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Modeling and analysis of butterfly loops via Preisach operators and its application in a piezoelectric material

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***Abstract*— We present modeling and analysis of the *so-called* butterfly hysteresis behavior, based on the use of the Preisach operator. The desired butterfly loop properties can be obtained under some mild conditions on the weighting function that defines the Preisach operator. The proposed framework is used to model the electric-field dependence of the strain in a piezoelectric material purposely designed to exhibit asymmetric butterfly loops with remnant deformation.**

I. Introduction

Hysteresis is a nonlinear phenomenon that often involves memory and is present in a variety of physical systems, such as ferromagnetic materials, ferroelectric and piezoelectric materials, shape-memory alloys and friction-induced mechanical systems. Generally speaking, hysteresis represents a quasi steady-state input-output behavior whose phase portrait follows a particular curve, which is typically known as hysteresis loop. An example of such hysteresis loop is illustrated in Figure 1, which can describe the magnetization-magnetic field relation in a ferromagnetic material or the polarization-electric field relation in a ferroelectric material.

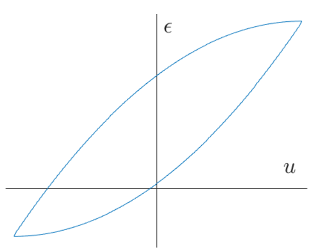

Fig. 1. Simple hysteresis loop found typically in ferroelectric or ferromagnetic materials. The variable u represents electric or magnetic field and ε denotes the magnetization (for the ferromagnetic case) or polarization (for the ferroelectric case).

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Its presence in nonlinear control systems has presented some control design challenges which are mainly due to the lack of accurate physico-mathematical description of the phenomenon. In literature, a number of phenomenological models of hysteresis and their basic mathematical properties have been reviewed in [1], [2], [3].

One important class of phenomenological models of hysteresis is the Preisach operator [4]. It is an infinite-dimensional operator whose infinitesimal element is given by a weighted relay operator. Some important mathematical characterizations of this operator and their applicability in the stability analysis of hysteretic systems have been thoroughly discussed, among many others, in [1], [3], [5]. Another large class of phenomenological models of hysteresis is the Duhem operator [1], [6], which encapsulates the Jiles-Atherton model [7] for describing magnetization process in ferromagnets, the Bouc-Wen model [8], [9] for modeling restoring forces in material, the Coleman-Hodgdon model [10] for describing magnetic hysteresis phenomenon and the Chua-Stromsmoe model [11]. This class of hysteresis operator is described by a non-smooth integro-differential equation, which enables the systems to follow a different path whenever the input changes its direction; enabling the creation of typical hysteresis loops.

In these works, the mathematical analysis focuses mainly on the hysteresis behavior corresponding to the simple hysteresis loop as shown in Figure 1. For instance, the analysis of counterclockwise input-output behavior in a hysteresis loop has been presented in literature for different hysteresis models. In [12], [13], it is shown that the counterclockwise behavior is given for Preisach operator with positive semi-definiteness of hysterons' weights. Correspondingly, such condition is also related to the monotonicity property of hysteresis operator as discussed in [5]. Related works on counterclockwise property for the Duhem hysteresis operator is given in [6], [14] where it is related to the existence of a storage function that characterizes counterclockwise property. On the other hand, mathematical characterization of clockwise input-output behavior has been rigorously discussed only for the Duhem operator [15]; despite the fact that such behavior can also be exhibited using Preisach operator with negative weights as in [16].

On the basis of these characterization studies, various stability analysis and control design methods for hysteretic systems have been investigated in literature. One popular control design approach is to construct an inverse hysteresis operator which can (approximately) linearize the nonlinearity introduced by the hysteretic systems. This approach has been proposed for both the Preisach model (such as, the work in [17]), as well as, for Duhem model (for example, the method discussed in [18] for the Bouc-Wen model). Another approach is to exploit some inherent properties (such as, monotonicity, sector bound condition, dissipativity, etc.) of the underlying hysteresis operators that can be incorporated explicitly in the control design. For example, it has been shown in [19] that the monotonicity constant can be used to design a stabilizing PID controller. Alternatively the dissipativity property (that is related to the counterclockwise or clockwise behavior) can accommodate the design of PID or state feedback controllers as discussed in [15], [20], [21]. Using nonsmooth analysis combined with the classical circle criterion theorem, one can appropriately define a sector bound condition for hysteresis operator and use it in the stability analysis of hysteretic systems [22].

Despite these recent progresses, these approaches are no longer applicable for systems that exhibit more complex hysteresis loops. This is the case of physical systems that can present an input-output map with two loops in opposite direction and connected at one crossover point. Such hysteresis curve is known as butterfly loop (from the resemblance of the input-output map with the wings of a butterfly). Such butterfly hysteresis loop is, for instance, found in the dependence of the strain with the electric field in piezoelectric materials, as illustrated in Fig. 2.

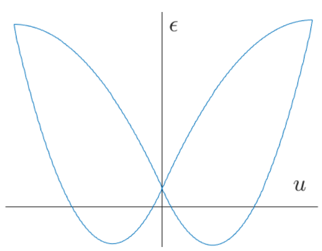


Fig. 2. Butterfly hysteresis loop that is commonly found in piezoelectric materials. In this case, the variable u represents electrical field and ε represents the strain variable.

Although butterfly hysteresis loops have been reported in various experimental studies in literature, the mathematical modeling, characterization and analysis of such hysteretic behavior are not widely studied. A recent work reports on the modeling of a hysteresis operator exhibiting butterfly loops and the analysis of systems containing such operator [23]. The authors study if there exists a (nonlinear) function that can transform a hysteresis operator with butterfly hysteresis loops into one with a simple hysteresis loop, such that the aforementioned analysis and control design for simple hysteresis loops can be carried out immediately to butterfly loops. However the existence of such function is not trivial and it does not provide further understanding on the mathematical properties of butterfly hysteresis operators. Consequently, the lack of such insights has limited the development of appropriate control methods.

Motivated by the development of novel piezoelectric materials that can exhibit remnant displacement after the removal of electrical field and are suitable for new deformable mirror systems [24], we present in this paper a preliminary study on modeling and analysis of butterfly hysteresis operators. We develop a material with an asymmetric (biased) loop, that exhibits remnant deformation (non-zero strain upon the removal of the electric field) [25]. Its development is driven by the end application of a novel hysteretic deformable mirror for wavefront correction in active optics systems [24]. The new material consists of Nb-doped PZT ceramics close to the Morphotropic Phase Boundary (MPB) mixed with ZrO_2 particles [25]. Based on Preisach operators with sign-indefinite weighting functions, we introduce in Section III a number of sufficient conditions on the weighting functions that allow us to obtain butterfly hysteresis loops. Based on the insights that we have obtained in the previous sections, we are able to model the measured butterfly loop from our novel piezoelectric material and it is presented in Section IV. For completeness of our exposition, we provide preliminaries to various relevant notions and concepts in Section II.

II. Preliminaries

A. Hysteresis operators

We denote the space of absolute continuous function by AC. Let us consider an operator $\Phi : AC(\mathbb{R}_+) \to AC(\mathbb{R}_+)$. We will define general hysteresis operator following the work in [26] as follows.

A function $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is called a *time transformation* if $\phi(t)$ is continuous, increasing and satisfies $\phi(0) = 0$ and $\lim_{t\to\infty} \phi(t) = \infty$. The operator Φ is said to be *rate independent* if

$$\big(\Phi(u \circ \phi)\big)(t) = \Phi(u) \circ \phi(t)$$

holds for all $u \in AC(\mathbb{R}_+)$, $t \in \mathbb{R}_+$ and all admissible time transformation ϕ. The operator Φ is said to be *causal* if for all $\tau > 0$ and all $u_1, u_2 \in C(\mathbb{R}_+)$

$$\begin{aligned} & u_1(t) = u_2(t) \quad \forall t \in [0, \tau] \\ & \qquad \Rightarrow \big(\Phi(u_1)\big)(t) = \big(\Phi(u_2)\big)(t) \quad \forall t \in [0, \tau]. \end{aligned}$$

Definition 2.1: [5] The operator Φ is called a *hysteresis operator* if Φ is causal and rate-independent.

B. Counterclockwise, clockwise and butterfly loop

For the past decade, the dynamic behavior of hysteresis operators has been analyzed according to their input-output behavior. The phase portrait of input-output pair of particular classes of hysteresis operators have been shown to exhibit counterclockwise or clockwise behavior [6], [12], [13], [14], [15]. This knowledge can be useful for the analysis and control design of systems with hysteresis in the feedback loop as pursued in [15]. Let us formalize the notion of clockwise and counterclockwise loops following the exposition of Padthe, Oh and Bernstein in [12]. For simplicity of presentation, we will focus on a particular time interval where the input-output pair forms a loop.

Consider an input-output pair $y, u \in AC(\mathbb{R}_+)$ with $y = \Phi(u)$ and two time instances $T_2 > T_1 > 0$ such that $C := \{(y(t), u(t)) \mid t \in [T_1, T_2]\}$ defines a closed curve; thus, $(y(T_1), u(T_1)) = (y(T_2), u(T_2))$ holds. For such input-output pair, the signed area enclosed by C is given by

$$\mathbb{A} := \frac{1}{2}\int_{T_1}^{T_2} \big(u(\tau)\dot{y}(\tau) - y(\tau)\dot{u}(\tau)\big)\mathrm{d}\tau. \tag{1}$$

As presented in [12], the characterization of counterclockwise and clockwise loop can be given in terms of the signed area $\mathbb{A}$ as in (1). The curve C is a *counterclockwise loop* if $\mathbb{A} > 0$ and correspondingly, it is a *clockwise loop* if $\mathbb{A} < 0$.

Based on these notions then we can say that a hysteresis operator Φ exhibits a *counterclockwise hysteresis behavior* if there exists an input-output pair y, u with $y = \Phi(u)$ that contains a counterclockwise loop (as defined above). Analogously, Φ exhibits a *clockwise hysteresis behavior* if there exists an input-output pair y, u with $y = \Phi(u)$ that has a clockwise loop.

In a similar manner, we can characterize a butterfly loop using the signed area $\mathbb{A}$. However, it is important to note that a butterfly loop is more complex because it is composed of two loops being one of them a *counterclockwise loop* and the other a *clockwise loop* (c.f., Figure 2). Thus the total area enclosed by the curve C of a butterfly loop is given by the sum of the signed area of each individual loop. More precisely, we can define butterfly hysteresis operators as follows.

Definition 2.2: The hysteresis operator Φ is called a *butterfly hysteresis operator* if there exist an input-output pair of signals $y, u \in AC(\mathbb{R}_+)$, where u is periodic with period of $T > 0$ and $y = \Phi(u)$, and a constant $T_0 > 0$ such that $\mathbb{A} = 0$, where $\mathbb{A}$ is defined as in (1), holds for all $T_1 > T_0$ and $T_2 = T_1 + T$.

In this definition, the time T_0 can be regarded as the time when the influence of initial condition is no longer influencing the output y so that y becomes a periodic signal in the time interval $[T_0, \infty)$ with the same period as the input. For the Preisach hysteresis operator, which will be presented shortly below, the time T_0 coincides with T. This is due to the wipe-out and minor loop properties of Preisach operator. However, for other class of hysteresis operator, such as the Duhem operators, a number of cycles may be required before the input-output pair forms a closed curve. Such property is known as the *accommodation* property [27] and is inherent in hysteresis operators described by integro-differential equation. For such operators, the periodic cycle will revolve around the so-called *anhysteresis curve* [6], [14].

Roughly speaking, the above definition of butterfly hysteresis operator asks for the existence of two connected loops where one loop is a counterclockwise loop and the other one is a clockwise loop, and moreover, the signed area of each loop has the same amplitude but with an opposite sign of each other. The curve C in this case is *non-simple*, unlike that for the clockwise or the counterclockwise case. Although this definition seems restrictive where we only concern with input-output pair such that $\mathbb{A} = 0$, this does not preclude that we can have asymmetric butterfly loops using other input-output pairs. In Definition 2.2, we only need to look for an admissible pair of input-output signals such that $\mathbb{A} = 0$. As will be shown later, if Φ exhibits an (asymmetric) butterfly loop then we can adjust the magnitude of the input signal accordingly in order to enlarge or to reduce the area of one of the two loops in the butterfly loop such that both loops have the same area.

C. Preisach hysteresis operator

One important class of hysteresis operators is the Preisach operator which is, roughly speaking, an integration of weighted relay operators. In order to define Preisach operator properly, we need the notion of Preisach plane $P \subset \mathbb{R}^2$ where the relay operators are defined and it is defined by $P := \{(\alpha, \beta) \mid \alpha \geq \beta\}$. For any given input signal $u \in AC(\mathbb{R}_+)$, the Preisach operator $y = \Phi(u)$ as proposed in [4] is given by

$$\Phi(u) := \iint_{(\alpha,\beta)\in P} \mu(\alpha, \beta)\ \mathcal{R}^{\circlearrowleft}_{\alpha,\beta}(u)\ \mathrm{d}\alpha\mathrm{d}\beta \tag{2}$$

where $\mu \in C(P)$ is a weighting function and $\mathcal{R}^{\circlearrowleft}_{\alpha,\beta}$ is the standard (counterclockwise) relay operator parametrized by $\alpha > \beta$ and defined by

$$\big(\mathcal{R}^{\circlearrowleft}_{\alpha,\beta}(u)\big)(t) := \begin{cases} 1 & \text{if } u(t) > \alpha \\ -1 & \text{if } u(t) < \beta \\ \big(\mathcal{R}^{\circlearrowleft}_{\alpha,\beta}(u)\big)(t_-) & \text{otherwise} \end{cases} \tag{3}$$

with t_- be the left-handed limit of t.

This relay operator element is called hysteron and is illustrated in Fig. 3.

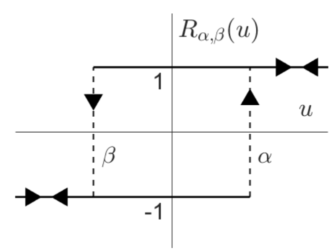


Fig. 3. An input-output phase plot of a standard relay $\mathcal{R}^{\circlearrowleft}_{\alpha,\beta}(u)$ as in (3) with two switching moments β and α. The resulting input-output loop follows a counterclockwise loop.

When an input signal u is applied to such operator, all relay operators in (2) will respond simultaneously to the variation in u at every time instances. As a result, the Preisach plane P can be divided into two disjoint regions P_- and P_+ that depends on the history of past input u and the initial conditions of each relay. In the region P_-, all relays $\mathcal{R}_{\alpha,\beta}$ with $(\alpha, \beta) \in P_-$ are in -1 state, while in P_+, all relays $\mathcal{R}_{\alpha,\beta}$ with $(\alpha, \beta) \in P_+$ are in 1. For every time instance $t > 0$, we can define a staircase line, so-called the interface $L(t)$, which divides these regions. Its vertices coincide with every local minima and maxima of the truncated input signal $\{u(\tau)|0 \leq \tau \leq t\}$. Using the interface $L(t)$, we can define P_- as the region above $L(t)$ while P_+ as the region below $L(t)$. Figure 4 illustrates these notion on the Preisach plane P at a particular time instance t. Interested readers are referred to [3], [5] for detailed discussion on Preisach operators.

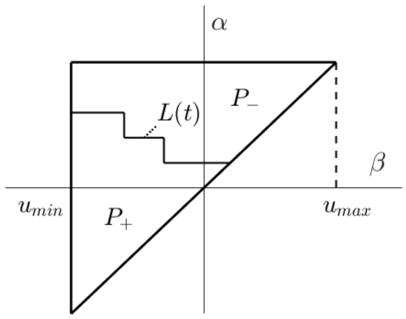


Fig. 4. Plot of a Preisach plane P used in the Preisach hysteresis operator (2) where P_+ defines the region of relays that are in $+1$ state while P_- defines the region of relays that are in -1 state. The line $L(t)$ describes the boundary between these two different regions and it changes dynamically according to the applied input signal $u(t)$ which has a maximum value of $u_{\max}$ and a minimum value of $u_{\min}$.

As studied in [5], Preisach operator has a number of nice properties which can be useful for dynamical analysis of systems containing such operator with *positive* weighting function μ in the feedback loop. These properties include invariance w.r.t. the Sobolev input space $W^{1,1}_{\text{loc}}$, Lipschitz continuity and monotonicity, e.g.,

$$\big(\overbrace{\Phi(u)}^{\cdot}\big)(t)\dot{u}(t) \geq 0.$$

Other well-known properties of this operator are the wiping-out property and the congruency property. For these two properties, we refer interested reader to the exposition in [3].

Although considering positive weighting function can become useful for the dynamical analysis of linear systems with hysteresis in the feedback loop [5], the definition of Preisach operator in (2) does not, in general, impose any positivity assumption on μ.

In fact, when the weighting function $\mu(\alpha,\beta)$ is constrained to be positive, the corresponding Preisach operator gives only *counterclockwise* dynamics (see, for instance, [13]). On the other hand, Zamboni and Visone in [16] show that we can obtain a *clockwise* dynamics using Preisach operator simply by restricting μ to be negative. The latter can also be interpreted as having a modified Preisach operator as in (2) where, instead of using the counterclockwise relay operator $\mathcal{R}^{\circlearrowleft}_{\alpha,\beta}$ as in (3), we use *clockwise* relay operators $\mathcal{R}^{\circlearrowright}_{\alpha,\beta}$ defined by

$$\big(\mathcal{R}^{\circlearrowright}_{\alpha,\beta}(u)\big)(t) := \begin{cases} -1 & \text{if } u(t) > \alpha \\ 1 & \text{if } u(t) < \beta \\ \big(\mathcal{R}^{\circlearrowright}_{\alpha,\beta}(u)\big)(t_-) & \text{otherwise.} \end{cases} \qquad (4)$$

Such *clockwise* hysteron element is illustrated in Fig. 5.

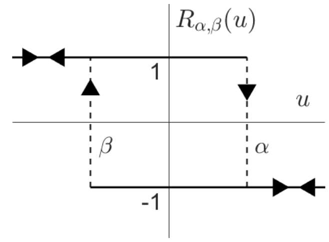


Fig. 5. An input-output phase plot of a clockwise relay $\mathcal{R}^{\circlearrowright}_{\alpha,\beta}(u)$ as in (4) with two switching moments β and α.

III. Preisach butterfly hysteresis operator

As discussed before, the Preisach operators as in (2) can be shown to exhibit simple *counterclockwise* or *clockwise* input-output behavior simply by constraining the weighting function $\mu(\alpha,\beta)$ to positive function or to negative function, respectively. However, following Definition 2.2, a *butterfly hysteresis operator* requires that it exhibits both *clockwise* as well as *counterclockwise* loops.

Early work in the modeling of butterfly loop in magnetostriction effect using Preisach operator is presented in [28]. In this work, the weighting function μ is not constrained to positive nor negative function. By removing this constraint, the Preisach operator can be fitted to the experimental measurement and it is capable of exhibiting *butterfly* input-output behavior. In this subsection, we will formalize and provide analysis of such Preisach operators.

Let us introduce a class of Preisach operators with two-sided weighting function μ. By two-sided, we mean that there exists a simple curve B that divides the Preisach plane P into two disjoint domains B_- and B_+ such that $B_- \cup B_+ \cup B = P$. The domain B_- will refer to case where $\mu(\alpha,\beta) < 0$ for all $(\alpha,\beta) \in B_-$. Similarly, B_+ refers to the case where $\mu(\alpha,\beta) > 0$ for all $(\alpha,\beta) \in B_+$. The curve B is called *monotonically decreasing* if for any given two pairs $(\alpha_1,\beta_1),(\alpha_2,\beta_2) \in B$ we have $\frac{\beta_2-\beta_1}{\alpha_2-\alpha_1} < 0$.

In the following proposition, we provide sufficient condition on B and μ such that Φ is a butterfly hysteresis operator.

Proposition 3.1: Consider a Preisach operator Φ as in (2) with μ be a two-sided piecewise continuous function. Suppose that the first order lower and upper partial moments of μ satisfy

$$\int_r^\infty \mu(\alpha,\beta)\beta \mathrm{d}\beta = \infty \qquad (5)$$

$$\int_{-\infty}^r \mu(\alpha,\beta)\alpha \mathrm{d}\alpha = \infty, \qquad (6)$$

for all $(\alpha,\beta) \in P$. Assume that the boundary curve B is monotonically decreasing. Then Φ is a butterfly hysteresis operator.

In Proposition 3.1, we consider a general case where μ can be any two-sided function, as long as, its decay to zero is not too fast. In this proposition, the decay of $|\mu|$ to zero must be slower than $1/\alpha^2$ or $1/\beta^2$. By considering this assumption on μ, we are not restricting the amplitude of periodic input signal to obtain a butterfly hysteresis operator according to Definition 2.2. Otherwise, we may not be able to find an input-output pair of signals such that the total signed-area $\mathbb{A}$ will be zero.

This requirement can in fact be weakened by focusing on a small neighborhood close to the meeting point of B and the line $\{(\alpha, \beta) \mid \alpha = \beta\}$. In this case, we can only show a local existence of a butterfly loop.

However, we can define another interesting class of Preisach butterfly operator as given in the following proposition.

Proposition 3.2: Consider a Preisach operator Φ as in (2) with μ be a two-sided piecewise continuous function. Assume that the boundary curve of μ is given by $B = \{(\alpha, \beta) \in P \mid \alpha = -\beta + \kappa\}$ where $\kappa \in \mathbb{R}$ is an offset and μ is anti-symmetric with respect to B, i.e., $\mu(\alpha, \beta) = -\mu(-\alpha, -\beta)$ holds for all $(\alpha, \beta) \in P$. Then Φ is a butterfly hysteresis operator.

The proofs of both Proposition 3.1 and 3.2 are omitted in this paper due to space limitations. However, we refer interested readers to [29] for a complete exposition.

IV. Modeling of novel hysteretic piezoelectric material

In this section, we will apply our characterization of Preisach butterfly hysteresis operator described in previous sections to the modeling of a novel piezoelectric material. In contrast to the standard piezoelectric material made of doped Lead Zirconate Titanate (PZT) ceramics where the hysteresis loop is symmetric (see Figure 2), we develop a material with an asymmetric (biased) loop, that exhibits remnant deformation (non-zero strain upon the removal of the electric field). Its development is driven by the end application of a novel hysteretic deformable mirror for wavefront correction in active optics systems. Piezoelectric materials with remnant deformation (strain memory effect) present advantages such as fast responses, compared to temperature shape memory alloys, and lower power by using electric fields compared to magnetically-driven shape memory alloys. The new material consists of Nb-doped PZT ceramics close to the Morphotropic Phase Boundary (MPB) mixed with ZrO_2 particles. The experimental strain data (as shown in Fig. 6 with solid black line) is obtained by interferometer measurements on a sample of such ceramics, which has been coated by a 100nm Au and a 5nm/100nm Ti/Pt layers as top and bottom electrodes,respectively. We use electrode materials with different work function values to induce an internal bias and induce the strain memory effect of the piezoelectric material. In this measurement, where a high electric field periodic input signal is applied, we can observe in Fig. 6 that the strain measurement exhibits an asymmetric butterfly loop with remnant deformation (non-zero strain at zero electric field).

We fitted the experimental data to a Preisach operator based on least square approach without constraining the

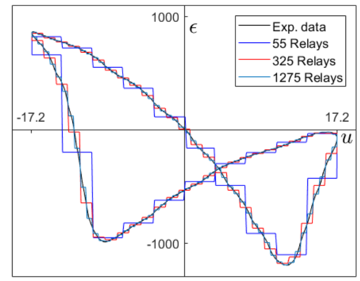


Fig. 6. The phase portrait of electrical field (kV/cm) and the strain (nm) of the new piezoelectric material. The black solid line is the measurement data using interferometer. The dark-blue, red and light-blue solid lines are the fitted Preisach hysteresis operator for different discretization level.

weighting function to a sign definite function. The infinite-dimensional Preisach operator is firstly discretized by discretizing the Preisach plane P into equal grid size. Fig. 6 shows the fitting result and particularly we can observe the effect of discretization where smaller grid size (i.e., more weighted relay operators are used) leads to a better fitting with the data.

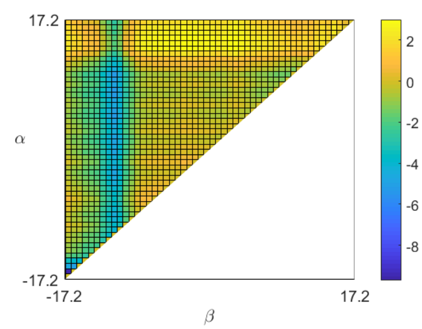
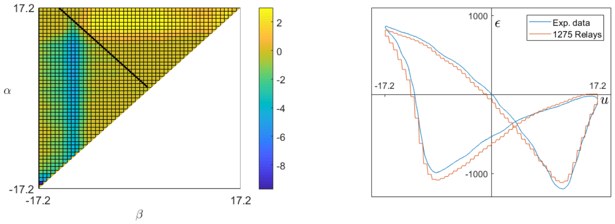


Fig. 7. The plot of fitted weighting function $\hat{\mu}(\alpha, \beta)$ based on the experimental data and without sign definite constraint. The fitted function shows a distinctive clustering of regions with positive and negative weight value.

Let us now analyze the fitted discretized weighting function $\hat{\mu}$ as shown in Fig. 7. As can be observed in this figure, the fitted weighting function $\hat{\mu}$ almost fits to the main hypothesis of Proposition 3.1 and 3.2 where two-sided piecewise continuous function μ is assumed. In particular, a predominantly positive and a predominantly negative area of $\mu(\alpha, \beta)$ can be distinguished. When we truncate the fitted weighting function $\hat{\mu}$ such that it has B with a slope of -1 and it satisfies $\hat{\mu}(\alpha, \beta) < 0$ for all $(\alpha, \beta) \in B_-$ and $\hat{\mu}(\alpha, \beta) > 0$ for all $(\alpha, \beta) \in B_+$ as shown in Fig. 8(a), the discretized Preisach operator using this truncated weighting function can still fit with the data as depicted in Fig. 8(b).

Consistent to the Definition 2.2, we can construct a periodic input signal u for the fitted Preisach butterfly hysteresis operator such that the resulting signed area is equal to zero. One of such construction is shown in Fig. 9 where we obtain a butterfly loop whose signed area is approximately zero. In this case the error is mainly caused by the discretization error.



(a) Truncated weighting function (b) Input-output phase portrait

Fig. 8. The resulting Preisach butterfly hysteresis operator which is fitted to the experimental data of novel piezoelectric material and satisfies the main hypothesis in Proposition 3.2.

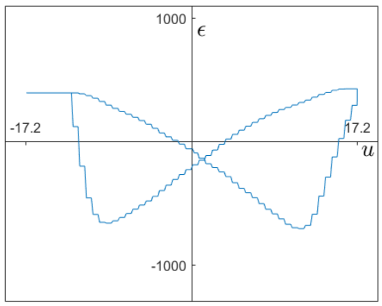


Fig. 9. The phase portrait of a butterfly loop using the fitted Preisach operator whose signed area is approximately close to zero.

V. Conclusions

We have presented a mathematical modeling framework for describing butterfly hysteresis behavior. This description was lacking and has become a limiting factor in the design of advanced instrumentation, high-precision mechatronic systems or other high-tech systems. By admitting a two-sided weighting function in the standard Preisach operator, we are able to obtain a butterfly hysteresis operator. The resulting Preisach butterfly hysteresis operator is also capable of describing non-standard, asymmetric butterfly loops with non-equal minimum points, as those exhibited in the lab by a piezoelectric material purposely-designed by us. This result generalizes existing modeling tools for describing specifically the butterfly hysteresis phenomena.

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