THE UNIQUE RECONSTRUCTION OF A SCALAR OR VECTORIAL FIELD FROM ITS VALUES ON AN ARBITRARY SURFACE TO WHICH IT HAS PROPAGATED THROUGH AN ARBITRARY MEDIUM

HOENDERS, BJ

Published in:
IEEE Transactions on Antennas and Propagation

DOI:
10.1109/TAP.1981.1142589

Publication date:
1981

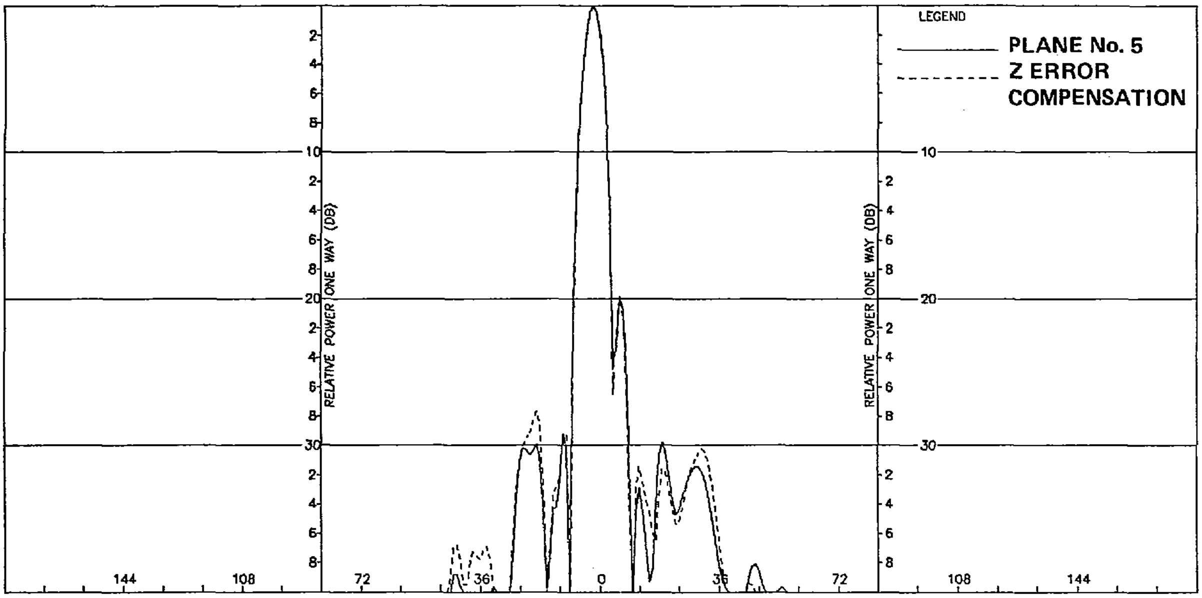

Fig. 4. Superposition of far-field elevation principal plane cuts calculated from near field on plane 5 (solid) and near field after Z probe positioning error compensation (dash).

this technique in enhancing the accuracy of far-field pattern calculation from near-field measurements where known probe positioning errors exist.

The computer time required to solve the inverse problem is a strong function of the maximum deviation of the nearby surface from the reference planar surface. The result presented was for a 64-point by 64-point near-field data set with a maximum deviation of one-half wavelength. The computation time was approximately two hours on a CDC Cyber 74 series computer. Smaller deviations require significantly less computation time.

REFERENCES

[1] D. T. Paris, W. M. Leach, and E. B. Joy, "Basic theory of probe-compensated near-field measurements," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 373–379, May 1978.

[2] K. Tanabe, "Projection method for solving a singular system of linear equations and its applications," *Numer. Math.*, vol. 17, pp. 203–214, 1971.

The Unique Reconstruction of a Scalar or Vectorial Field from its Values on an Arbitrary Surface to Which it has Propagated Through an Arbitrary Medium

B. J. HOENDERS

Abstract—**The problem of the determination of the values of a field on a surface to which it has propagated through an arbitrary medium is shown to have a unique solution if the field satisfies any linear elliptic partial differential equation.**

Manuscript received April 21, 1980; revised October 13, 1980.

The author is with the Department of Applied Physics, State University at Groningen, Groningen, The Netherlands.

ANALYSIS

Suppose that a scalar field is the solution of a linear elliptic partial differential equation

$$\sum_{i,k}^{n} \frac{\partial}{\partial x_k}\left(a_{ik}(\mathbf{r})\frac{\partial\psi(\mathbf{r})}{\partial x_i}\right) + \sum_{i} b_i(\mathbf{r})\frac{\partial\psi(\mathbf{r})}{\partial x_i} + c(\mathbf{r})\psi(\mathbf{r}) = 0 \tag{1}$$

in a domain D bounded by two closed surfaces S_1 and S_2. The field outside S_1 is uniquely determined by its values on S_1, (1), and Sommerfeld's radiation condition. It is our aim to show that the values of ψ on *any* arbitrary, not necessarily closed, surface, say S_2', are sufficient to determine the values of ψ on S_1. *We therefore do not need to know the values of both ψ and $\partial\psi/\partial n$ on S_2'!*

To this end we need a theorem, derived by Beckert [1] (see also Hoenders [2]) which states that (1) admits a solution $\phi^{(1)}$ existing on the inside S_2 such that

$$\phi^{(1)} \cong 0 \text{ and } \frac{\partial\phi^{(1)}}{\partial n} \cong \delta_r(\mathbf{r}-\mathbf{y}) \quad \text{if } \mathbf{r} \text{ and } \mathbf{y} \in S_1'. \tag{2}$$

The function $\delta_r(\mathbf{r} - \mathbf{y})$ denotes a regularization of the δ function, and the curve S_1' is drawn in the figure with small spheres. The vector $\mathbf{r}$ denotes the variable, and the vector $\mathbf{y}$ a parameter. We will restrict ourselves to fields satisfying the Helmholtz equation with a variable index of refraction:

$$(\nabla^2 + k^2 n^2(\mathbf{r}))\psi(\mathbf{r}) = 0. \tag{3}$$

Let D_2 denote the union of the domain D_1, bounded by $\partial D_1 = S_1' \cup S_2$, and the domain D_3, bounded by $S_1 : D_2 = D_1 \cup D_3$. The domain D_2 is bounded by $\partial D_2 \equiv S_2' + S_1''$. The curve S_1'' is indicated in Fig. 1 with the chain of small lines.

Suppose that $\phi^{(2)}$ is a solution of (3) in the domain D_2

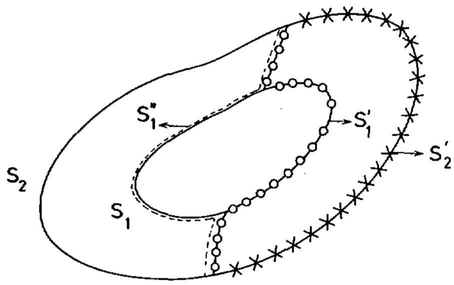

Fig. 1. Geometry of surfaces.

with

$$\phi^{(2)}(\mathbf{r}, \mathbf{y}) \cong 0 \quad \frac{\partial}{\partial n}\phi^{(2)}(\mathbf{r}, \mathbf{y}) \cong \delta_r(r-y) \quad \mathbf{r} \in S_1''. \tag{4}$$

Green's theorem

$$\int_{\partial D_j} \left\{ \psi \frac{\partial}{\partial n} \phi^{(j)} - \phi^{(j)} \frac{\partial}{\partial n} \psi \right\} d\sigma = 0, \quad j = 1, 2, \tag{5}$$

for the domains D_1 and D_2, together with the representation

$$\psi(\mathbf{r}') = \int_{S_1} \psi(\mathbf{r}) G(\mathbf{r}, \mathbf{r}')\, d\sigma \tag{6}$$

where G denotes the outgoing surface Green's function, leads to

$$\psi(\mathbf{y}) \cong \int_{S_1} \psi(\mathbf{r}) K^{(j)}(\mathbf{r}, \mathbf{y})\, d\sigma_r + f^{(j)}(\mathbf{y}) \quad \mathbf{y} \in S_1{}^{(j)} \tag{7a}$$

$$K^{(j)}(\mathbf{r}, \mathbf{y}) = -\int_{S_2'} \phi^{(j)}(\mathbf{r}', \mathbf{y}) \frac{\partial}{\partial n} G(\mathbf{r}, \mathbf{r}')\, d\sigma_{r'} \tag{7b}$$

$$f^{(j)}(\mathbf{y}) = \int_{S_2'} \psi(\mathbf{r}) \phi^{(j)}(\mathbf{r}, \mathbf{y})\, d\sigma_{r'} \qquad j = 1, 2, \tag{7c}$$

if $S_1{}^{(1)} \equiv S_1{}'$ and $S_2{}^{(2)} \equiv S_2{}''$.

The reason why we had to apply Green's theorem twice is apparent from the set of linear Fredholm integral equations of the second kind (7a). Equation (7a) with $j = 1$ is a linear relation between $\psi(\mathbf{y})$ if $\mathbf{y} \in S_1{}'$ and $\psi(\mathbf{y})$ if $\mathbf{y} \in S_1$, and an additional equation, viz., (7a) with $j = 2$, is therefore needed which gives a relation between the values of $\psi(\mathbf{y})$ if $\mathbf{y} \in S_1{}''$ (or at least those values of $\psi(y)$ on the intersection of $S_1{}''$ and S_1) and the values of $\psi(\mathbf{y})$ if $\mathbf{y} \in S_1$.

The set of coupled linear Fredholm equations (7a) can, in general, be solved uniquely for the unknown functions $\psi^{(1)}(\mathbf{y}) \equiv \psi(\mathbf{y})$ if $\mathbf{y} \in S_1{}'$ and $\psi^{(2)}(\mathbf{y}) \equiv \psi(\mathbf{y})$ if $\mathbf{y} \in S_1{}''$. The problem of stability is *not* considered (see, for this problem, e.g., Shewell and Wolf [3] and Sherman [4].

REFERENCES

[1] H. Beckert, "Eine bemerkenswerte Eigenschaft der Lösungen des Dirichletsen Problems bei linearen elliptischen Differentialgleichungen," *Math. Annal.*, vol. 139, pp. 255–264, 1960.

[2] B. J. Hoenders, "The unique solution of the inverse diffraction problem," *Opt. Comm.*, vol. 30, pp. 327–328, 1979.

[3] J. R. Shewell and E. Wolf, "Inverse diffraction and a new reciprocity theorem," *J. Opt. Soc. Amer.*, vol. 58, pp. 1596–1603, 1968.

[4] G. C. Sherman, "Inverse diffraction," *J. Opt. Soc. Amer.*, vol. 57, pp. 1490–1498, 1967.

Inverse Problems and Coherence

H. P. BALTES AND H. A. FERWERDA

***Abstract*—A summary of current inverse problems of statistical optics is presented together with a short guide to the pertinent review-type literature. The retrieval of structural information from the far-zone degree of coherence and the average intensity distribution of radiation scattered by a superposition of random and periodic scatterers is discussed.**

I. INTRODUCTION

Inverse optical problems have been intensively studied for the case of coherent radiation, i.e., in terms of deterministic wave amplitudes [1], [2]. Comprehensive reviews of the various aspects of the deterministic inverse problems are available [3]–[8]. Of more recent interest are the corresponding inverse problems for partially coherent sources or stochastic scatterers (such as fluctuating continua, random distributions of discrete scatterers, or rough surfaces) characterized by correlation functions or statistical distributions rather than by deterministic field amplitudes [3], [6], [9]–[16].

II. SUMMARY

The various inverse problems of statistical optics can be summarized as follows:

1) the phase reconstruction problem for time and space coherence functions [3];
2) reconstruction of intensity profile and coherence properties of a (direct or indirect) source from far-zone average intensity distribution and degree of coherence [9];
3) the question of uniqueness of the above reconstruction, [17] and the related problem of nonradiating current correlations [18];
4) reconstruction of the statistics of the photons received by a detector from the raw-data of photoelectric detection [12];
5) retrieval of statistical features of disordered scattering systems or fluctuating sources from properties of the scattered field [6], [10], [11];
6) reconstruction of the photon statistics from a few moments or correlations under given constraints, i.e., the moment problem of quantum optics [19].

With respect to the problem 1) for the case of quasi-homogeneous planar sources (i.e., sources whose intensity profile varies much slower in space than the coherence function), we recall the generalized Van Cittert–Zernike theorem: this theorem establishes a Fourier-transform relationship 1) between the far-zone coherence properties and the source-plan intensity profile and 2) between the far-zone intensity distribution and the source-plane coherence properties (see, e.g., [9] and references therein). This theorem however does not always hold when the source has, in addition, a deterministic phase profile [20].

Manuscript received April 2, 1980; revised October 12, 1980.

H. P. Baltes is with Zentrale Forschung und Entwicklung, Landis & Gyr Zug AG, CH-6301 Zug, Switzerland.

H. A. Ferwerda is with Department of Applied Physics, State University at Groningen, Nijenborgh 18, NL-9747AG Groningen, The Netherlands.