The shape of the dark matter halo in the early-type galaxy NGC 2974

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ABSTRACT

We present H I observations of the elliptical galaxy NGC 2974, obtained with the Very Large Array. These observations reveal that the previously detected HI disc in this galaxy is in fact a ring. By studying the harmonic expansion of the velocity field along the ring, we constrain the elongation of the halo and find that the underlying gravitational potential is consistent with an axisymmetric shape.

We construct mass models of NGC 2974 by combining the H I rotation curve with the central kinematics of the ionized gas, obtained with the integral-field spectrograph SAURON. We introduce a new way of correcting the observed velocities of the ionized gas for asymmetric drift, and hereby disentangle the random motions of the gas caused by gravitational interaction from those caused by turbulence. To reproduce the observed flat rotation curve of the H I gas, we need to include a dark halo in our mass models. A pseudo-isothermal sphere provides the best model to fit our data, but we also tested an NFW halo and modified Newtonian dynamics, which fit the data marginally worse.

The mass-to-light ratio \( M/L_i \) increases in NGC 2974 from 4.3 M⊙/L⊙,I at one effective radius to 8.5 M⊙/L⊙,I at 5 \( R_e \). This increase of \( M/L \) already suggests the presence of dark matter: we find that within 5 \( R_e \) at least 55 per cent of the total mass is dark.

Key words: galaxies: elliptical and lenticular, cD – galaxies: haloes – galaxies: individual: NGC 2974 – galaxies: kinematics and dynamics – dark matter.

1 INTRODUCTION

Although the presence of dark matter dominated haloes around spiral galaxies is well established (e.g. van Albada et al. 1985), there is still some controversy about their presence around early-type galaxies. Spiral galaxies often contain large regular H I discs, which allow us to obtain rotation curves out to large radii, and therefore we can constrain the properties of their dark haloes. But these discs are much rarer in elliptical galaxies (e.g. Bregman, Hogg & Roberts 1992), so that for this class of galaxies we are often required to use other tracers to obtain velocity measurements, such as stellar kinematics, planetary nebulae or globular clusters. These tracers however are not available for all early-type galaxies, and give mixed results (e.g. Rix et al. 1997; Romanowsky et al. 2003; Bridges et al. 2006).

With the increase in sensitivity of radio telescopes, it has been discovered that many early-type galaxies in the field do contain H I gas, though with smaller surface densities than in spiral galaxies (e.g. Morganti et al. 2006). The average H I surface density in the Morganti et al. sample is around 1 M⊙ pc⁻², which is far below the typical value for spiral galaxies (4–8 M⊙ pc⁻², e.g. Cayatte et al. 1994). This would explain why previously only the most gas-rich early-type galaxies were detected in H I. Morganti et al. find that H I can be present in different morphologies: H I discs seem to be as common as offset clouds and tails, though they occur mostly in the relatively gas-rich systems.

Recently rotation curves of H I discs in low surface brightness galaxies and dwarf galaxies, complemented with Hz observations, have been used not only to confirm the existence of dark matter haloes, but also to obtain estimates on the inner slope of the density profiles of the haloes (e.g. van den Bosch et al. 2000; Weldrake, de Blok & Walter 2003). Simulations within a cold dark matter (CDM) cosmology yield haloes with cusps in their centres (NFW profiles, see Navarro, Frenk & White 1996), but observations...
suggest core-dominated profiles (e.g. de Blok & Bosma 2002; de Blok 2005).

Detailed studies of rotation curves of early-type galaxies that contain H1 discs are sparser, due to lack of spatial resolution: to detect low H1 surface densities, larger beams are needed. Also, only a small-scale two-dimensional gas velocity map in the centre of spectrograph SAURON (Bacon et al. 2001). This combination of (VLA), with that of ionized gas, obtained with the integral-

Table 1. Properties of NGC 2974. The values are taken from the Lyon/Meudon Extragalactic Database and corrected for the distance modulus, which is taken from the surface brightness fluctuation measurements by Tonry et al. (2001). Note that 0.06 mag is subtracted to adjust to the Cepheid zero-point of Freedman et al. (2001); see Mei et al. (2005), Section 3.3, for a discussion. The effective radius is taken from Cappellari et al. (2006).

<table>
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<th>Parameter</th>
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</tr>
<tr>
<td>Effective radius</td>
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</tr>
</tbody>
</table>

ments of a rotation curve ranging from 100 pc within the centre of the galaxy to 10 kpc at the edges of the H1 ring. We use this rotation curve, together with ground- and space-based optical imaging, to determine the dark matter content in NGC 2974, and to constrain the shape of the dark halo.

In Section 2, we discuss the two data sets and their reduction, and describe the H1 ring. We concentrate on the analysis of the velocity maps in Section 3. Section 4 is devoted to the rotation curve that we extract from the velocity maps, and in Section 5 we show mass models with various halo models, and find the best fit to the rotation curve. Section 6 summarizes our results.

2 OBSERVATIONS AND DATA REDUCTION

2.1 VLA observations

Earlier VLA observations (Kim et al. 1988) of NGC 2974 showed that this galaxy contains a significant amount of H1 that, in their observations, appears to be distributed in a regularly rotating disc. Given the modest spatial and velocity resolution of those observations, we re-observed NGC 2974 with the VLA C-array while also using a different frequency set-up that allows us to study this galaxy at both higher spatial and higher velocity resolution. The observations were performed on 2005 September 11 and 19 with a total on-source integration time of 15 h. In each observation, two partially overlapping bands of 3.15 MHz and 64 channels were used. The two bands were offset by 500 km s−1 in central velocity. This frequency set-up allows us to obtain good velocity resolution over a wide range of velocities (about 1080 km s−1).

The data were calibrated following standard procedures using the MIRIAD software package (Sault, Teuben & Wright 1995). A spectral-line data cube was made using robust weighting (robustness = 1.0) giving a spatial resolution of 19.9 × 17.0 arcsec2 and a velocity resolution of 20.0 km s−1 (after Hanning smoothing). The noise in the final data cube is 0.23 mJy beam−1.

To construct the total H1 image, a mask was created using a data cube that was smoothed to about twice the spatial resolution and that was clipped at twice the noise of that smoothed data cube. The resulting total H1 is shown in Fig. 1, and our observations show that the H1 is distributed in a regular rotating ring instead of a filled disc. The inner radius of the ring is approximately 50 arcsec (∼5 kpc) and extends to 120 arcsec, which corresponds to 12 kpc, or 5 effective radii (1 Re = 24 arcsec).

The H1 velocity field was derived by fitting Gaussians to the spectra at those positions where signal is detected in the total H1 image. The resulting velocity map is shown in Fig. 2. Typical errors on this map are 5–10 km s−1.

We find a total mass of 5.5 × 108 M⊙ for the H1 gas content of the ring, which is in agreement with Kim et al. (1988), if we correct for the difference in assumed distance modulus. The amount and morphology of the H1 observed in NGC 2974 are not unusual for early-type galaxies. Oosterloo et al. (2007) have found that between 5 and 10 per cent of early-type galaxies show H1 masses well above 108 M⊙, while the fraction of detections increases further for lower H1 masses (Morganti et al. 2006). The majority of the H1-rich systems have the neutral hydrogen distributed in disc/ring like structures (often warped) with low surface brightness density and no or little ongoing star formation, as observed in NGC 2974. However, there is a region in the north-east of the H1 ring where the surface density is higher, and the gas could be forming stars. Jeong et al. (2007) published ultraviolet (UV) imaging of NGC 2974, obtained with GALEX. Their images reveal indeed a region of increased star...
in Emsellem et al. (2004) and Sarzi et al. (2006), respectively, and we refer the reader to these papers for the methods of data reduction and extraction of the kinematics.

In Fig. 2 we compare both the SAURON velocity maps of stars and \([\text{[O III]}]\) with the velocity map of the H\textsc{i} ring. Stars and gas are well aligned, and the transition between the ionized and the neutral gas seems to be smooth, suggesting that they form one single disc. The twist in the velocity map of the ionized gas in the inner 4 arcsec is likely caused by the inner bar of this galaxy (Emsellem et al. 2003; Krajnović et al. 2005).

3 ANALYSIS OF VELOCITY FIELDS

We used kinemetry (Krajnović et al. 2006) to analyse the SAURON and VLA velocity maps. In our application to a gas disc, kinemetry reduces to the tilted-ring method (Begeman 1978). The velocity along each elliptical ring is expanded in Fourier components (e.g. Franx, van Gorkom & de Zeeuw 1994; Schoenmakers, Franx & de Zeeuw 1997):

\[
V_{\text{los}}(R, \phi) = V_{\text{sys}}(R) + \sum_{n=1}^{N} c_n(R) \cos n\phi + s_n(R) \sin n\phi, \tag{1}
\]

where \(V_{\text{los}}\) is the observed velocity, \(R\) is the length of the semimajor axis of the elliptical ring, \(\phi\) the azimuthal angle, measured from the projected major axis of the galaxy, \(V_{\text{sys}}\) the systemic velocity of the ring and \(c_n\) and \(s_n\) are the coefficients of the harmonic expansion. The \(c_1\) term relates to the circular velocity \(V_c\) in the disc, so that \(c_1 = V_c \sin i\), where \(i\) is the inclination of the gas disc. Assuming that motions in the ring are intrinsically circular and that the ring is infinitely thin, the inclination can be inferred from the flattening \(q\) of the fitted ellipse: \(\cos i = q\).

If a gas disc only displays pure circular motions, all harmonic terms other than \(c_1\) in equation (1) are zero. Non-circular motions, originating from e.g. inflows caused by spiral arms or bars, or a triaxial potential, will cause these terms to deviate from zero. Alternatively, also wrong input parameters of the ring (which are flattening \(q\), position angle \(\Gamma_1\) and the coordinates of the centre of the ellipse) will result in specific patterns in these terms, see e.g. van der Kruit...
& Allen (1978), Schoenmakers et al. (1997) and also Krajnović et al. (2006) for details. Therefore, the flattening and position angle of each ring are determined by minimizing \( s_1, s_3 \) and \( c_1 \) along that ring. The centre is kept constant and is chosen to coincide with the position of maximal flux in the galaxy.

3.1 Non-circular motions

Fig. 3 shows the properties of the elliptic rings that were fitted to the SAURON and VLA velocity fields, and Fig. 4 shows the resulting harmonic terms. The data points of the VLA data are separated by approximately one beam size. Error bars were calculated by constructing 100 Monte Carlo realizations of the velocity fields, where the measurement errors of the maps were taken into account.

Both the position angles and the inclinations of the rings show some variation in the SAURON field, but are very stable in the VLA field. The dashed line in the top two panels of Fig. 3 indicates the mean value of the position angle and inclination of the HI data, which are \( \Gamma = 47 \pm 1^\circ \) and \( i = 60 \pm 2^\circ \). Here, \( \Gamma \) is the position angle of the receding side of the galaxy, measured north through east. The systemic velocities (lower panel of Fig. 3) have been corrected for barycentric motion and are in good agreement. For the SAURON field we find a systemic velocity of \( 1891 \pm 3 \) km s\(^{-1}\), while for the VLA field we find \( 1888 \pm 2 \) km s\(^{-1}\). The dashed lines give both these mean velocities. Both the inclination and the systemic velocity that we find are in agreement with previous studies (Cinzano & van der Marel 1994; Emsellem et al. 2003; Krajnović et al. 2005).

The harmonic terms are shown in Fig. 4. All terms are normalized with respect to \( c_1 \). From \( c_1 \) we see that the velocity curve of the gas rises steeply in the centre, but flattens out at larger radii. This already suggests that a dark halo is present around this galaxy. In Section 4 we will analyse the rotation curve in more detail.

The other terms have small amplitudes, and are small compared to \( c_1 (< 4 \) per cent). We do not observe signatures that could indicate incorrect ring parameters, as described in Schoenmakers et al. (1997) and Krajnović et al. (2006).

3.2 Shape of the gravitational potential

Following Schoenmakers et al. (1997), we calculate the elongation of the potential from the harmonic terms. Using epicycle theory these authors showed that a \( \cos m\phi \)-term perturbation of the potential results in signal in the \( m-1 \) and \( m+1 \) coefficients of the harmonic expansion in equation (1).

We assume that the potential of NGC 2974 is affected by an \( m=2 \) perturbation, which could correspond to a perturbation by a bar. We assume that the galaxy is not affected by lopsidedness, warps or spiral arms. To first order, the potential of the galaxy in the plane of the gas ring can then be written as

\[
\Phi(R, \phi) = \Phi_0(R) + \Phi_2(R) \cos 2\phi, \tag{2}
\]

with \( \Phi_2(R) \ll \Phi_0(R) \). As explained in Schoenmakers et al. (1997), the elongation of the potential \( \epsilon_{\text{pot}} \) in the plane of the gas is in this case given by

\[
\epsilon_{\text{pot}} \sin 2\phi = \frac{(s_1 - s_3)}{c_1} \frac{(1 + 2q^2 + 5q^4)}{(1 - q^2)}, \tag{3}
\]

where \( \phi \) is one of the viewing angles of the galaxy, namely the angle between the minor axis of the galaxy and the observer, measured in...
the plane of the disc. This viewing angle is in general unknown, so that from this formula only a lower limit on the elongation can be derived. Schoenmakers (1998) used this method in a statistical way and found an average elongation \( \epsilon_{\text{pot}} = 0.044 \) for a sample of eight spiral galaxies.

We calculated the elongation at different radii in NGC 2974, and the result is plotted in Fig. 5. As in Schoenmakers et al. (1997), we did not fix \( \Gamma \) and \( q \) when determining the harmonic terms, because an offset in \( \Gamma \) or \( q \) introduces extra signal in \( c_1, s_1 \) and \( s_3 \), that would then be attributed to the elongation of the potential.

Although the ionized gas has high random motions (see also Section 4) and therefore the calculated elongation is probably only approximate, it is striking that the elongation changes sign around 10 arcsec. The potential in the inner 10 arcsec has a rather high elongation \( \epsilon_{\text{pot}} \approx 0.1 \), while outside this region the elongation as measured from the ionized gas is \( \epsilon_{\text{ion}} = -0.047 \). The change of sign could be the result of the bar system in NGC 2974, with the direction along which the potential is elongated changing perpendicularly. It is worth mentioning here that Krajnović et al. (2005) find a ring in the \([\text{OIII}]\) equivalent width map, with a radius of 9 arcsec. Their data suggest also the presence of a (pseudo-)ring around 28 arcsec, and Jeong et al. (2007) find a ring with a radius of \( \sim 60 \) arcsec in their GALEX UV map, which is where our H\textsc{i} starts. Assuming that these three rings are resonances of a single bar, Jeong et al. (2007) deduce a pattern speed of 78 \pm 6 km s\(^{-1}\) kpc\(^{-1}\). In addition to the large scale bar, Emsellem et al. (2003) postulate a small nuclear bar (\( \sim 3 \) arcsec).

The H\textsc{i} gas is more suitable for measuring the elongation of the potential, since the cold gas has a small velocity dispersion (typical values \( < 10 \) km s\(^{-1}\)) and is on nearly circular orbits. Taking the mean value of the elongation as obtained from the H\textsc{i} field, we find \( \epsilon_{\text{pot}} \sin 2 \varphi = 0.016 \pm 0.022 \). We conclude that the potential of NGC 2974 is well approximated by an axisymmetric one.

4 ROTATION CURVE

To find the rotation curve of NGC 2974, we subtract the systemic velocities from the ionized and neutral gas velocity fields separately. Next, we fix \( \Gamma = 47^\circ \) and \( q = 0.50 \) (or equivalently \( i = 60^\circ \)) of the ellipses to the mean values obtained from the neutral gas, and rerun kinemetry on both the velocity maps, now forcing \( \Gamma \) and flattening to be the same everywhere in the gas disc. Also, because velocity is an odd moment, the even terms in the harmonic expansion should be zero, and are not taken into account during the fit (see Krajnović et al. 2006). The rotation curve of the ionized gas is shown in Fig. 6 (open diamonds).

The ionized gas has a high observed velocity dispersion \( \sigma_{\text{obs}} \), exceeding 250 km s\(^{-1}\) in the centre of the galaxy. Three phenomena can contribute to the observed velocity dispersion of a gas: thermal motions, turbulence and gravitational interactions:

\[
\sigma_{\text{obs}}^2 = \sigma_{\text{thermal}}^2 + \sigma_{\text{turb}}^2 + \sigma_{\text{grav}}^2.
\]  

The thermal velocity dispersion is always present, and caused by the thermal energy of the gas molecules:

\[
\sigma_{\text{thermal}}^2 = \frac{k T}{m},
\]

where \( k \) is Boltzmann’s constant, \( T \) the temperature of the gas and \( m \) the typical mass of a gas particle. The contribution of \( \sigma_{\text{thermal}} \) to the total velocity distribution in ionized gas is small: a typical temperature for ionized gas is \( 10^4 \) K, which implies \( \sigma_{\text{thermal}} \approx 10 \) km s\(^{-1}\).

Turbulence can be caused by e.g. internal motions within the gas clouds or shocks induced by a non-axisymmetric perturbation to the potential, such as a bar. This increases the dispersion, but has a negligible effect on the circular velocity of the gas. In contrast, gravitational interactions of individual gas clouds not only increase random motions of the clouds and therefore their dispersion, but also lower the observed velocity. To correct for this last effect, we need to apply an asymmetric drift correction to recover the true circular velocity.

Unfortunately, it is not possible a priori to determine which fraction of the high velocity dispersion in the ionized gas is caused by turbulence and which by gravitational interactions. We therefore now first investigate the effect of asymmetric drift on the rotation curve of the ionized gas.

4.1 Asymmetric drift correction

Due to gravitational interactions of gas clouds on circular orbits, the observed velocity is lower than the circular velocity connected...
to the gravitational potential. Since we are interested in the mass distribution of NGC 2974, we need to trace the potential, and therefore we have to increase our observed velocity with an asymmetric drift correction, to obtain the true circular velocity. We follow the formalism described in Appendix A, which is based on the Jeans equations and the higher order velocity moments of the collisionless Boltzmann equation.

We assume that the galaxy is axisymmetric, which is a valid approach given the low elongation of the potential that we derived in Section 3.2. Further we assume that the gas lies in a thin disc.

We fit the prescription that Evans & de Zeeuw (1994) used for their power-law models to the rotation curve extracted from the ionized gas,

\[ v_{\text{mod}} = \frac{V_c R}{R_{\text{mod}}}, \tag{6} \]

where \( V_c \) is the rotation velocity at large radii, and we introduce

\[ R_{\text{mod}}^2 = R^2 + \kappa^2, \tag{7} \]

with \( R \) the core radius of the model. This is equation (A14) evaluated in the plane of the disc \((z = 0)\), with a flat rotation curve at large radii \((\beta = 0)\). Since we observe the gas only in the equatorial plane of the galaxy, we cannot constrain the flattening of the potential \(q_\phi\). We therefore assumed a spherical potential \(q_\phi = 1\), which is not a bad approximation even if the density distribution is flattened, since the dependence on \(q_\phi\) is weak. Moreover, even though the density distribution of most galaxies is clearly flattened, the potential is in general significantly rounder than the density. For example, an axisymmetric logarithmic potential is only about a third as flattened as the corresponding density distribution (e.g. section 2.2.2 of Binney & Tremaine 1987).

To be able to fit the observed velocity we need to convolve our model with the point spread function (PSF) of the observations, and take the binning into account that results from the finite pixel size of the CCD. We therefore constructed a two-dimensional velocity field of the extracted rotation curve, such that

\[ V(R, \phi) = v_{\text{mod}} \cos \phi \sin i, \tag{8} \]

and we convolved this field with a kernel as described in the appendix of Qian et al. (1995). This kernel takes into account the blurring caused by the atmosphere and the instrument (FWHM = 1.4 arcsec, for the SAURON observations of NGC 2974; see Emsellem et al. 2004) and the spatial resolution of the reduced observations (0.8 arcsec for SAURON). We extracted the velocity along the major axis of the convolved velocity model and used the resulting rotation curve to fit our observations. The best fit is shown in Fig. 6, and has a core radius \( R_c = 2.1 \) arcsec (~0.2 kpc).

Under the assumptions of equation (6), the asymmetric drift correction of equation (A17) reduces to

\[ V_c^2 = \sqrt{V_c^2} - \sigma_{R,\text{rad}} \left[ \frac{\partial \ln \Sigma}{\partial \ln R} + \frac{\partial \ln \Sigma}{\partial \ln R} + \frac{R^2}{2R_{\text{mod}}^2} \right. \]

\[ + \left. \frac{\kappa R^2}{\kappa (2R_{\text{mod}}^2 - R^2) + R^2} \right], \tag{9} \]

where \( V_c \) is the observed velocity, \( \Sigma \) is the surface brightness of the ionized gas and \( \sigma_{R,\text{rad}} \) the radial dispersion of the gas. The last two terms in the equation are connected to the shape of the velocity ellipsoid, with \( \kappa \) indicating the alignment of the ellipsoid, see Appendix A.

To determine the slope of the surface brightness profile, we run kinemetry on the [OIII] flux map, extracting the surface brightness along ellipses with the same position angle and flattening as the ones used to describe the velocity field. To decrease the noise we fit a double exponential function to the profile,

\[ \Sigma(R) = \Sigma_0 e^{-R/R_0} + \Sigma_1 e^{-R/R_1}, \tag{10} \]

and determine the slope needed for the asymmetric drift correction from this parametrization. The observed surface brightness profile and its fit are shown in Fig. 7. As with the velocity profile, we convolved our model of the surface brightness during the fit with the kernel of Qian et al. (1995) to take seeing and sampling into account.

\[ \sigma_R \] can be obtained from the observed velocity dispersion \( \sigma \) using equation (A13). Along the major axis, and under the assumptions made above, this expression simplifies to

\[ \sigma_{R,\text{obs}}^2 = \sigma_{R,\text{mod}}^2 \left[ 1 - \frac{R^2 \sin^2 i}{2R_{\text{mod}}^2} + \frac{R^2 \cos^2 i}{\kappa R_{\text{mod}}^2 (2 - R^2/R_{\text{mod}}^2) + R^2} \right]. \tag{11} \]

with \( R_{\text{mod}} \) defined in equation (7), and adopting \( R_c = 2.1 \) arcsec from the velocity profile.

We choose \( \kappa = 0.5 \), which is a typical value for a disc galaxy (e.g. Kent & de Zeeuw 1991), but we also experimented with other values for this parameter. Varying \( \kappa \) between 0 and 1 resulted in differences in \( V_c \) of approximately 10 km s\(^{-1}\), and we adopt this value into the error bars of our final rotation curve.

To obtain the slope of \( \sigma_R \) we follow the same procedure as for the surface brightness, extracting the profile of \( \sigma_{R,\text{obs}} \) from the velocity dispersion map with kinemetry. We assume for the moment that turbulence is negligible in the galaxy (\( \sigma_{\text{turb}} = 0 \)) and subtract quadratically \( \sigma_{\text{thermal}} = 10 \) km s\(^{-1}\) from \( \sigma_{R,\text{obs}} \). We convert the resulting \( \sigma_{R,\text{obs}} \) into \( \sigma_R \) using the relation in equation (11). We parametrize this profile by

\[ \sigma_R(R) = \sigma_0 + \sigma_1 e^{-R_{\text{mod}}/R_0}, \tag{12} \]

This profile has a core in the centre (introduced by \( R_{\text{mod}} \)), so that we can better reproduce the flattening of the profile towards the centre. Again, we convolved our model to take seeing and sampling into account during the fit. The top panel of Fig. 8 shows the resulting profile and fit, as well as the observed velocity dispersion.

We first assume that turbulence plays no role in this galaxy, and we use \( \sigma_R \) as computed above to calculate the asymmetric drift correction (equation 9). The resulting rotation curve, as well as the
from equation (11), where we inserted a core radius \( R_c \) with the stellar velocity dispersion from the Schwarzschild model, and agrees very well with the stellar velocity model.

To check the validity of our thin-disc approximation for our model of \( R_c \), we extract this quantity from the Schwarzschild model of Krajnović et al. (2005), for \( \theta = 84^\circ \), close to the \( z = 0 \) plane. The resulting profile is smoothed and shown as the upper dotted blue line in Fig. 8. It is not a fit to the data, but derived independently from the Schwarzschild model, and agrees very well with the stellar \( \sigma_R \) we got from kinemetry. Also, \( \sigma_{\text{obs}} \) derived from the Schwarzschild model (lower dotted blue line) agrees with the observed rotation curve of the ionized gas, as shown in the top panel of Fig. 9.

To check our asymmetric drift corrected rotation curve of the ionized gas, we compare it with the asymmetric drift corrected stellar rotation curve. Stars do not feel turbulence and are not influenced by thermal motions like the gas, and therefore their observed velocity dispersion contains only contributions of gravitational interactions:

\[
\sigma_{\text{obs}} = \sigma_{\text{grav}}
\]

If we are correct with our assumption that turbulence does not play a role in the ionized gas, then the stellar corrected rotation curve should overlap with the corrected curve of the gas. If it does not, then we know that we should not have neglected the turbulence.

To derive the asymmetric drift correction of the stars, we obtain the observed rotation curve, surface density and velocity dispersion of the stars from our SAURON observations with kinemetry, and parametrize them in the same way as we did for the gas (see Figs 6–8 for the observed profiles and their models). The models were convolved during the fitting as described for the ionized gas. Because for the stars \( \sigma_{\text{obs}} = \sigma_{\text{grav}} \) we do not need to subtract \( \sigma_{\text{thermal}} \) as we did for the ionized gas and hence can calculate \( \sigma_R \) directly from equation (11), where we inserted a core radius \( R_c = 3.0 \) arcsec from the stellar velocity model.

In the above, we assumed that the stars lie in a thin disc, which is the case in NGC 2974. To check the validity of our thin-disc approximation for our model of \( R_c \), we extract this quantity from the Schwarzschild model of Krajnović et al. (2005), for \( \theta = 84^\circ \), close to the \( z = 0 \) plane. The resulting profile is smoothed and shown as the upper dotted blue line in Fig. 8. It is not a fit to the data, but derived independently from the Schwarzschild model, and agrees very well with the stellar \( \sigma_R \) we got from kinemetry. Also, \( \sigma_{\text{obs}} \) derived from the Schwarzschild model (lower dotted blue line) agrees with the observed rotation curve of the ionized gas (black diamonds) and stars (black stars). The dotted red line denotes the radial dispersion \( \sigma_R \) as determined from the Schwarzschild model (lower dotted blue line) agrees with the observed radial dispersion \( \sigma_R \) as extracted from the Schwarzschild model of Krajnović et al. (2005). Because for the stars \( \sigma_{\text{obs}} = \sigma_{\text{grav}} \) we do not need to subtract \( \sigma_{\text{thermal}} \) as we did for the ionized gas and hence can calculate \( \sigma_R \) directly from equation (11), where we inserted a core radius \( R_c = 3.0 \) arcsec from the stellar velocity model.

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\[
\sigma_{\text{obs}} = \sigma_{\text{grav}}
\]

If we are correct with our assumption that turbulence does not play a role in the ionized gas, then the stellar corrected rotation curve should overlap with the corrected curve of the gas. If it does not, then we know that we should not have neglected the turbulence.

To derive the asymmetric drift correction of the stars, we obtain the observed rotation curve, surface density and velocity dispersion of the stars from our SAURON observations with kinemetry, and parametrize them in the same way as we did for the gas (see Figs 6–8 for the observed profiles and their models). The models were convolved during the fitting as described for the ionized gas. Because for the stars \( \sigma_{\text{obs}} = \sigma_{\text{grav}} \) we do not need to subtract \( \sigma_{\text{thermal}} \) as we did for the ionized gas and hence can calculate \( \sigma_R \) directly from equation (11), where we inserted a core radius \( R_c = 3.0 \) arcsec from the stellar velocity model.

In the above, we assumed that the stars lie in a thin disc, which is the case in NGC 2974. To check the validity of our thin-disc approximation for our model of \( R_c \), we extract this quantity from the Schwarzschild model of Krajnović et al. (2005), for \( \theta = 84^\circ \), close to the \( z = 0 \) plane. The resulting profile is smoothed and shown as the upper dotted blue line in Fig. 8. It is not a fit to the data, but derived independently from the Schwarzschild model, and agrees very well with the stellar \( \sigma_R \) we got from kinemetry. Also, \( \sigma_{\text{obs}} \) derived from the Schwarzschild model (lower dotted blue line) agrees with the observed rotation curve of the ionized gas, as shown in the top panel of Fig. 9.

To check our asymmetric drift corrected rotation curve of the ionized gas and stars, we compare it with the asymmetric drift corrected stellar rotation curve. Stars do not feel turbulence and are not influenced by thermal motions like the gas, and therefore their observed velocity dispersion contains only contributions of gravitational interactions:

\[
\sigma_{\text{obs}} = \sigma_{\text{grav}}
\]

If we are correct with our assumption that turbulence does not play a role in the ionized gas, then the stellar corrected rotation curve should overlap with the corrected curve of the gas. If it does not, then we know that we should not have neglected the turbulence.

To derive the asymmetric drift correction of the stars, we obtain the observed rotation curve, surface density and velocity dispersion of the stars from our SAURON observations with kinemetry, and parametrize them in the same way as we did for the gas (see Figs 6–8 for the observed profiles and their models). The models were convolved during the fitting as described for the ionized gas. Because for the stars \( \sigma_{\text{obs}} = \sigma_{\text{grav}} \) we do not need to subtract \( \sigma_{\text{thermal}} \) as we did for the ionized gas and hence can calculate \( \sigma_R \) directly from equation (11), where we inserted a core radius \( R_c = 3.0 \) arcsec from the stellar velocity model.

In the above, we assumed that the stars lie in a thin disc, which is the case in NGC 2974. To check the validity of our thin-disc approximation for our model of \( R_c \), we extract this quantity from the Schwarzschild model of Krajnović et al. (2005), for \( \theta = 84^\circ \), close to the \( z = 0 \) plane. The resulting profile is smoothed and shown as the upper dotted blue line in Fig. 8. It is not a fit to the data, but derived independently from the Schwarzschild model, and agrees very well with the stellar \( \sigma_R \) we got from kinemetry. Also, \( \sigma_{\text{obs}} \) derived from the Schwarzschild model (lower dotted blue line) agrees with the observed rotation curve of the ionized gas, as shown in the top panel of Fig. 9.
10 km s$^{-1}$, we can at each radius calculate the corresponding $\sigma_R$ with equation (9). Using equation (11) we obtain the observed velocity dispersion, which in this case consists only of $\sigma_{\text{grav}}$. Since we know $\sigma_{\text{obs}}$, we can subtract quadratically $\sigma_{\text{grav}}$ and $\sigma_{\text{thermal}} = 10 \, \text{km s}^{-1}$ to obtain $\sigma_{\text{turb}}$.

Fig. 11 shows $\sigma_{\text{obs}}$ (deconvolved model) and its components $\sigma_{\text{thermal}}, \sigma_{\text{grav}}$ and $\sigma_{\text{turb}}$. We fitted a single exponential function (equation 10) with $R_e = 2.1$ arcsec to the inner 15 arcsec of $\sigma_{\text{turb}}$ and find that with this parametrization we can get a decent fit. We find a length-scale of 5.0 arcsec for the turbulence. The fit is also shown in Fig. 11.

5 MASS MODEL AND DARK MATTER

In this section we combine the corrected rotation curve of the ionized gas with the rotation curve of the neutral gas. The rotation curve of NGC 2974 rises quickly to a maximal velocity and then declines to a somewhat lower velocity, after which it flattens out (see e.g. Fig. 15). Unfortunately, we lack the data to study this decline in more detail, because our HI ring is not filled. The behaviour of our rotation curve is similar to what is seen in other bright galaxies with a concentrated light distribution (Casertano & van Gorkom 1991; Noordermeer et al. 2007). The decline of the rotation curve in such systems could indicate that the mass distribution in the centre is dominated by the visible mass and that the dark halo only takes over at larger radii. In contrast, in galaxies where the light distribution is less concentrated, such as low-luminosity later type galaxies, the rotation curves does not decline (e.g. Catinella, Giovanelli & Haynes 2006; Spekkens & Giovanelli 2006).

We separately model the contribution of the stars, neutral gas and dark halo to the gravitational potential. Also we derive the total $M/L$ as a function of radius, and obtain a lower limit on the dark matter fraction in NGC 2974.

In our model, we do not take the weak bar system of NGC 2974 into account. Emsellem et al. (2003) find that the perturbation of the gravitational potential caused by the inner bar in their model of this galaxy is less than 2 per cent. Also, we find that the harmonic coefficients that could be influenced by a large scale bar ($s_1, s_2$ and $c_2$) are small compared to the dominant term $c_1 (< 4$ per cent). We therefore conclude that although the rotation curve probably is affected by the presence of the bar system, this effect is small, and negligible compared to the systematic uncertainties introduced by the asymmetric drift correction. Furthermore, the largest constraints in our models come from the rotation curve at large radii, where we showed that the elongation of the potential is consistent with axisymmetry.

5.1 Stellar contribution

The contribution of the stellar mass to the gravitational potential and the corresponding circular velocity can be obtained by deprojecting and modelling the surface photometry of the galaxy. We use the multi-Gaussian expansion (MGE) method for this purpose, as described in Cappellari (2002).

Krajnović et al. (2005) presented an MGE model of NGC 2974, based upon the PC part of a dust-corrected WFPC2/F814W image and a ground-based $J$-band image obtained at the 1.0-m Jacobus Kapteyn Telescope (JKT). This image was however not deep enough to yield an MGE model that is reliable out to 5 $R_e$ or 120 arcsec, which is the extent of our rotation curve. We therefore construct another MGE model, replacing the JKT $J$-band image with a deeper one obtained with the 1.3-m McGraw-Hill Telescope at the MDM Observatory (see Table 2). This image is badly contaminated by a bright foreground star, so we do not include the upper half of the image in the fit. Since our model is axisymmetric, enough signal remained to get a reliable fit. We also exclude other foreground stars and bleaching from the image. The parameters of the PSF for the WFPC2 image were taken from Krajnović et al. (2005).

Table 2. Properties of the space- and ground-based imaging of NGC 2974, used to model the stellar contribution to the potential. The MDM image was constructed of three separate exposures, resulting in a total integration time of 1500 s.

<table>
<thead>
<tr>
<th>Filter band</th>
<th>HST/WFPC2</th>
<th>MDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure time (s)</td>
<td>250</td>
<td>1500</td>
</tr>
<tr>
<td>Field of view</td>
<td>$32 \times 32$ arcsec$^2$</td>
<td>$17.4 \times 17.4$ arcmin$^2$</td>
</tr>
<tr>
<td>Pixel scale (arcsec)</td>
<td>0.0455</td>
<td>0.508</td>
</tr>
<tr>
<td>Date of observation</td>
<td>1997 April 16</td>
<td>2003 March 26</td>
</tr>
</tbody>
</table>
We match the ground-based MDM image to the higher resolution WFPC2 image, and use it to constrain the MGE fit outside 15 arcsec. Outside 200 arcsec, the signal of the galaxy dissolves into the background and we stop the fit there. We are therefore confident of our MGE model out to a radius of at least 120 arcsec, which is the extent of the observed HI rotation curve. The goodness of fit can be examined as a function of radius in Fig. 12.

We forced the axial ratios \( q_j \) of the Gaussians to lie in the interval \([0.58, 0.80]\) (which is the same range as Krajnović et al. (2005) used in their paper), maximizing the number of allowed inclinations and staying as close as possible to a model with constant ellipticity, without significantly increasing the \( \chi^2 \) of the fit. This resulted in an MGE model consisting of 12 Gaussians, whose parameters can be found in Table 3. The parameters of the inner Gaussians agree very well with the ones in the model of Krajnović et al., which is not surprising as we used the same dust-corrected WFPC image. The outer Gaussians deviate, where their JKT image is replaced by our MDM image.

Table 3. Parameters of the Gaussians of the MGE model of NGC 2974. From left- to right-hand side: number of the Gaussian, central intensity, width (standard deviation), axial ratio and total intensity.

<table>
<thead>
<tr>
<th>( j )</th>
<th>( I_j \left( L_\odot \text{ pc}^{-2} \right) )</th>
<th>( \sigma_j ) (arcsec)</th>
<th>( q_j )</th>
<th>( L_j \left( \times 10^9 L_\odot \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>187 628.0</td>
<td>0.037 6306</td>
<td>0.580 000</td>
<td>0.0099</td>
</tr>
<tr>
<td>2</td>
<td>44 798.9</td>
<td>0.092 3321</td>
<td>0.800 000</td>
<td>0.0197</td>
</tr>
<tr>
<td>3</td>
<td>25 362.4</td>
<td>0.184 352</td>
<td>0.800 000</td>
<td>0.0445</td>
</tr>
<tr>
<td>4</td>
<td>28 102.0</td>
<td>0.343 100</td>
<td>0.586 357</td>
<td>0.1251</td>
</tr>
<tr>
<td>5</td>
<td>23 066.0</td>
<td>0.607 222</td>
<td>0.722 855</td>
<td>0.3964</td>
</tr>
<tr>
<td>6</td>
<td>9694.88</td>
<td>1.209 84</td>
<td>0.774 836</td>
<td>0.7089</td>
</tr>
<tr>
<td>7</td>
<td>5019.87</td>
<td>3.567 54</td>
<td>0.659 952</td>
<td>2.7186</td>
</tr>
<tr>
<td>8</td>
<td>1743.48</td>
<td>9.232 67</td>
<td>0.580 000</td>
<td>5.5578</td>
</tr>
<tr>
<td>9</td>
<td>329 832</td>
<td>16.951 11</td>
<td>0.770 081</td>
<td>4.7057</td>
</tr>
<tr>
<td>10</td>
<td>111 091</td>
<td>30.572 1</td>
<td>0.800 000</td>
<td>3.8829</td>
</tr>
<tr>
<td>11</td>
<td>96 2559</td>
<td>44.0573</td>
<td>0.770 081</td>
<td>8.6440</td>
</tr>
<tr>
<td>12</td>
<td>16 7257</td>
<td>103.0850</td>
<td>0.800 000</td>
<td>9.1678</td>
</tr>
</tbody>
</table>

The deviations are however small, and we conclude that the MGE model is a good representation of the galaxy surface brightness.

5.2 Gas contribution

The contribution of the HI ring to the gravitational potential is small compared to the stars and halo \((5.5 \times 10^8 M_\odot)\), three orders of magnitude smaller than the stellar mass) but still included in our mass models. We include a factor of 1.3 in mass to account for the helium content of the ring. The mass of the ionized gas is estimated at only \(2.2 \times 10^8 M_\odot\) (Sarzi et al. 2006), and therefore can be neglected in our models.

5.3 Mass-to-light ratio

By comparing the observed rotation curve and the light distribution from the MGE model, we can already calculate the \( M/L \) in NGC 2974. The enclosed mass within a certain radius \( r \) in a spherical system follows directly from the circular velocity:

\[
M(< r) = \frac{V_r^2 r}{G},
\]

with \( G \) the gravitational constant. Here we assume that the gravitational potential of the total galaxy is spherical symmetric. This is clearly not the case for the neutral gas, which resides in a thin disc. However, the total mass of the gas is three orders of magnitudes smaller than the total mass, and therefore can be neglected. Also, the stars reside in a flattened potential, as can be shown from their MGE model. But since we cannot disentangle the contributions of the stars and the dark matter to the observed rotation velocity a priori, we will for the moment assume that also the stellar mass density can be approximated by a spherical distribution.

Since we know the mass within a sphere of radius \( r \), we also need to calculate the enclosed \( I \)-band luminosity within a sphere. We first obtain the gravitational potential of our MGE model as a function of radius (see appendix A of Cappellari et al. 2002). Here, we take the flattening of the separate Gaussians into account. We subsequently calculate the corresponding circular velocity, with an arbitrary \( M_*/L \). To find the luminosity enclosed in a sphere we calculate the spherical mass needed to produce this circular velocity with equation (13), and convert this mass back to a luminosity using the same \( M_*/L \) that we used to calculate the velocity curve. This way we have replaced the luminosity within a flattened axisymmetric
ellipsoid (oblate sphere) by a sphere with radius equal to the long axis of the ellipsoid.

With this method we arrive at a mass-to-light ratio $M/L_I = 8.5 \, M_\odot/L_\odot$, at 5 effective radii ($1R_e = 24$ arcsec). In the literature, this value is usually expressed in $B$-band luminosities. Using an absolute magnitude of $M_B = -20.07$ for NGC 2974 (see Table 1), we find that $M/L_B = 14 \, M_\odot/L_\odot$. We checked that $M_B$ is consistent with our MGE model, adopting a colour $B-I = 2.13$ for NGC 2974 (see Tonry et al. 2001 and Table 1). H\textsc{i} studies of other early-type galaxies yield similar numbers (Morganti et al. 1997, and references therein). For example, Franx et al. (1994) find $M/L_B = 16 \, M_\odot/L_\odot$ at $6.5 \, R_e$, using the H\textsc{i} ring around IC 2006, and Oosterloo et al. (2002) report $M/L_B = 18 \, M_\odot/L_\odot$ for NGC 3108 at $6 \, R_e$.

Fig. 14 shows the increase of $M/L_I$ with radius. We find that within $1R_e, M/L_I = 4.3 \, M_\odot/L_\odot$, which agrees with the results from Schwarzschild modelling of Krajnović et al. (2005) and Cappellari et al. (2006). The increase of $M/L$ indicates that the fraction of dark matter grows towards larger radii.

5.4 Dark matter fraction

To calculate the dark matter fraction, we need to know the stellar mass-to-light ratio $M_*/L_I$. An upper limit on $M_*/L_I$ can be derived by constructing a maximal disc model. From the MGE model we calculate a rotation curve (taking the flattening of the potential into account, as in Cappellari et al. 2002), and we increase $M_*/L_I$ until the calculated curve exceeds the observed rotation curve. This way, we find that $M_*/L_I$ cannot be larger than $3.8 \, M_\odot/L_\odot$. We plotted the rotation curve of the maximal disc model, together with the observed rotation curve in Fig. 15. The rotation curve of the model has been convolved to take seeing and the resolution of the observations into account, as described in Section 4.1. The contribution of the neutral gas to the gravitational potential has been included in the model, but has only a negligible effect on the fit.

It is clear that even in the maximal disc model, a dark matter halo is needed to explain the flat rotation curve of the H\textsc{i} gas at large radii. From this model, we can calculate a lower limit to the dark matter fraction in NGC 2974. We then find that within $1R_e$, 12 per cent of the total mass is dark, while within $5R_e$, this fraction has grown to 55 per cent.

There is however no reason to assume that the stellar $M/L$ is well represented by its maximal allowed value. Cappellari et al. (2006) find $M_*/L_I = 2.34 \, M_\odot/L_\odot$ for NGC 2974, measured from line-strength values using single stellar population models. The formal error that they report on this $M/L$ is $\sim 10$ per cent, but they warn that this value is strongly assumption dependent. Secondary star formation in a galaxy can result in an underestimation of $M_*/L_I$, and the GALEX observations of Jeong et al. (2007) indeed show evidence for recent star formation in NGC 2974. The population models of Cappellari et al. (2006) are based on a Kroupa initial mass function (IMF), but if instead a Salpeter IMF is used, their
$M_\ast/L_\ast$ values increase by $\sim$40 per cent, which for NGC 2974 would result in $M_\ast/L_\ast = 3.3 M_\odot/L_\odot$, Cappellari et al. (2006) discard the Salpeter IMF based models, because for a large part of their sample their models then have $M_\ast/L_\ast > M_{\text{sa}}/L_\ast$, which is unphysical.

If we adopt $M_\ast/L_\ast = 2.34 M_\odot/L_\odot$ from the stellar population models, then 46 per cent of the total mass within $1 R_e$ is dark. The dark matter fraction increases to 72 per cent within $5 R_e$. See Fig. 16 for the change in dark matter fraction as a function of radius, and the comparison with the lower limits derived above.

Gerhard et al. (2001) and Cappellari et al. (2006) find an average dark matter fraction of $\sim$30 per cent within one effective radius in early-type galaxies, but we note that NGC 2974 is an outlier in the sample of Cappellari et al. The value of 47 per cent that we find is a bit high compared to this average, though the minimal fraction of dark matter is 12 per cent in our galaxy. Without an accurate determination of $M_\ast/L$ we cannot give a more precise estimate on the dark matter fraction in NGC 2974.

5.5 Halo models

We now include a dark halo in our model, to explain the flat rotation curve that we extracted from the H I ring. We explore two different halo models: the pseudo-isothermal sphere and the NFW profile.

The pseudo-isothermal sphere has a density profile given by

$$\rho(r) = \frac{\rho_0}{1 + (r/r_c)^2},$$

where $\rho_0$ is the central density of the sphere, and $r_c$ is the core radius.

The velocity curve resulting from the density profile of the pseudo-isothermal sphere is straightforward to derive analytically, and given by

$$V_c^2(r) = 4\pi G \rho_0 r_c^2 \left[ 1 - \frac{r_c}{r} \arctan \left( \frac{r}{r_c} \right) \right].$$

The NFW profile was introduced by Navarro et al. (1996) to describe the haloes resulting from simulations, taking a CDM cosmology into account. This profile has a central cusp, in contrast to the pseudo-isothermal sphere which is core-dominated. Its density profile is given by

$$\rho(r) = \frac{\rho_0}{r/r_c \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^\gamma},$$

with $\rho_0$, the characteristic density of the halo and $r_c$ a characteristic radius. The velocity curve of the NFW halo is given by

$$V_c^2(r) = \frac{V_{200}^2}{x \ln(1 + c x) - c/(1 + c)} \left[ \ln(1 + c x) - c/(1 + c) \right].$$

where $x = r/r_{200}$ and $c$ the concentration parameter defined by $c = r_{200}/r_s$. $r_{200}$ is defined such that within this radius the mean density is 200 times the critical density $\rho_{\text{crit}}$, and $V_{200}$ is the circular velocity at that radius. These parameters depend on the assumed cosmology.

We construct mass models of NGC 2974 including a dark matter halo with the observed stellar and gaseous mass. We then calculate the circular velocity resulting from our models, by adding the circular velocities resulting from the separate components:

$$V_c^2(r) = V_{c,\text{halo}}^2 + V_{c,\text{stars}}^2 + V_{c,\text{gas}}^2,$$

and fit these to our observed rotation curve. The inner 25 arcsec of our model rotation curve, which are based on the SAURON ionized gas measurements, are convolved with a kernel to take seeing and sampling into account, as described in Section 4.1.

For both profiles, we found that we could not constrain the stellar $M_\ast/L$ in our models because of degeneracies: for each $M_\ast/L_\ast$ below the maximal disc value of $3.8 M_\odot/L_\odot$, we could get a decent fit. We therefore show two fits for each model, with $M_\ast/L$ values that are justified by either line-strength measurements and single stellar population models ($M_\ast/L_\ast = 2.34 M_\odot/L_\odot$) or the observed rotation curve itself ($M_\ast/L_\ast = 3.8 M_\odot/L_\odot$). This last case would be a model requiring a minimal halo.

The best-fitting models for a dark halo described by a pseudo-isothermal sphere is shown in Fig. 17. The model in the top panel has a fixed $M_\ast/L_\ast = 2.34 M_\odot/L_\odot$, while the bottom panel shows the model with $M_\ast/L_\ast = 3.8 M_\odot/L_\odot$. The first model fits the SAURON measurement of the rotation curve well, but has a small slope at the outer part, where the observations show a flat rotation curve. Nevertheless, this model provides a good fit, with a minimal $\chi^2 = 27$ for $27 - 2 = 25$ degrees of freedom. We find for this model...
\[ \rho_0 = 19 \text{M}_\odot \text{pc}^{-3} \text{ and core radius } r_c = 2.3 \text{arcsec} = 0.23 \text{kpc}. \]

The second model with \( M_*/L_\odot = 3.8 \text{M}_\odot/L_\odot \) provides a better fit to the H I measurements, but has problems fitting the central part of the rotation curve. The model has a lower central density \( \rho_0 = 0.06 \text{M}_\odot \text{pc}^{-3} \) and larger core radius \( r_c = 54 \text{arcsec} = 5.4 \text{kpc} \). This fit is worse than for the previous model, with \( \chi^2 = 133 \).

Fig. 18 shows the best-fitting models with an NFW dark halo. This model fits the data less well than the pseudo-isothermal sphere: for the model with \( M_*/L_\odot = 2.34 \text{M}_\odot/L_\odot \) (top panel) we find a minimal \( \chi^2 = 44 \) for 27 – 2 degrees of freedom. The corresponding parameters of the density function are \( \rho_s = 1.1 \text{M}_\odot \text{pc}^{-3} \) and \( r_s = 21 \text{arcsec} = 2.1 \text{kpc} \). For \( M_*/L_\odot = 3.8 \text{M}_\odot/L_\odot \) the fit is worse (\( \chi^2 = 144 \)) but the outer part of the rotation curve is better fitted. We find \( \rho_s = 1.1 \times 10^{-9} \text{M}_\odot \text{pc}^{-3} \) and \( r_s \approx 1300 \text{arcsec} \), which corresponds to approximately 130 kpc.

Adopting \( H_0 = 73 \text{km s}^{-1} \text{Mpc}^{-1} \), the critical density is given by \( \rho_{\text{crit}} = 3H_0^2/8\pi G = 1.5 \times 10^{-7} \text{M}_\odot \text{pc}^{-3} \). We calculate the concentration parameter \( c \), given that

\[
\frac{\rho_s}{\rho_{\text{crit}}} = \frac{200}{3} \frac{c^3}{\ln(1+c) - c/(1+c)},
\]

and find \( c = 71 \) and 4.7 for the NFW profiles in the \( M_*/L_\odot = 2.34 \text{M}_\odot/L_\odot \) and \( M_*/L_\odot = 3.8 \text{M}_\odot/L_\odot \) models, respectively. These values are quite deviant from the value that is expected from cosmological simulations \( (c \sim 10, \text{ Bullock et al. 2001}) \). When fixing \( c = 10 \) and fitting again an NFW halo to our observations with \( M_*/L_\odot \) and the scale radius as free parameters, we arrive at the model shown in Fig. 19. We find \( M_*/L_\odot = 3.3 \text{M}_\odot/L_\odot \) and \( r_s \approx 380 \text{arcsec} \approx 38 \text{kpc} \), with a minimal \( \chi^2 \) value of 87 for 27 – 2 degrees of freedom. We regard this model as more realistic than the two other NFW profiles mentioned above, but since also here the fit is not perfect, we cannot conclude that therefore \( M_*/L_\odot = 3.3 \text{M}_\odot/L_\odot \) is a better estimate for the stellar \( M/L \) in NGC 2974, than the value from the stellar population models.

The results of the halo models discussed above are summarized in Table 4.
5.6 MOND

An alternative to including a dark matter halo in a galaxy to explain its rotation curve at large radii, is provided by modified Newtonian dynamics (MOND, Milgrom 1983). In this theory, Newtonian dynamics is no longer valid for small accelerations (\(a \ll a_0\)), but instead the acceleration \(a\) in a gravitational field is given by

\[
\mu \frac{a(\alpha/a_0)}{a_0} = \mu_0, \tag{20}
\]

where \(a_0\) is the Newtonian acceleration and \(\mu\) is an interpolation function, such that \(\mu(x) = 1\) for \(x \gg 1\) and \(\mu(x) = x\) for \(x \ll 1\). Given the stellar \(M/L\) of a galaxy, MOND predicts its rotation curve. An overview of properties and predictions of MOND is offered by Sanders & McGaugh (2002).

We fitted our rotation curve of NGC 2974 with \(M_*/L_0\) as a free parameter. For \(a_0\) we adopted the value of \(1.2 \times 10^{-4}\) cm s\(^{-2}\), which was derived by Begeman, Broeils & Sanders (1991) from a sample of spiral galaxies. The contribution of the neutral gas is included in our model in the same way as described before, as well as a convolution to take seeing and sampling into account.

NGC 2974 is an ideal candidate to study the transition between the Newtonian and MOND regime, since the Newtonian acceleration reaches \(a_0\) at a radius of approximately 95 arcsec if we adopt a stellar \(M/L\) of 2.34 \(M_\odot/L_\odot\). For larger \(M_*/L_0\), this radius increases, and for the maximum disc value of 3.8 \(M_\odot/L_\odot\), \(a_0\) is reached around 120 arcsec. This means that a large part of the observed rotation curve lies in the transition region, and we could therefore use NGC 2974 to discriminate between interpolation function.

We first constructed a model with the standard interpolation function of MOND,

\[
\mu(x) = \frac{x}{\sqrt{1 + x^2}}. \tag{21}
\]

The resulting fit is shown as Model I in Fig. 20. This model has the same \(M_*/L_0\) value as the maximal disc model, 3.8 \(M_\odot/L_\odot\), but does clearly not provide a good fit to the data.

We constructed a second model, with an alternative interpolation function explored by Famaey & Binney (2005),

\[
\mu(x) = \frac{x}{1 + x}. \tag{22}
\]

This function makes the transition between the Newtonian and the MOND region less abrupt than the standard interpolation function and requires a lower \(M_*/L_0\). The fit provided by this model to the data is much better (Model II in Fig. 20), but formally less good than a model with a dark matter halo. This model requires \(M_*/L_0 = 3.6 \ M_\odot/L_\odot\), and the fit yields \(\chi^2 = 98\), for 27 − 1 degrees of freedom.

Famaey & Binney (2005) find that their simple interpolation function provides better constraints to the terminal velocity of the Milky Way than the standard function. Famaey et al. (2007) fitted the rotation curves of a sample of galaxies with Hubble types ranging from small irregular dwarf galaxies to large early-type spirals, and report that both interpolating functions fit the data equally well. However, Sanders & Noordermeer (2007) find that for their sample of early-type disc galaxies the simple interpolation function yields more sensible values for \(M_*/L\) than the standard one.

It would be interesting to see whether there is a preference for the simple interpolation function over the standard one in early-type galaxies. In this scenario, the challenge for MOND would be to provide a universal interpolation function that would fit rotation curves of all galaxy types along the Hubble sequence. So far, mostly spirals and dwarf galaxies have been confronted with MOND, but with more early-type galaxies getting detected in HI and more rotation curves becoming available, the sampling in morphology should become less biased to late-type galaxies.

6 SUMMARY

We obtained HI observations of the early-type galaxy NGC 2974 and found that the neutral gas resides in a ring. The ring starts around 50 arcsec, and extends to 120 arcsec, which corresponds to 12 kpc or 5 \(R_e\). The total mass of the neutral gas is \(5.5 \times 10^8 \ M_\odot\).

We compared the velocity field of the HI ring with the kinematics of the ionized gas. We found that both velocity fields are very regular and nicely aligned, indicating that they could form a single disc. A harmonic decomposition of the velocity field showed that at large radii the gravitational potential is consistent with an axisymmetric shape.

We introduced a new way to correct the rotation curve of the ionized gas for asymmetric drift. We found that the correction approaches a constant value and this enabled us to remove the effect of turbulence on the rotation curve, assuming a constant asymmetric drift correction throughout the galaxy. We confirmed that this assumption is valid in NGC 2974, by comparing with the asymmetric drift corrected rotation curve of the stars (which was not affected by turbulence). An interesting question is whether other galaxies show the same behaviour. If this is the case, then with our method we would be able to investigate rotation curves of ionized gas and the effects of turbulence in more detail in other galaxies, and search for connections with e.g. spiral structure and bars. Although in

<table>
<thead>
<tr>
<th>Halo profile</th>
<th>(M_*/L_0)</th>
<th>(\mu_0), (\mu_h)</th>
<th>(r_c, r_s) (kpc)</th>
<th>(c)</th>
<th>(\chi^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-thermal</td>
<td>2.34</td>
<td>19</td>
<td>0.23</td>
<td>–</td>
<td>27</td>
</tr>
<tr>
<td>NFW</td>
<td>3.8</td>
<td>0.06</td>
<td>5.4</td>
<td>–</td>
<td>133</td>
</tr>
</tbody>
</table>

Table 4. Comparison of the best-fitting models with a dark matter halo, as described in the text.
principle we could for NGC 2974 also have used the stellar rotation curve together with the H1 to constrain the mass models, this will not be the case for all galaxies. For instance, in low surface brightness galaxies stellar kinematics are not easy to obtain, and even in high surface brightness galaxies, the absorption line kinematics need to be binned to higher signal-to-noise ratio than the emission line kinematics, provided that ionized gas is present.

It is clear from the rotation curve of NGC 2974 that dark matter is required to explain the observed velocities. We found that the total $M/L$ increases from $4.3 \, M_{\odot} / L_{\odot} / R$ at $1 R_e$ to $8.5 \, M_{\odot} / L_{\odot} / R$ at $5 R_e$. This last value would correspond to $14 \, M_{\odot} / L_{\odot} / R$ in $B$ band. Even in the maximal disc model, 55 per cent of the total mass is dark, and an additional dark halo needs to be included.

We constructed mass models of NGC 2974, where we modelled both the stellar and gaseous contribution to the gravitational potential. The latter is negligible compared to the stars ($M_{\text{gas}} \approx 0.001 \, M_*$), but still included in our models. For the dark halo, we tested two different profiles: the core-dominated pseudo-isothermal sphere and the cuspy NFW profile. We experimented with different values for the stellar $M/L$, but found that we cannot constrain this value with just the rotation curve: for most $M_*/L$ smaller than the maximal disc value, we could obtain a decent fit for both the pseudo-isothermal sphere and the NFW profile. If we compare models with $M_*/L$ from single stellar population models ($2.34 \, M_{\odot} / L_{\odot} / R$) with maximal disc model ($M_*/L = 3.8 \, M_{\odot} / L_{\odot} / R$), then the first provide better fits to the data. Especially the inner data points are better reproduced with $M_*/L = 2.34 \, M_{\odot} / L_{\odot} / R$, but we note that the H1 data points are better fitted in the models with the larger $M_*/L$ value. The pseudo-isothermal sphere fits our data marginally better than the NFW profile, but the difference is not significant. With MOND we can also reproduce the observed rotation curve, but not as well as with models that include a dark matter halo.

The largest uncertainty in our analysis is the stellar $M/L$. We can only derive an upper limit on this ratio from the maximal disc model or e.g. from Schwarzschild modelling, since $M_*/L$ should always be equal to or smaller than the dynamical $M/L$. Values for $M_*/L$ from stellar population synthesis models depend significantly on the model assumptions: we mentioned already that going from a Kroupa to a Salpeter IMF can increase $M_*/L$ by as much as 40 per cent. Also, even low-level secondary star formation can affect $M_*/L$ severely. Furthermore, there is no reason why the $M_*/L$ should remain constant over $5 R_e$, which we assumed when modelling the stellar mass. If the stellar $M/L$ were known, we would be able to determine the dark matter fraction in the galaxy with more accuracy, and either rule out or confirm the maximal disc hypothesis. Also, since $M_*/L$ is the only free parameter when fitting a rotation curve in MOND, knowing this value would provide us with a rotation curve that can be compared to the data directly, providing a clear test for MOND.

We have shown in this paper that it is possible to combine rotation curves of neutral and ionized gas, correcting the latter one for asymmetric drift using the Jeans equations and the higher order velocity moments of the collisionless Boltzmann equations. Our method to correct for the asymmetric drift therefore does not require a cold disc assumption ($\sigma \ll V_c$). With more early-type galaxies getting detected in H1, and more high-quality rotation curves becoming available, we can now study the shape of their dark matter haloes.

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REFERENCES

APPENDIX A: ASYMMETRIC DRIFT CORRECTION IN A THIN DISC

In this appendix we derive expressions for the asymmetric drift correction in a stationary axisymmetric system, using the velocity moments of the collisionless Boltzmann equation. We then evaluate this expression in a thin-disc approximation. Our method does not require that the velocity dispersion be small compared to the circular velocity ($\sigma / v_c \ll 1$) and is comparable to the ‘hot disc model’, (see e.g. Häring-Neumayer et al. 2006).

A1 The velocity ellipsoid

To derive the asymmetric drift correction we start from the collisionless Boltzmann equation for a stationary axisymmetric galaxy and using cylindrical coordinates $r = (R, \phi, z)$:

$$v_R \frac{\partial f}{\partial R} + v_z \frac{\partial f}{\partial z} + \left( \frac{v_\phi^2}{R} - \frac{\partial \Phi}{\partial R} \right) \frac{\partial f}{\partial \phi} - \nu v_R v_\phi \frac{\partial f}{\partial v_R} - \nu \frac{\partial f}{\partial v_\phi} = 0,$$

with $\Phi(R, z)$ the underlying potential and $\nu(v)$ the (luminosity) density given by $\int \nu f(R,v) \, dv$.

We multiply the above equation by $v_R$ and subsequently integrate over all velocities. We then obtain the Jeans equation:

$$\frac{\partial (\nu v_R^2)}{\partial R} + \frac{\partial (\nu v_R v_\phi)}{\partial z} + \frac{\nu}{R} \left( v_R^2 - v_\phi^2 + \frac{\partial \Phi}{\partial R} \right) = 0.\quad (A2)$$

Since our system is axisymmetric, we set $\partial v_\phi / \partial z = 0$ by symmetry. Substituting the circular velocity $v_c^2 = R (\partial \Phi / \partial R)$, we arrive at equation (4-33) of Binney & Tremaine (1987):

$$v_R^2 = \frac{\nu v_\phi^2}{\sigma_R^2} - \sigma_\phi^2 \left[ \frac{\partial \ln \nu}{\partial \ln R} + \frac{\partial \ln \sigma_R^2}{\partial \ln R} + 1 - \frac{\sigma_\phi^2}{\sigma_R^2} + R \frac{\partial (\nu v_R v_\phi)}{\partial R} \right].$$

(A3)

with $\sigma_\phi^2 = \nu - \sigma_R^2$. $\sigma_Z^2 = v_\phi^2$ and $\sigma_\phi^2 = \nu v_\phi^2$. The observed velocity field gives $\sigma_R^2$, and the remaining terms in equation (A3) form the asymmetric drift correction.

The last term in the asymmetric drift correction depends on the alignment of the velocity ellipsoid. In case of alignment with the cylindrical coordinate system ($R, \phi, z$) we have $\nu v_R v_\phi = 0$, while in case of alignment with the spherical coordinate system ($r, \theta, \phi$) we have $\nu v_R v_\phi = \sigma_\phi^2 / \sigma_R^2 |(1 - \cos^2 \theta)|$, which becomes proportional to $z/R$ close to the disc plane. These are two extreme situations, and we introduce the parameter $\kappa$ to find a compromise:

$$\nu v_R v_\phi = \kappa (\sigma_\phi^2 / \sigma_R^2) \frac{z}{R} / \sigma_R^2, \quad 0 \leq \kappa \leq 1,$$

(A4)

where a typical value for $\kappa$ is 0.5 for disc galaxies (e.g. Kent & de Zeeuw 1991).

To evaluate the asymmetric drift correction, we need expressions for $\sigma_z / \sigma_R$ and $\sigma_\phi / \sigma_R$. We use higher order velocity moments of the collisionless Boltzmann equation to derive these expressions.

Starting again from equation (A1), we multiply by $v_R v_\phi$ and integrate over all velocities:

$$\frac{\partial (\nu v_R^2 v_\phi)}{\partial R} + \frac{\partial (\nu v_R v_\phi v_\phi)}{\partial z} + \frac{\nu}{R} \left( 2 v_R^2 v_\phi - v_\phi^2 - v_R^2 \frac{\partial \Phi}{\partial R} \right) = 0.\quad (A5)$$

Aligning the velocity ellipsoid in the azimuthal direction we have $\nu v_R v_\phi = 0$ and $\nu v_R v_\phi v_\phi = 0$, so that $\nu v_R v_\phi = \sigma_R^2 v_\phi^2$ and $\nu v_R v_\phi v_\phi = \nu v_\phi^2 v_\phi$. We substitute these relations in equation (A5), and subtract $\sigma_R^2 v_\phi$ times the Jeans equation (A2):

$$v_R \sigma_R^2 \frac{\partial v_\phi}{\partial R} + v_\phi \sigma_R^2 \frac{\partial v_\phi}{\partial R} + \frac{\nu}{R} \left[ \sigma_R^2 v_\phi - (v_\phi^2 - v_R v_\phi) \right] = 0.\quad (A6)$$

We substitute $v_R^2 - v_\phi^2 v_\phi = 2 \sigma_R^2 v_\phi^2 + (v_\phi - v_R v_\phi)$ and $\tau_R^2 v_\phi$ from equation (A4) to arrive at

$$\sigma_R^2 \frac{\partial v_\phi}{\partial z} = \frac{1}{2} \left[ 1 + \alpha_R + \kappa \left( \frac{1}{1 - \kappa} \right) \alpha_z \right] \left[ \nu v_R v_\phi - \nu v_R v_\phi - \nu v_R v_\phi \right],$$

(A7)

where we have introduced the logarithmic slopes

$$\alpha_R = \frac{\partial \ln \tau_R}{\partial \ln R} \quad \text{and} \quad \alpha_z = \frac{\partial \ln \tau_R}{\partial \ln z}.$$

(A8)

To obtain an expression for $\sigma_z / \sigma_R$ we again start with the collisionless Boltzmann equation, but now multiply with $v_c (v_\phi - v_R)$.
before integrating over all velocities:
\[ \nu_{\nu_\text{R}/z} \frac{\nu_\text{R}}{R} \left( 1 + \frac{\partial \ln \nu_\text{R}}{\partial \ln R} \right) + \nu_\text{R} \frac{\partial \ln \nu_\text{R}}{\partial \ln z} = 0. \]  
(A9)

Substituting equation (A4) we find

\[ \frac{\sigma_v^2}{\sigma_R^2} = \frac{\kappa z^2 (1 + \alpha_R)}{\kappa z^2 (1 + \alpha_R) - (R^2 - z^2)\alpha_z}. \]  
(A10)

The above expressions can be inserted into equation (A3) to obtain the asymmetric drift correction and therefore the true circular velocity. In practice, we often apply the asymmetric drift correction in the thin-disc approximation, because from observations the \( z \) dependence is not straightforward to derive.

In the thin-disc approximation, we have \( z \ll R \), and therefore we can write equation (A4) as

\[ \nu_{\nu_\text{R}/z} = \kappa (\sigma_R^2 - \sigma_z^2) \frac{z}{R} \]  
(A11)

and following the same reasoning as before, we see that the expressions in equations (A7) and (A10) simplify slightly: in the first expression the one-to-last term disappears, and for the second one, \( (R^2 - z^2) \) gets replaced by \( R^2 \) in the denominator of the expression. Furthermore, the derivative of \( \nu_{\nu_\text{R}/z} \) simplifies considerably.

We find the following expression for the asymmetric drift correction in the thin-disc approximation, after substitution in equation (A3):

\[ V_z^2 = \nu_{\nu_\text{R}/z}^2 - \sigma_R^2 \left[ \frac{\partial \ln \nu_\text{R}}{\partial \ln R} + \frac{\partial \ln \sigma_R^2}{\partial \ln R} + \frac{1}{2} (1 - \alpha_R) \right. \\
+ \left. \frac{1}{2} \left( \nu_{\nu_\text{R}/z} - \nu_\text{R} \right)^2 \right] - \frac{\kappa \nu_\text{R} \sigma_R}{\sigma_R^2} - \frac{\kappa R^2 \alpha_z}{\kappa z^2 (1 + \alpha_R) - R^2 \alpha_z}. \]  
(A12)

The one-to-last term vanishes in the case of a velocity ellipsoid symmetric around \( \nu_\text{R} = \nu_{\nu_\text{R}/z} \). This need not necessarily be the case, and the exact form of \( (\nu_{\nu_\text{R}/z} - \nu_\text{R})^2 \) depends on the underlying distribution function, which in general cannot be constrained easily (e.g. Kuijken & Tremaine 1991). However, since this term is a factor of \( \sigma_R^2 \) smaller than the other terms, it can be safely ignored for most purposes.

\section*{A2 Observables}

Here we investigate how in the thin-disc approximation we can correct our observed velocity field for asymmetric drift, to obtain the true circular velocity \( V_z \). This quantity traces the potential and therefore the mass of the galaxy.

In a thin disc, we can replace \( \partial \ln v / \partial \ln R \) by the slope of the surface brightness \( \partial \ln \Sigma / \partial \ln R \). This slope can be obtained directly from observations.

The observed velocity and velocity dispersion of an axisymmetric thin disc seen under an inclination \( i \) is given by

\[ V = v_{\text{sys}} + v_\nu \cos \phi \sin i, \]

\[ \sigma^2 = \sigma_R^2 \sin^2 \phi \sin^2 i + \sigma_z^2 \cos^2 \phi \sin^2 i + \sigma_R^2 \cos^2 i - v_\nu \nu \sin \phi \sin 2i. \]  
(A13)

It is straightforward to obtain \( v_\nu \) from the observed velocity field, and though \( \sigma_R \) can be estimated rather well, \( \sigma_z \) is less easy to constrain. Therefore, we fit to \( v_\nu \) the prescription of Evans & de Zeeuw (1994) for power-law models:

\[ v_{\text{mod}} \propto \frac{R}{(R_c^2 + R^2 + z^2/q_\phi^2)^{1/2} + \beta^2}, \]  
(A14)

where \( R_c \) is the core radius, \( q_\phi \) the flattening of the potential and \( \beta \) the logarithmic slope of the rotation curve at large radii (such that \( \beta = 0 \) implies a flat rotation curve).

For the slopes of \( v_\nu \) we find that

\[ \alpha_R = 1 - \frac{(1 + \beta/2)R^2}{R_c^2 + R^2 + z^2/q_\phi^2}, \]

\[ \alpha_z = - \frac{(1 + \beta/2)z^2/q_\phi^2}{R_c^2 + R^2 + z^2/q_\phi^2} = - \frac{z^2}{q_\phi^2 R_c^2} (1 - \alpha_R), \]  
(A15)

so that with \( \nu_{\nu_\text{R}/z} = 0 \) in the disc plane, we obtain

\[ \sigma_z^2 = \sigma_R^2 \left[ 1 - \frac{1}{2} (1 - \alpha_R) \cos^2 \phi \sin^2 i \right. \\
\left. - \frac{(1 + \beta/2)R^2}{q_\phi^2 (R_c^2 + R^2)(1 + \alpha_R) + (1 + \beta/2)R^2} \cos^2 i \right]. \]  
(A16)

When evaluated along the major axis, \( \cos^2 \phi = 1 \).

Assuming that the velocity ellipsoid is symmetric around \( v_\nu = v_\nu \), the corresponding term in equation (A12) vanishes. Inserting the relations obtained from the power-law model, we arrive at the following expression for the circular velocity:

\[ V_z^2 = \nu_{\nu_\text{R}/z}^2 - \sigma_R^2 \left[ \frac{\partial \ln \Sigma}{\partial \ln R} + \frac{\partial \ln \sigma_R^2}{\partial \ln R} + \frac{1}{2} (1 - \alpha_R) \right. \\
+ \left. \frac{\kappa (1 - \alpha_R)}{\kappa (2R_c^2 - R^2) + R^2} \right]. \]  
(A17)

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