Energy-level structure of Sn$^{3+}$ ions

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Laser-produced Sn plasma sources are used to generate extreme ultraviolet light in state-of-the-art nanolithography. An ultraviolet and optical spectrum is measured from a droplet-based laser-produced Sn plasma, with a spectrograph covering the range 200–800 nm. This spectrum contains hundreds of spectral lines from lowly charged tin ions Sn$^{1+}$–Sn$^{5+}$ of which a major fraction was hitherto unidentified. We present and identify a selected class of lines belonging to the quasi-one-electron, Ag-like ([Kr]4d$^{10}$n$l$ electronic configuration), Sn$^{3+}$ ion, linking the optical lines to a specific charge state by means of a masking technique. These line identifications are made with iterative guidance from COWAN code calculations. Of the 53 lines attributed to Sn$^{3+}$, some 20 were identified from previously known energy levels, and 33 lines are used to determine previously unknown level energies of 13 electronic configurations, i.e., 7p, (7, 8)d, (5, 6)f, (6–8)g, (6–8)h, (7, 8)i. The consistency of the level energy determination is verified by the quantum-defect scaling procedure. The ionization limit of Sn$^{3+}$ is confirmed and refined to 328 908.4 cm$^{-1}$, with an uncertainty of 2.1 cm$^{-1}$. The relativistic Fock-space coupled-cluster (FSCC) calculations of the measured level energies are generally in good agreement with experiment but fail to reproduce the anomalous behavior of the 5d$^2$D and 6s$^2$F terms. By combining the strengths of the FSCC calculations, COWAN code calculations, and configuration interaction many-body perturbation theory, this behavior is shown to arise from interactions with doubly excited configurations.

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I. INTRODUCTION

Emission of light by neutral tin atoms and lowly charged tin ions, SnI–SnV, is abundant in a wide variety of plasmas, ranging from laser-produced extreme-ultraviolet (EUV) light generating Sn plasma for nanolithography [1,2], divertor plasma when using tin-containing materials in future thermonuclear fusion reactors [3–5], and discharge plasma between tin whiskers causing short circuits [6] to astrophysical environments [7–15]. Spectroscopic investigations of these kinds of plasmas can help characterize plasma parameters [16–22] such as ion and electron densities and temperatures by study of the observed line strengths and their shapes. However, spectroscopic information on the relevant charge states Sn$^{3+}$ and Sn$^{4+}$, i.e., SnIV and SnV, is rather scarce, because of the poorly known electronic structure of these ions.

Sn$^{3+}$, with its ground electronic configuration [Kr]4d$^{10}$ 5s, belongs to the Ag-like isoelectronic sequence. Remarkably, only the lowest eight singly excited 4d$^{10}$nl, the doubly excited 4d$^2$5s$^2$, and three 4d$^2$ 5s 5p levels in Sn$^{3+}$ are tabulated in the National Institute of Standards and Technology (NIST) Atomic Spectra Database (ASD) [23]. Level energies originate from unpublished work by Shenstone [24], while wavelengths are given in another compilation by the National Bureau of Standards [25]. The assessment of energy levels by Shenstone is based on extended and revised work by Lang and others [26–29]. Since the early compilation [24] of almost 60 years ago, the only extension of the electronic energy-level structure of SnIV stems from EUV spectroscopy by Ryabtsev and coworkers [30] in which they extend the ns series from $n = 8$ up to $n = 10$ and add the 7d$^2$D term. A more extensive list of SnIV lines is given in an otherwise unpublished master’s thesis [31]. In other works, beam-foil techniques have been used to determine lifetimes [32–34]. Besides the singly excited levels, some doubly excited energy levels belonging to the 4d$^2$ 5s 5p configuration are identified in laser- and vacuum-spark-produced tin plasmas [30,35–39]. Theoretical level energies and transition probabilities [40,41] have been calculated for Ag-like ions. The narrow, inverted fine structure of the 4f $^2$F term in Ag-like Sn$^{3+}$ has been
addressed in detail by theory [40–44]. In spite of all these efforts, knowledge of the electronic structure of Sn IV is mostly limited to its lowest energy levels.

To obtain the electronic structure of Sn$^{3+}$, we have studied its line emission in the wavelength range of 200–800 nm. The optical lines belonging to Sn IV are identified among the hundreds of optical lines stemming from a laser-produced droplet-based Sn plasma, by taking spectra as a function of the laser intensity. The method to single out transitions belonging to ions in a specific charge state relies on the strongly changing ratio between line intensity and background emission from the plasma as a function of the laser intensity.

In the following we first introduce and detail a convenient method to obtain charge-state-resolved optical spectra from a laser-produced plasma. Of the more than 350 lines observed in the visible spectral range, 53 are identified as stemming from Sn$^{3+}$. Of these, 33 lines are new determinations. Thereafter, line identification is discussed. On the basis of these line identifications an extended level diagram for Sn$^{3+}$ is constructed. The consistency of the highly excited levels is checked by quantum-defect scalings. In the final section, Fock-space coupled-cluster (FSCC) and configuration interaction many-body perturbation theory (CI+MBPT) calculations are employed to explain the anomalous behavior of the 5$d^2D$ and $nf^{2}F$ terms.

II. EXPERIMENTAL SETUP

An overview of the experimental setup is depicted in Fig. 1. A more detailed explanation is provided in Ref. [45]. The experimental laser-produced plasma source consists of a vacuum vessel (about $10^{-7}$ mb) equipped with a droplet generator from which a 10-kHz stream of liquid tin microdroplets is ejected. The droplets have a diameter of about 45 μm. A 10-Hz pulsed Nd:YAG laser, operating at its fundamental wavelength of 1064 nm, is used to irradiate the droplets in order to generate a plasma. The laser energy is varied without changing the beam shape by using the combination of a half-wave plate ($\lambda/2$) and a thin-film polarizer (TFP), reflecting part of the light into a beamdump (BD). The laser beam is circularly polarized by a quarter-wave plate ($\lambda/4$); hereafter the beam is focused onto the droplet. This results in a Gaussian full-width-at-half-maximum (FWHM) beam size of 115 μm at the droplet position. The laser has a 10-ns FWHM pulse length. Light reflected by the droplet falling through a helium-neon (HeNe) laser sheet is detected by a photon-multiplier tube (PMT) used to trigger the laser.

The light emitted from the plasma is observed through a viewport perpendicular to the laser beam propagation and 30° above the horizontal plane. A biconvex lens images the plasma onto a quartz fiber that is used to guide the light to the spectrometer (Princeton Isoplane SCT 320). The entrance side of the fiber consists of 19 cores with a diameter of 200 μm in a hexagonal configuration, while at the exit side the cores are oriented in a linear configuration to efficiently guide light through the spectrometer slit. The spectrometer is laid out in a Czerny-Turner configuration with a focal length of 320 mm. The grating has 1200 lines per millimeter and is blazed at 500 nm, leading to a significantly reduced grating diffraction efficiency below 300 nm. A CCD camera (Princeton Pixis 2KBUV) optimized for the ultraviolet and visible regime recorded the diffracted light. By rotating the grating, thus changing the spectral detection range, the full spectral range from 200 to 800 nm is covered in steps of approximately 50 nm, overlapping by about 10 nm. From the shortest to the longest wavelength the linear dispersion decreases from 0.033 to 0.028 nm per pixel.

The wavelength axis is calibrated using neon-argon and mercury lamps. The FWHM line widths of the calibration lines are smaller than 0.1 nm. The total uncertainties of the mid positions of the Sn$^{3+}$ lines are better than 0.01 nm over all observed laser energies and wavelengths. The emitted light is space- and time-integrated by summing the intensity resulting from the various fiber cores and taking an integration time of 10 s, corresponding to 100 laser shots.

Measurements are performed with, and without, edge-pass filters to distinguish second-order lines from first-order ones. This enables filtering out the second-order lines appearing at wavelengths longer than 400 nm. Additionally, closely packed lines in the ultraviolet below 300 nm can be resolved in second order at a higher resolution. Weakly appearing lines at first order, due to the low grating response below 300 nm, are observed with a higher intensity at second order.

III. CHARGE STATE IDENTIFICATION

We performed passive spectroscopy measurements on the laser-produced tin plasma for a series of laser energies ranging from 0.5 to 370 mJ. Figure 2 shows example measurements over a selected wavelength range for three laser energies, where it is shown that the number of lines increases with the laser energy. This is a signature of an increasing number of contributing charge states to the measured spectrum. A closer inspection indicates that indeed sets of lines appear with increasing laser energy that exhibit similar changes in intensity. As demonstrated below, each of these sets of lines

FIG. 1. Schematic top view of the main components of the laser-produced plasma source from which the spectroscopic data were taken. For details see Sec. II.
The observable increase in the background level is due to increased continuum emission from the plasma at higher laser energies. Spectra shown are taken without spectral filters and, thus, include second-order contributions.

can be singled out by considering their intensities with respect to the continuum background, increasing strongly with the laser energy.

To illustrate the procedure we select a well-known line of each of the charge states Sn—I—SnV. These are the Sn I $^5S_{0} − ^5P_{1}$ ($\lambda = 452.60$ nm [46]), Sn II $^5D_{3/2} − ^2F_{5/2}$ ($\lambda = 580.05$ nm [47]) (the transition from the $^4f^2F_{5/2}$ at 579.85 nm is also visible), Sn III $^5S_{0} − ^5P_{1}$ ($\lambda = 522.64$ nm [48]), Sn IV $^5S_{1/2} − ^2P_{1/2}$ ($\lambda = 421.73$ nm), and Sn V $^5D_{3} − ^5P_{3}$ ($\lambda = 315.6$ nm, based on level energies taken from Refs. [49] and [50]). The scaled intensity is defined as $I_{i}/I_{bg} - 1$, with $I_{i}$ the line intensity and $I_{bg}$ the (local value of the) continuum background level. For direct comparison, the scaled intensity of each individual line is normalized to its maximum value, as shown in Fig. 3. In Fig. 3, it is shown that for the lowest charge states Sn—I and Sn II the normalized, scaled intensity maximizes for the lowest laser energy of 0.5 mJ, while for the highest observed charge state Sn V a laser energy of 10 mJ is optimal. Sn IV, the ion of interest here, maximizes at 2 mJ. This demonstrates that the contributions of higher charge states to the spectrum increase with increasing laser energy. Preliminary nanosecond time-resolved spectroscopic measurements revealed that spectral line emission is mostly observed in the late-time evolution of the plasma. Traces of line broadening are observed in the time-integrated spectra presented in this work, e.g., the Sn IV line shape in the 2-mJ spectrum. Analysis of line-broadening mechanisms and the time evolution of these plasmas is left for future work, as they

FIG. 2. Experimentally obtained Sn$^{+1}$ spectra for laser energies of 0.5 mJ (blue; lower), 2 mJ (yellow; middle), and 10 mJ (green; upper). The specific transitions shown are described in the text.

FIG. 3. Spectral intensity scaling as a function of the wavelength (vacuum; in nm) for varying laser energies. Here, the intensity is normalized to their (local) continuum background level, and unity is subsequently subtracted. The specific transitions shown are described in the text.
TABLE I. Vacuum wavelengths (in nm) of Sn IV lines between levels previously known. Wavelengths determined in this work are compared with literature values taken from an otherwise unpublished master’s thesis [31]. The upper and lower energy levels indicating the transition represent the \( nl \) one-electron orbital outside the [Kr]4d\(^{10}\) core configuration. The 5s\(^2\) indicates the doubly excited 4d\(^{10}\) 5s\(^2\) configuration. Wavelengths determined in this work are averaged centroid positions of Gaussian fits in spectra taken at different laser energies. The intensity \( I \) represents the area under the curve of this line in the 30-mJ spectrum [in arbitrary (arb.) units]. \( gA \) factors from the upper level result from analysis with the COWAN code.

<table>
<thead>
<tr>
<th>( \lambda ) (nm)</th>
<th>( I ) (arb. units)</th>
<th>( gA ) ( (10^4 \text{ s}^{-1}) )</th>
<th>Lower</th>
<th>Upper</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>( nl )</td>
<td>( J )</td>
<td>( nl )</td>
</tr>
<tr>
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</tr>
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<td>677</td>
<td>3.5</td>
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The COWAN code [52], one of the most widely applied electronic structure codes, is used to calculate the energies of yet unestablished Sn IV levels. The COWAN code produces radial wave functions using a quasirelativistic Hartree-Fock method. The electrostatic single-configuration radial integrals \( F_k \) and \( G_k \) (Slater integrals), configuration interaction, Coulomb radial integrals, and spin-orbit parameters are calculated from the obtained wave functions. Subsequently, level energies and intermediate coupling eigenvectors are extracted.

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Other Sn I–Sn V lines show a similar dependence on the laser energy. The same lines as presented in Fig. 3 are used. Other Sn I–Sn V lines show a similar dependence on the laser energy.

Furthermore, values for the transition probabilities and wavelengths are obtained. The values of the electrostatic integrals are scaled by a factor of between 0.7 and 0.85 as well as the scaling of spin-orbit parameters to optimally fit the thus far understood experimental spectrum. The outcome of this parameter scaling procedure yields a useful interpretation of the experimental spectrum. This enables predictions for lines between not yet experimentally established energy levels. These predictions include their relative strength expressed as the $gA$ factor: the Einstein coefficient $A$ multiplied with the statistical weight $g$ of the upper state. Due to the one-electron nature of the system, the number of allowed transitions between two terms is only three (or two if a $2S$ term is involved). However, we might observe only two lines since transitions between equal angular momenta typically have a small $gA$ factor. Experimental lines that lie close to a predicted transition and have relative line strengths similar to those determined on the specific transition. This provides an enlarged set of levels that can be used to fine-tune the calculations in the next step.

B. Quantum defect

The energy levels of quasi-one-electron systems approach a hydrogenlike level structure, especially for high principal and angular quantum numbers. For such systems the energy levels which are shifted towards slightly higher binding energies can be well described by introducing the so-called quantum defect $\delta_l$ as a correction to the Bohr formula. The position of the energy level $E_{nl}$ (relative to the ionization limit) is defined by [51]

$$E_{nl} = -\frac{Z_c^2}{(n - \delta_l)^2}.$$  

The quantum defect becomes smaller with increasing angular momentum. For high-$n$ values the first term dominates and the minute change in $\delta_l$ as a function of $1/n^2$, with $n^*$ being the apparent principal quantum number $n^* = n - \delta_l$, with quantum defect $\delta_l$,

$$\delta_l = a \left( \frac{1}{n^{*2}} \right) + b \left( \frac{1}{n^{*4}} \right)^2 + \cdots. \quad (2)$$

The consistency of the energies of levels within a specific $l$ series is verified by determining the respective quantum defects. Quantum defects are calculated using Eq. (1) with the level energies found relative to the ionization limit. Therefore an accurate value of this limit is needed.

The ionization limit of 328 550 (300) cm$^{-1}$, tabulated in the NIST ASD [23], is based on the determination of the series limit of the $ns$ levels ($n = 5–7$). Ryabtsev et al. [30] extended this $ns$ series with $8s$, $9s$, and $10s$. Using the extended $ns$ series they were able to refine the ionization limit to 328 910 (5) cm$^{-1}$. The ionization limit can be further improved by taking additional levels into account. Configurations which are prone to shifting of the level energies by configuration interaction effects ($np$, $nd$, and $nf$, further described in Sec. V C) are, however, unsuitable for determining the series limit. Therefore we only use the $ng$, $nh$, and $ni$ configurations to refine the ionization limit. The results from analysis with the POLAR code [54] is 328 908.4 cm$^{-1}$, with a statistical error of 0.3 cm$^{-1}$. Combining this in quadrature with the uncertainty of the $6s$ anchor level, we arrive at a total uncertainty of 2.1 cm$^{-1}$. Figure 5 presents the quantum defects of the SnIV levels as a function of $1/n^2$. Overall, a smooth dependence is found for all angular quantum numbers from $l = 0$ (s) up to $l = 6$ (i), underpinning the consistency of our identifications.

V. RESULTS AND DISCUSSION

Using the iterative guidance from the COWAN code as described above, we assign the newly found lines. The new SnIV line assignments are summarized in Table II. The level energies of excited states are determined with respect to the 6s, which is used as an anchor level considering that transitions to the 5s ground state are outside our detection region. The optimization of level energies is performed using Kramida’s code LOPT [53], and the final results are presented in Table III.

The consistency of the energies of levels within a specific $l$ series is verified by determining the respective quantum defects. Quantum defects are calculated using Eq. (1) with the level energies found relative to the ionization limit. Therefore an accurate value of this limit is needed.

The ionization limit of 328 550 (300) cm$^{-1}$, tabulated in

FIG. 4. Normalized scaled intensities of Sn I–Sn V lines as a function of the laser energy. The same lines as presented in Fig. 3 are used. Other Sn I–Sn V lines show a similar dependence on the laser energy.

\begin{align*}
E_{nl} &= -\frac{Z_c^2}{(n - \delta_l)^2}, \quad (1) \\
\delta_l &= a \left( \frac{1}{n^{*2}} \right) + b \left( \frac{1}{n^{*4}} \right)^2 + \cdots. \quad (2)
\end{align*}
under the curve of this line in the 30-mJ spectrum [in arbitrary (arb.) units].

The excitation energies of the 8s, 9s, and 10s levels were determined in EUV spectroscopy experiments by Ryabtsev et al. [30] in which transitions to the 5p2P1/2,3/2 terms were measured. The 5p2P1/2,3/2 terms can be populated from the nd series; this nd series is known up to n = 7. The highest known np configuration so far was 6p. All transitions in the optical spectral range (Table II) between both ns and nd and 6p agree with the literature excitation energies of the respective levels.

The 7p2P1/2,3/2 levels are found by considering all possible transitions from the 7d2D3/2,5/2 and 8s2S1/2 to the 7p2P1/2,3/2 levels. These levels can decay via emission in the visible to 6d2D3/2,5/2. The excitation energies of the 7p configuration are established using seven transitions to three surrounding energy terms, providing a reliable assessment of the 7p2P1/2,3/2 level energies. For the 7d2D5/2 good agreement with Ref. [30] is found, while for the 7d2D3/2 level a difference of about 30 cm−1 is observed. The 8d is successively determined using two transitions to the 7p, where discussed and explained separately. Finally, Fig. 6 depicts the extended level diagram of SnIV as a concise summary of our results.

A. I ≤ Icore configurations

The 5s, 6s, and 7s levels are included in Moore’s tables [24]. The excitation energies of the 8s, 9s, and, 10s levels were determined in EUV spectroscopy experiments by Ryabtsev et al. [30] in which transitions to the 5p2P1/2,3/2 terms were measured. The 5p2P1/2,3/2 terms can be populated from the nd series; this nd series is known up to n = 7. The highest known np configuration so far was 6p. All transitions in the optical spectral range (Table II) between both ns and nd and 6p agree with the literature excitation energies of the respective levels.

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the transition between equal angular momenta is likely too weak to be observed in our spectra.

The quantum defects for $l \leq l_{\text{core}}$ are shown in Fig. 5, calculated using the refined ionization limit. All data points exhibit a linear behavior, with only the lowest 5f configurations slightly deviating, reflecting a signature of the small quadratic term in Eq. (2). The most remarkable observation is the almost-equal quantum defects of the 5d $^2D_{3/2,5/2}$ levels, indicative of an anomalously small fine-structure interval of the 5d $^2D$ term. This anomaly is further discussed in Sec. VC.

### B. $l > l_{\text{core}}$ configurations

Of the high-$l$ configurations, i.e., nf, ng, nh, and ni, only the level energies of the 4f $^2F$ and 5g $^2G$ terms were known thus far. Our measurements confirm the small inverted fine-structure splitting of approximately 60 cm$^{-1}$ of the 4f $^2F$ term.
and of about 0.5 cm\(^{-1}\) of the 5\(g\) \(^2\)\(G\) by direct comparison of the \(\Delta J = 0\) and \(\Delta J = -1\) transitions. The 4\(f\) \(^2\)\(F\) and 5\(g\) \(^2\)\(G\) terms form the main basis on which the excitation energies of the high-\(l\) configurations are determined. Fine-structure splittings of the \(ng^2G\) \((n \geq 6)\), \(nh^2H\), and \(ni^2I\) terms are too small to be determined, implying that their fine-structure splitting is less than 0.5 cm\(^{-1}\). The fine-structure splittings of the \(nf\) terms are presented in Table IV and discussed in detail in Sec. VC.

The first level of the \(nh\) series, \(6h\), is found by assigning the strong transition from this level to \(5g\). The \(6h\) is the lower level of the transitions determining the \(7i\) and \(8i\). The \(7h\) and \(8h\) are found by transitions to the \(6g\), which is based on the transition to the \(5f\) \(^2\)\(F\). The \(ng^2G\) \((n \geq 7)\) are determined from their transitions to the \(5f\) \(^2\)\(F\) and \(6f\) \(^2\)\(F\) terms. The \(5f\) and \(6f\) terms are defined by transitions to their lower-lying \(nd^2D\) counterparts.

The relative values of the quantum defects for the \(ng\), \(nh\), and \(ni\) series are in good agreement with the \(ni\) scaling laws for \(l > l_{core}\) as presented by Edlén [51]. The quantum defects for the \(nf\) series are about a factor of 3 to 4 larger than expected from these scaling laws. In addition, relatively large fine-structure splittings are observed for the \(5f\) \(^2\)\(F\) and \(6f\) \(^2\)\(F\) terms. Both effects may be a signature of an enhanced interaction with core-electron configurations.

### C. Anomalous fine-structure effects in the 5\(d^2D\) and \(nf^2F\) terms

Table IV summarizes experimental and theoretical fine-structure intervals in Sn IV. We have performed Fock-space coupled-cluster and configuration interaction many-body perturbation theory in order to address the aforementioned anomalous values of the fine-structure intervals in the 5\(d^2D\) and \(nf^2F\) terms.

FSCC calculations of the transition energies were performed within the framework of the projected Dirac-Coulomb-Breit (DCB) Hamiltonian [55].

\[
H_{DCB} = \sum_i h_D(i) + \sum_{i<j} (1/r_{ij} + B_{ij}).
\]  

(3)

Here, \(h_D\) is the one-electron Dirac Hamiltonian,

\[
h_D(i) = c \alpha_i \cdot p_i + c^2 \beta_i + V_{nucl}(i),
\]  

(4)

and \(V_{nucl}\) is the nuclear potential. The \(\alpha_i\) and \(\beta_i\) are the Dirac matrices, \(c\) is the velocity of light, and \(V_{nucl}\) is the nuclear potential. The interaction \(B_{ij}\) is the Breit interaction, \(r_{ij}\) is the internuclear distance, and \(\alpha_i\) and \(\beta_i\) are the Dirac matrices.

**FIG. 5.** Quantum defect values as a function of the 1/\((n - \delta)^2\) of the Sn IV energy levels, for \(l \leq l_{core}\) (top) and \(l > l_{core}\) (bottom). Quantum defects are calculated using Eq. (1) and the refined ionization limit of 328 908.4 cm\(^{-1}\). Black lines are linear fits of the data points where the lowest level is excluded (except for the \(n\) configuration).
The basis set was composed of 37 $s$, 31 $p$, 26 $d$, 21 $f$, 16 $g$, 11 $h$, and 6 $i$ functions; the convergence of the obtained transition energies with respect to the size of the basis set was verified. All the electrons were correlated.

FSCC calculations were performed using the Tel-Aviv Relativistic Atomic FSCC code (TRAFS-3C) [59]. To account for the QED corrections to the transition energies we applied the model Lamb shift operator of Shabaev and coworkers [60] to the atomic no-virtual-pair, many-body DCB Hamiltonian as implemented in the QEDMOD program. Our implementation of the model Lamb shift operator formalism into the Tel Aviv atomic computational package allows us to obtain the vacuum polarization and self-energy contributions beyond the usual mean-field level, namely, at the DCB-FSCCSD level.

The FSCC results are compared to the experimental level energies and several results from previous theoretical work in Table III and are overall in good agreement. Typical differences from experiment are about 100 to 300 cm$^{-1}$, which is on the $10^{-3}$ level of the calculated excitation energies. Concerning the measured anomalous fine-structure intervals of the 5$d^2D$ and 5$f^2F$ and 6$f^2F$ terms, listed in Table IV, the apparent narrowing of the fine-structure interval of the 5$d^2D$ term and the widening of the 5$f^2F$ and 6$f^2F$ term intervals are not reproduced by the FSCC calculations. The FSCC intervals are similar to those presented in earlier theoretical investigations [40–44].

For the 5$d^2D$ term the fine-structure interval is measured at 107 cm$^{-1}$, while all theoretical results are higher by a factor of approximately 7 (see Table IV). Upon inspecting the level diagram (Fig. 6) one notes that the 5$d^2D$ term might suffer from configuration interaction of the doubly excited 4$d^95s^2$ levels.

To quantify the strength of the configuration interaction we employ CI+MBPT calculations using the AMBiT code. Details of the AMBiT code can be found in Refs. [61–64]. To begin our discussion of the AMBiT treatment of the problem, first consider the 5$d^2D_{5/2}$ and 4$d^95s^2D_{5/2}$ levels as a two-level system. In the absence of interaction between them, they have theoretical energies $\epsilon_1$ and $\epsilon_2$, respectively. $\epsilon_1$ is, to a good approximation, the FSCC value of the 5$d^2D_{5/2}$ level, since in that calculation the one-hole two-particle 4$d^95s^2D_{5/2}$ level is not explicitly included. If we now add an interaction $V$, then the states mix and the levels repel each other.

The Hamiltonian of this two-level system is

$$\hat{H} = \begin{pmatrix} \epsilon_1 & V \\ V & \epsilon_2 \end{pmatrix}.$$ 

Writing $\Delta \epsilon = \epsilon_2 - \epsilon_1 > 0$, the 5$d^2D_{5/2}$ level shifts down by an amount

$$\delta = \frac{\Delta E - \Delta \epsilon}{2} = \frac{\Delta \epsilon}{2} \left( \sqrt{1 + \frac{4V^2}{\Delta \epsilon^2}} - 1 \right) = b^2 \Delta E,$$

where $\Delta E = E_2 - E_1$ is the difference between the eigenvalues of $H$ and $b$ is the smaller component of the normalized eigenvector $(a, b)^T$.

Using AMBiT, we calculate theoretical values for the parameters $\Delta \epsilon_{\text{th}}$ and $b$, from which we can obtain the interaction $V = -ab \Delta \epsilon_{\text{th}}$. However, the values of $\Delta \epsilon_{\text{th}}$ and $b$ are sensitive to details of the calculations. In particular, it

![Figure 6](image-url)

**FIG. 6.** Level diagram of Sn$^{3+}$, drawn from 150 000 cm$^{-1}$ to the ionization limit at 328 908.4 cm$^{-1}$. The ground state 5$s$, the 5$p^2p$ term, and multiply excited configurations lying near the ionization limit are omitted, as transitions to these levels occur outside the detection range of this study. The levels determined in this study are shown in the boxed area. Other levels are based on Refs. [24] and [30].
is challenging to match theoretical with experimental level energies at a good level of accuracy. On the other hand, the values of $V$ that we obtain are highly stable since they are not sensitive to the separation. If we use the experimental separation $\Delta E_{\text{exp}} = 3823 \text{ cm}^{-1}$ and $|V| = 1523 \text{ cm}^{-1}$ from AMBiT, we find the energy shift of the $5d^2D_{5/2}$ level due to interaction with the hole state,

$$\delta = \frac{\Delta E_{\text{exp}}}{2}
\left(1 - \frac{4V^2}{\Delta E_{\text{exp}}} \right), \quad (8)$$

yielding $−755 \text{ cm}^{-1}$. The $5d^2D_{5/2}$ level also shifts down due to interaction with the $4d^95s^2D_{3/2}$ hole level. However, the energy difference is three times larger, and since $b \sim 1/\Delta E$, the energy shift is smaller by approximately a factor of three. We calculate $|V| = 1498 \text{ cm}^{-1}$ for this pair of levels, so Eq. (8) gives a level shift of $−181 \text{ cm}^{-1}$. The change in the $5d$ fine-structure splitting is therefore $−574 \text{ cm}^{-1}$, which is close to the difference between experiment and the FSCC calculation of $−629 \text{ cm}^{-1}$.

In support of the role of the configuration interaction in the $5d^2D$ fine structure, FSCC calculations were performed for isoelectronic $\text{In}^{2+}$ ions, in which there is a much larger energy difference between the $5d^2D$ term and the doubly excited $4d^95s^2D_{3/2}$ level [24]. Thus a better agreement with FSCC calculations is expected. In comparison to $5\text{Sn}^{2+}$, for $\text{In}^{2+}$ the difference between experiment [24] and the FSCC indeed reduces strongly, from a factor of 7 to only 30% (298 versus 398 cm$^{-1}$).

Before discussing the impact of configuration interaction on the fine structure of the $5f^2F$ and $6f^2F$ terms we note that the $4f^2F$ exhibits an inverted fine structure, with the $J = 7/2$ level being more strongly bound than the $J = 5/2$ level by approximately 60 cm$^{-1}$. The occurrence of this inversion of the fine structure and the actual value of the fine-structure interval results from an intricate balance between relativistic, spin-orbit, and core polarization effects and has been the subject of a variety of theoretical approaches calculating the $4f^2F$ fine structure along the isoelectronic sequence of $\text{Ag}$-like ions [40–44].

We confirm the fine structure of the $4f^2F$ term by measuring the wavelengths of the transitions from the $5g^6G$ levels and the transitions to the $5d^2D$ levels. Weaker transitions from the $6d^2D$ term to the doubly excited $4d^95s^2D_{3/2}$ level are observed in addition. A comparison of the fine-structure splitting of the observed $nf^2F$ levels with theoretical calculations is given in Table IV. Both measurements and theoretical calculations agree on an inverted fine-structure splitting for these $nf^2F$ terms. However, the magnitudes of the fine-structure splitting of the $5f$ and $6f$ terms are much smaller than our experimental ones.

In similar fashion to the $5d^2D$ levels, the $5f^2F$ fine-structure splitting is strongly affected by interaction with holol states. However, the $5f^2F$ case is more complicated because the $4d^95s5p$ configuration has seven configuration state functions (CSFs) with $J = 5/2$ and four CSFs with $J = 7/2$. The CSFs tend to be strongly mixed with each other and, also, have small contributions of CSFs belonging to other configurations. Therefore, rather than treat the system as a few-level system, we use the approach of perturbation theory.

| $E_i$ (cm$^{-1}$) [30] | $J$ | $|V_{i,5f}|$ | $\delta_{5f}$ |
|------------------------|---|-------------|----------|
| 226 363                | 5/2| 3           | 0        |
| 231 318                | 5/2| 484         | 11       |
| 239 582                | 5/2| 765         | 47       |
| 242 203                | 5/2| 622         | 39       |
| 249 541                | 5/2| 41          | 1        |
| 263 718                | 5/2| 111         | −1       |
| 269 440                | 5/2| 1341        | −104     |
| 231 090                | 7/2| 72          | 0        |
| 239 920                | 7/2| 417         | 15       |
| 246 851                | 7/2| 601         | 72       |
| 260 398                | 7/2| 2404        | −676     |

Using AMBiT we obtain energies for the $4d^95s5p$ levels as well as a mixing coefficient. At first order in perturbation theory, the coefficient of $\psi_{5f}$ in level $\psi_i$ is simply

$$b_{5f} = \frac{V_{i,5f}}{E_i - E_{5f}}.$$

From our AMBiT values of $b_{5f}$ and $E_i$, we extract values for the matrix element $V_{i,5f}$. Again, these are relatively stable for different calculations, even though the energies and $b$ coefficients can change dramatically.

The corresponding energy shift of the $5f$ level is $\delta_{5f} = V_{i,5f}/\Delta E_{\text{exp}}$. Unfortunately we do not have precise experimental determinations of most of the interaction $4d^95s5p$ levels. Instead we use the results of COWAN calculations [30] to obtain an approximation to the level shifts. The results are presented in Table V.

TABLE V. Sm$^2+$ $4d^95s5p$ level energies as candidates for possible configuration interaction with the $nf$ levels. Level energies are obtained from COWAN code calculations published in Ref. [30]. Matrix elements $|V_{i,5f}|$ are calculated using the AMBiT code. The resulting shift of the $5f$ level by configuration interaction by the level is given by $\delta_{5f}$. The $J = 5/2$ levels interact with the $5f^2F_{5/2}$, similarly, the $J = 7/2$ levels, with the $5f^2F_{7/2}$. We see that while each of the $5f$ levels is shifted by interactions with the hole levels, the change in the fine-structure splitting is dominated by the interaction of the $5f^2F_{7/2}$ level with a hole state at 260 398 cm$^{-1}$. The final expected shift is $580 \text{ cm}^{-1}$, overestimating the actual difference between experimental fine-structure and FSCC calculations of 270 cm$^{-1}$. Nevertheless, given the uncertainties in our estimation of $V$ and the location of the doubly excited levels, we arrive at a plausible explanation for the observed anomaly.

A similar explanation can be given for the observed difference in fine-structure splitting for the $6f^2F$ term, interacting with the high-lying levels in the same series of hole states. Because the energy differences between these levels are larger, this effect may reasonably be expected to be smaller than for the $5f^2F$ levels. Likewise, the $7p^2P$ is expected to interact with several doubly-excited levels.

We also performed COWAN calculations to investigate the terms described above. The number of fitted parameters in this case is reduced by tying the Hartree-Fock ratios to the spin-orbit parameters for the $np$, $nd$, and $nf$ levels. The $4d^95s5p$ levels are taken from Ref. [30] (included in Table V). The
two $4^9 5s^2 2D$ levels were calculated with two adjustable parameters. All interaction parameters were fixed at 0.8 of their Hartree-Fock values. The number of levels determined was insufficient to more accurately fit the parameters. Results of the calculation are listed in Table IV and show good agreement with the experimental results, although the inversion of the $4f^2 2F$ level is not reproduced. The agreement for all other $nf$ and the $np$ and $nd$ levels underlines the significant role of configuration interaction in a quasi-one-electron system like Sn$^{3+}$.

VI. CONCLUSION

Optical techniques are useful diagnostics in plasma sources of EUV light in nanolithography. We present the ultraviolet and optical spectra of a laser-produced tin plasma. The lines belonging to Sn$^{3+}$ are identified using a convenient masking technique. The 33 newly found lines are used to determine 13 new configurations with iterative guidance from Cowan code calculations. The level energies are verified using a quantum-defect scaling procedure, leading to the refinement of the ionization limit to 328 908.4 cm$^{-1}$, with an uncertainty of 2.1 cm$^{-1}$. FSCC calculations are generally in good agreement with the present measurements. The anomalous behavior of the $5d^2 2D$ and $nf^2 2D$ terms is shown to arise from configuration interaction with doubly excited levels by joining the strengths of the FSCC, Cowan, and Cl+MBPT approaches.

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