.07 and 1.23 ± 0.05 ± 0.03, respectively, where the first uncertainties are statistical and the second are systematic.

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measurements of the production cross section and EMFF ratios are highly desirable.

In this work, the cross section of the reaction $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$ is measured at four c.m. energies: $\sqrt{s} = 4574.5$, 4580.0, 4590.0, and 4599.5 MeV. At each c.m. energy, ten Cabibbo-favored hadronic decay modes, $\Lambda_c^+ \rightarrow pK^-\pi^+$, $pK^0_S$, $\Lambda\pi^+$, $pK^-\pi^-\pi^0$, $pK^0_S\pi^0$, $\Lambda\pi^+\pi^-\pi^0$, $\Lambda\pi^-\pi^+\pi^-$, $\Sigma^+\pi^-$, and $\Sigma^+\pi^+\pi^-$, as well as the ten corresponding charge-conjugate modes are independently used to reconstruct $\Lambda_c^+$ or $\bar{\Lambda}_c^-$. Each mode will produce one measurement of the cross section, and the total cross section is obtained from a weighted average over the 20 individual measurements. In addition, the higher statistics data samples at $\sqrt{s} = 4574.5$ and 4599.5 MeV enable the study of the polar angle distribution of $\Lambda_c$ in the c.m. system. From these distributions, the ratios between the electric and the magnetic form factors, i.e., $|G_E/G_M|$, are extracted for the first time.

The data samples are collected with the BESIII detector [12] at BEPCII. The detector has a geometrical acceptance of $93\%$ of the $4\pi$ solid angle. It contains a small-celled, helium-based main drift chamber (MDC), a time-of-flight system (TOF) based on plastic scintillators, an electromagnetic calorimeter (EMC) made of CsI(Tl) crystals, a muon system (MUC) made of resistive plate chambers, and a superconducting solenoid magnet.

Monte Carlo (MC) simulations based on GEANT4 [13] are performed to determine detection efficiencies, optimize selection criteria, extract signal shapes, and study backgrounds. The $e^+e^-$ collisions are simulated by the KKMCS generator [14], which takes the beam energy spread and the ISR correction into account. The distribution of the $\Lambda_c$ polar angle is considered in the generator by parametrizing it with the function $f(\theta) \propto 1 + \alpha_{\Lambda_c} \cos^2 \theta$. After an iterative procedure, the values of $\alpha_{\Lambda_c}$ at $\sqrt{s} = 4574.5$ and 4599.5 MeV are obtained from real data (see Table IV) and at the remaining c.m. energies by a linear interpolation.

Using the branching fractions (BR) measured in Ref. [15], all tagged $\Lambda_c$ decays are simulated by weighting phase-space events according to the decay behavior observed in real data. The subsequent decays listed by the Particle Data Group (PDG) [16] are modeled with EVTGEN [17]. The inclusive MC samples include $\Lambda_c^+\Lambda_c^-$ pair production, $\ell^+\ell^-$ ($\ell = e, \mu, \tau$) events, open charm processes [18], ISR-produced low-mass $\psi$ states, and the continuum process $e^+e^- \rightarrow q\bar{q}(q = u, d, s)$.

Charged tracks as well as the intermediate states $\pi^0$, $K^0_S$, $\Lambda$, $\Sigma^0$, and $\Sigma^+$ are selected and reconstructed with the same method described in Ref. [15].

In the final states of decay modes $pK^0_S\pi^0$ and $pK^0_S\pi^+\pi^-$, potential background from $\Lambda \rightarrow p\pi^-$ is eliminated by rejecting events with $M_{p\pi}$ lying in the mass window $(1100, 1125)$ MeV/c$^2$, where $M_{p\pi}$ is the invariant mass of $p\pi^-$ combinations in the final state. For the decay mode $\Sigma^+\pi^-\pi^-$, the corresponding exclusion window is $(1110, 1120)$ MeV/c$^2$ due to the smaller observed width of the $M_{p\pi}$ peak in data. Similarly, background from the intermediate state $\Sigma^+$ is removed from the $pK^0_S\pi^0$ sample by rejecting events with $M_{p\pi}$ in the mass window $(1170, 1200)$ MeV/c$^2$. In modes $\Lambda\pi^-\pi^-\pi^+$ and $\Sigma^+\pi^-\pi^+$, events with $M_{p\pi}$ within $(490, 510)$ MeV/c$^2$ are rejected to suppress the $K^0_S$ background.

According to energy and momentum conservation, two discriminating variables, the energy difference $\Delta E$ and the beam-constrained mass $M_{BC}$, are utilized to identify the $\Lambda_c$ signals. The energy difference is defined as $\Delta E \equiv E - E_{\text{beam}}$, where $E$ is the energy of the $\Lambda_c$ candidate and $E_{\text{beam}}$ is the mean energy of the two colliding beams. In each tagged mode, the $\Lambda_c$ candidates are formed by all possible combinations of the final-state particles, and only the one with minimum $|\Delta E|$ is stored. In the following analysis, events are rejected if they fail the $\Delta E$ requirements specified in Ref. [15]. The beam-constrained mass is defined as $M_{BC} \equiv \sqrt{E_{\text{beam}}^2 - p^2 c^2}$, where $p$ is the momentum of the $\Lambda_c$ candidate. Both $\Delta E$ and $M_{BC}$ are calculated in the initial $e^+e^-$ c.m. system. In Fig. 1, the $M_{BC}$ distributions for $\Lambda_c^+ \rightarrow pK^-\pi^+$ at the four c.m. energies are shown. Clear peaks at the nominal $\Lambda_c^+$ mass are observed. Studies of the inclusive MC samples show that the cross feeds among the ten tagged modes are less than 1.5%, and the background shape can be described by the ARGUS function [19].

Performing an unbinned maximum likelihood fit to each $M_{BC}$ distribution gives the corresponding event yields, as partly illustrated in Fig. 1. The signal shape of the fit is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{Fit results of the $M_{BC}$ distribution of $\Lambda_c^+ \rightarrow pK^-\pi^+$ in data at (a) $\sqrt{s} = 4574.5$ MeV, (b) 4580.0 MeV, (c) 4590.0 MeV, and (d) 4599.5 MeV. Dots are Poisson averages of the data in the bins, and error bars represent one time of corresponding standard deviations; the blue solid curves are the sum of fit functions, while the barely visible green dashed lines are the background shapes. $N$ is the yield with statistical uncertainty of the $\Lambda_c^+$ signal, and $\epsilon$ represents the corresponding detection efficiency and uncertainty.}
\end{figure}
obtained from convolving the $M_{BC}$ shape of MC simulations with a Gaussian function to compensate a possible resolution difference between data and MC simulations. The background is described by an ARGUS function with the high-end truncation fixed. At $\sqrt{s} = 4599.5$ MeV, the parameters of the ARGUS and the Gaussian functions used in the convolution are obtained from the fit. At the remaining c.m. energies, all parameters obtained at the highest energy, except for the mean of the Gaussian, are used to fix parameters in the new fits. Yields are extracted from the signal region $2276\text{ MeV} < M_{BC} < 2E_{\text{beam}}$ in each fit. The detection efficiency of each decay mode is evaluated by MC simulations of the $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c$ process. Figure 1 gives the efficiencies of mode $pK^-\pi^+$ at the four c.m. energies.

The cross section of the $i$th mode is determined using

$$\sigma_i = \frac{N_i}{\varepsilon i L_{\text{int}} f_{\text{VP}} BR_i f_{\text{ISR}}},$$

where $N_i$ and $\varepsilon_i$ represent the yield and corresponding detection efficiency. The integrated luminosity $L_{\text{int}}$ is taken from Refs. [20,21]. The vacuum polarization (VP) correction factor $f_{\text{VP}}$ is calculated to be 1.055 at all four c.m. energies [22]. $BR_i$ represents the product of branching fractions of the $i$th $\Lambda_c$ decay mode and its subsequent decay (s). $f_{\text{ISR}}$ is the ISR correction factor derived in Ref. [23] and implemented in KKMC. Since the calculation of $f_{\text{ISR}}$ requires the cross-section line shape as input, an iterative procedure has been performed.

The systematic uncertainties of the cross section can be classified into reconstruction-related and general contributions. The reconstruction-related contributions are mode specific and mainly originate from tracking, PID, reconstruction of intermediate states, and total BRs. The uncertainties of $\Delta E$ and $M_{BC}$ requirements are negligible after correcting for the difference in resolution between simulated and real data samples. The uncertainties from the tracking and PID of charged particles are investigated using control samples from $e^+e^- \rightarrow \pi^+\pi^-\pi^\pm\pi^\mp$, $K^+K^-\pi^+\pi^-$, and $p\bar{p}\pi^+\pi^-$ collected at $\sqrt{s} > 4.0$ GeV [24]. The uncertainties are obtained after weighting according to the momenta of the corresponding final states. Reconstruction uncertainties of $K_0^0$, $\Lambda$, and $\bar{\Lambda}$ have been found to be 1.2%, 2.5%, and 1.0% [15]. Statistical uncertainties of detection efficiencies are considered as systematic uncertainties. The dependence of the reconstruction efficiency on the MC model for the ten decay modes also gives a small contribution to the systematic uncertainty [15]. Uncertainties originating from the total BRs of the tagged modes are quoted from Refs. [15,16]. A summary of the reconstruction-related systematic uncertainties is given in Table I. The total uncertainty at each energy has been calculated assuming

![Figure 2](https://via.placeholder.com/150)

**FIG. 2.** Cross section of $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c$ obtained by BESIII (this work) and Belle. The blue solid curve represents the input line shape for KKMC when determining the $f_{\text{ISR}}$. The dash-dotted cyan curve denotes the prediction of the phase-space (PHSP) model, which is parametrized by Eq. (1), but with $C = 1$ and flat $|G_M|$ with respect to $\sqrt{s}$. 

| Source | Tracking | PID | $K_0^0$ | $\Lambda$ | $\bar{\Lambda}$ | MC statistic | Signal model | Total BR
<table>
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</tr>
</thead>
<tbody>
<tr>
<td>$\rho K^-\pi^+$</td>
<td>3.2</td>
<td>4.6</td>
<td>0.2</td>
<td>6.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho K_0^0\pi^0$</td>
<td>1.3</td>
<td>0.5</td>
<td>0.6</td>
<td>0.2</td>
<td>5.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda\pi^+$</td>
<td>1.0</td>
<td>1.0</td>
<td>0.6</td>
<td>0.5</td>
<td>6.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pK^-\pi^+\pi^0$</td>
<td>3.0</td>
<td>7.6</td>
<td>1.0</td>
<td>2.0</td>
<td>8.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pK_0^0\pi^+\pi^0$</td>
<td>1.0</td>
<td>1.8</td>
<td>1.0</td>
<td>1.1</td>
<td>7.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda\pi^+\pi^0$</td>
<td>1.0</td>
<td>1.0</td>
<td>25</td>
<td>1.0</td>
<td>0.6</td>
<td>6.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$pK_0^0\pi^+\pi^+$</td>
<td>2.8</td>
<td>5.3</td>
<td>1.2</td>
<td>1.0</td>
<td>0.5</td>
<td>9.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda\pi^+\pi^+$</td>
<td>3.0</td>
<td>3.0</td>
<td>25</td>
<td>0.9</td>
<td>0.8</td>
<td>7.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma^0\pi^+$</td>
<td>1.0</td>
<td>1.0</td>
<td>25</td>
<td>1.1</td>
<td>1.7</td>
<td>6.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma^+\pi^+\pi^+$</td>
<td>3.0</td>
<td>4.0</td>
<td>1.0</td>
<td>0.8</td>
<td>0.8</td>
<td>7.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The general contributions to the systematic uncertainty originate from uncertainties in $f_{\text{ISR}}$, $f_{\text{VP}}$, and $L_{\text{int}}$ in Eq. (2) and are the same for all decay modes. The $f_{\text{ISR}}$ is obtained using the KKMC generator, which requires a cross-section line shape as input. The line shape is in turn obtained by an iterative fitting procedure of the cross-section data using Eq. (1). In the fit, the $|G_E/G_M|$ value at an arbitrary c.m. energy is assigned by linear interpolation between the two known values listed in Table IV. For simplicity, $|G_M|$ is assumed to be independent of the c.m. energy. To precisely describe the data, the $a$ in the Sommerfeld resummation factor is replaced by $a_i (= 0.25)$. In the line shape, the cross section at the c.m. energy region $(2m_{\Lambda_c}c^2, 4574.5)$ MeV is obtained from extrapolating the fit; below threshold it vanishes, as shown by the blue solid curve in Fig. 2.
The cross sections obtained in different decay modes are considered: First, the uncertainty of the calculation model is studied using a different algorithm mentioned in Ref. [25]. Second, the uncertainty associated with the input line shape is estimated using different fit functions. Third, the uncertainty of the model depends on the c.m. energy of the $e^+ e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ process. The uncertainty of the c.m. energy therefore contributes near the threshold. At the lowest energy point, the c.m. energy is measured to be $\sqrt{s} = 4574.5 \pm 0.72$ MeV [26]. Finally, the beam energy spread, which has been estimated as 1.55 ± 0.18 MeV, is important near threshold and contributes to the $f_{\text{ISR}}$ uncertainty. For the other, higher, energies, the effects from the c.m. energy uncertainty and the beam energy spread are less than 0.1% and can be neglected due to the flat line shape of the cross section. The uncertainty of $f_{\text{VP}}$ is calculated to be 0.5% at all four c.m. energies [22]. The uncertainty from the integrated luminosity has been found to be 0.7% at $\sqrt{s} = 4580.0$ and 4590.0 MeV, and 1.0% at $\sqrt{s} = 4574.5$ and 4599.5 MeV [20,21]. A summary of the general contributions to the systematic uncertainties is given in Table III.

The cross sections obtained in different decay modes are combined using the method mentioned in Ref. [27], in which the cross section is given by

$$\sigma = \sum_i w_i \sigma_i \text{ with } w_i = (1/\Delta \sigma_i) \left/ \left( \sum_i 1/\Delta \sigma_i \right) \right.$$ \hspace{1cm} (3)

Here, $w_i$ and $\Delta \sigma_i$ denote the weight and the total uncertainty, respectively, of the measured cross section $\sigma_i$ of mode $i$. The sum is performed over all 20 decay modes.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (MeV)</th>
<th>$\mathcal{L}_{\text{int}}$ (pb$^{-1}$)</th>
<th>$f_{\text{ISR}}$</th>
<th>$f_{\text{VP}}$</th>
<th>$\mathcal{L}_{\text{int}}$</th>
<th>$\sigma$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4574.5</td>
<td>47.67</td>
<td>0.45</td>
<td>236 ± 11</td>
<td>46</td>
<td>236 ± 11</td>
</tr>
<tr>
<td>4580.0</td>
<td>8.54</td>
<td>0.66</td>
<td>207 ± 17</td>
<td>13</td>
<td>207 ± 17</td>
</tr>
<tr>
<td>4590.0</td>
<td>8.16</td>
<td>0.71</td>
<td>245 ± 19</td>
<td>16</td>
<td>245 ± 19</td>
</tr>
<tr>
<td>4599.5</td>
<td>566.93</td>
<td>0.74</td>
<td>237 ± 3</td>
<td>15</td>
<td>237 ± 3</td>
</tr>
</tbody>
</table>

The systematic uncertainties of the $\alpha_{\Lambda_c}$ considered here are the contributions from the fit range and the bin size. A change of the fit range in cos $\theta_{\Lambda_c}$ from (−1.0, 1.0) to (−0.8, 0.8) and in the number of bins from 10 to 20 is performed, and the differences in the obtained $\alpha_{\Lambda_c}$ are regarded as the systematic uncertainty. Systematics of $\Lambda_c^+$ and $\bar{\Lambda}_c^-$ [28]. The combined uncertainty is calculated by

$$\Delta \sigma^2 = \sum_{i,j} w_i (M_{\alpha})_{ij} w_j,$$ \hspace{1cm} (4)

where $M_{\alpha}$ represents the covariance matrix of these cross-section measurements, in which the correlations between any two measurements $\sigma_i$ and $\sigma_j$ are considered. The resulting cross sections at the four c.m. energies are listed in Table III and shown in Fig. 2 together with the Belle data [11] for comparison.

The data sets collected at $\sqrt{s} = 4574.5$ and 4599.5 MeV are large enough to perform a detailed study in the c.m. frame of the $\Lambda_c$ polar angle $\theta_{\Lambda_c}$, which is defined as the angle between the $\Lambda_c$ momentum and the beam direction. The data fulfilling all selection criteria are divided into ten bins in cos $\theta_{\Lambda_c}$. In each cos $\theta_{\Lambda_c}$ bin, the total yield is obtained by summing the yields of all the ten tagged modes. The one-dimensional bin-by-bin efficiency corrections are applied on these total yields. The same procedure is performed by tagging $\bar{\Lambda}_c^-$ decay channels. The total yields of $\Lambda_c^+$ and $\bar{\Lambda}_c^-$ are combined bin-by-bin, and the shape function $f(\theta) \alpha(1 + \alpha_{\Lambda_c} \cos^2 \theta)$ is fitted to the combined data, as shown in Fig. 3. Table IV lists the resulting $\alpha_{\Lambda_c}$ parameters obtained from the fits, as well as the $|G_E/G_M|$ ratios extracted using the equation

$$|G_E/G_M|^2 (1 - \beta^2) = (1 - \alpha_{\Lambda_c})/(1 + \alpha_{\Lambda_c}).$$ \hspace{1cm} (5)

The systematic uncertainties of the $\alpha_{\Lambda_c}$ considered here are the contributions from the fit range and the bin size. A change of the fit range in cos $\theta_{\Lambda_c}$ from (−1.0, 1.0) to (−0.8, 0.8) and in the number of bins from 10 to 20 is performed, and the differences in the obtained $\alpha_{\Lambda_c}$ are regarded as the systematic uncertainty. Systematics
originating from the model dependencies in the efficiency correction are found to be negligible compared to the statistical uncertainties.

In summary, using data collected at $\sqrt{s} = 4574.5$, 4580.0, 4590.0, and 4599.5 MeV with the BESIII detector, the cross sections of $e^+e^- \to \Lambda_c^+\Lambda_c^-$ have been measured with high precision, by reconstructing $\Lambda_c^+$ and $\Lambda_c^-$ independently with ten Cabibbo-favored hadronic decay channels. The most precise cross-section measurement is achieved so far at $\sqrt{s} = 4574.5$ MeV, which is only 1.6 MeV above the threshold. The measured value is $(236 \pm 11 \pm 46)$ pb, which highlights the enhanced cross section near threshold and indicates the complexity of production behavior of the $\Lambda_c^-$. At $\sqrt{s} = 4574.5$ and 4599.5 MeV, the data samples are large enough to study polar angle distributions of $\Lambda_c^-$ and measure the $\Lambda_c$ form-factor ratio $|G_E/G_M|$ for the first time. These results provide important insights into the production mechanism and structure of the $\Lambda_c$ baryons.

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[28] At $\sqrt{s} = 4599.5$ MeV, the deviations between any two of the 20 individual Born cross section can be covered by corresponding total uncertainty. However, this is not the case for a few decay channels at the other three energies due to the low statistics of the data sample.