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DIFFUSE REFLECTANCE AND COHERENCE

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Abstract—Concepts and laws of diffuse reflectance are revisited in the light of the theory of partial coherence and modern photogoniometric measurements using lasers. The angular intensity distribution of radiation scattered by inhomogeneous media or rough surfaces is studied in terms of $K$ correlations and facets models. The existence of 'Lambertian' scatterers is discussed.

INTRODUCTION

Diffuse reflectance and transmittance in the infrared and other spectral regions is widely used for studying samples showing a rough surface or consisting of inhomogeneous material such as packed grains or fibres. Examples are building material, ceramics, minerals, polymers, textiles, paper and biological material. The accepted concepts and laws of conventional diffuse reflectance\(^1\)\(^2\) are based on the classical theory of radiative transfer, which deals with the propagation of intensities and neglects interference effects. As was first pointed out by Walther,\(^3\) the classical concepts of radiometry and radiative transfer hold only in the hypothetical limit of strict spatial incoherence (zero correlation length). Modern photogoniometric measurements of diffuse scatterers, however, are made using laser radiation and allow determination of the effective correlation length of the scatterer.\(^4\) Moreover, the degree of spatial coherence of a radiation source is known to be crucial for the 'radiant intensity', i.e. the angular distribution of the average far-zone intensity, even in the case of poor coherence (see e.g. the recent review by Baltes, Geist, and Walther\(^5\)).

In the present paper we revisit the laws and concepts of diffuse reflection in the light of the theory of partial coherence. We study the angular distribution of the average intensity of the scattered radiation. We consider ensemble averages and thus do not account for the speckle noise that occurs in the individual experiment. The statistical properties of the scattered radiation will be studied in a forthcoming comprehensive paper.\(^6\)

RADIANT INTENSITY AND EFFECTIVE-SOURCE CORRELATIONS

For maximum simplicity, we restrict our attention as follows. We consider only quasi-planar, statistically homogeneous and isotropic scatterers, which are described by two-dimensional spatial correlation and lead to 'circularly diffuse' radiant intensities $I(\Omega)$. We assume a fully coherent illuminating beam with perpendicular incidence. The beam waist is supposed to be large compared with the correlation length of the scatterer. This correlation length is assumed to be larger than the wavelength of the radiation. We use the scalar approximation and study the scattered radiation only in the far zone (Fourier optics).

The illuminated scatterer can be visualized as an indirect source. This source is characterized by the isotropic cross-spectral density

$$W(\rho) = \int \mathcal{D}\tau \exp(\rho u_\tau) \langle u^*(\mathbf{r}, t)u(\mathbf{r} + \rho, t + \tau)\rangle. \quad (1)$$
Here, \( u(\mathbf{r}, t) \) denotes the stochastic amplitude at the time \( t \) and at the position \( \mathbf{r} \) in a plane immediately in front of the scatterer, the brackets \( \langle \ldots \rangle \) indicate the ensemble average, and \( \rho = |\mathbf{\rho}| \). Function (1) depends also on the frequency \( \omega \). The corresponding radiant intensity \( I(\theta) \) is known to be essentially the Fourier Transform of the correlation \( W(\rho) \), viz.

\[
I(\theta) \propto \cos^2 \theta \int \rho \, d\rho \, J_0(\kappa \rho \sin \theta) W(\rho), \quad k = \omega/c.
\]

By inversion of relation (2), the 'effective-source' correlation (1) can, in principle, be obtained from goniometric measurement of \( I(\theta) \).

A large class of experimental results can be interpreted in terms of \( K \) correlations, i.e. correlations of the type

\[
W_1(\rho) \propto (\rho/l)^v K_v(\rho/l),
\]

where \( K_v(x) = \frac{\pi^{1/2}}{\Gamma(v + 1/2)} J_v(x) \) denotes the modified Bessel function of the second kind of order \( v \) and where \( l \) is an appropriate correlation length. Of particular physical interest are the functions with \( v = \frac{1}{2}, 1, \frac{3}{2} \) and \( \frac{5}{2} \). The case \( v = \frac{1}{2} \) leads to an exponential correlation corresponding to a Markffian spatial process with the well known objectionable derivative at \( \rho = 0 \). The corresponding radiant intensity of the scattered radiation reads

\[
I_1(\theta) \propto k^{2v} \cos^2 \theta (1 + k^2 l^2 \sin^2 \theta)^{-v-1}.
\]

We observe that two popular theoretical models are not members of the \( 'K \) family': the Gaussian correlation and the 'Lambertian' correlation \( (k \rho)^{-1} \sin k \rho \) leading to \( I(\theta) \propto \cos \theta \).

**K Correlations and Facet Models**

We now pursue the question how the above radiant intensities and effective-source-plane correlations are related to physical models of diffuse scatterers. An old, but still attractive, model is the facet or micro-area model due to Bouguer. In the modern version of this model, the scatterer is thought of as a collection of facets of diameter \( \xi \) with a linear variation of the phase \( \Phi \), viz.

\[
\Phi(\mathbf{r}) = m \, \mathbf{r}/\xi
\]

with \( m \) denoting the slope of the facet. The facets are assumed to contribute incoherently to the scattered radiation field since \( \xi \) is meant to play the role of the correlation length of the phase fluctuation produced by the stochastic scatterer. The stochastic aspect of the scatterer is taken care of in terms of the distribution \( P(m) \) of the slopes \( m \) of the facets. In this paper we assume that the rms slope \( m_0 \) and the size \( \xi \) are constant parameters of the model. Models with varying \( m_0 \) and \( \xi \) are considered elsewhere. Statistically, isotropic scatterers are described by isotropic slope distributions \( P(m) \) with \( m = m \). Moreover, we assume that the facets are not too small, viz. \( k^2 \xi^2 \gg 1 \).

It can be shown that not only the reduced radiant intensity \( I(\theta)/\cos^2 \theta \) and the correlation \( W(\rho) \), but also the correlation \( W(\rho) \) and the slope distribution \( P(m) \) are Fourier conjugates, viz.

\[
W(\rho) \propto \int m \, dm \, J_0(mp/\xi) P(m).
\]

Hence the relation between the radiant intensity and the slope distribution is very simple:

\[
I(\theta) \propto \cos^2 \theta \, P(k \xi \sin \theta).
\]

Thus a measured isotropic angular distribution of the average scattered intensity can immediately be interpreted in terms of the pertinent micro-area model by plotting.
$I(\theta)/\cos^2 \theta$ as a function of $\sin \theta$ and by substituting the variable $\sin \theta$ by $m/k\xi$. The $K$ correlations (3) correspond to facet models with slope distribution

$$P_v(m) \propto (1 + m^2/m_0^2)^{-v-1}, \quad m_0 \equiv \xi/l. \quad (8)$$

We observe that the effective source correlation length $l$ and the facet size $\xi$ are related by $l = \xi/m_0$.

**RADIOMETRY OF DIFFUSE SCATTERERS**

Although there is much current interest in revisiting classical radiometry in the light of the theory of partial coherence,\cite{5} relatively little is known about the radiometry of diffuse scatterers. Radiometric properties of random phase screens were hitherto investigated\cite{15} only in terms of the Gaussian effective-source correlation proposed by Jakesman and Pusey.\cite{9} In view of the above results, $K$ correlations and the underlying models seem to provide a useful basis of further studies of the radiometry of diffuse scatterers.

For a facet-type scatterer with slope distribution $P(m)$, the *generalized radiance* as defined by Walther\cite{3} reads

$$B \propto \cos \theta P(k\xi \sin \theta)I(r) \quad (9)$$

with $I(r)$ denoting the (slowly varying) average intensity profile in the effective source plane. The corresponding *generalized emittance* reads $E(r) = CI(r)$, where

$$C \propto \int \sin \theta d\theta \cos^2 \theta P(k\xi \sin \theta) \quad (10)$$

denotes the *radiation efficiency*.\cite{5} This is a figure of merit for the total average energy transfer by fluctuating scatterers. In the case of $K$ correlated scattering systems we obtain

$$C_v = \frac{3}{4}v k^2 l_2 \sum_{1, \frac{3}{2}, -k^2 l^2} \quad (11)$$

with $\sum$ denoting a hypergeometric function and with $C_v \to 1$ in the coherent-limit $k^2 l^2 \to \infty$. We plot $C_v$ in Fig. 1 for a number of cases. For comparison we recall that the radiation efficiency of the blackbody\cite{10} is $C_{BB} = \frac{1}{2}$.

Let us finally discuss the question whether a *Lambertian* radiance $B = \text{constant}$ can be produced by a facet-type scatterer with constant facet size $\xi$ and rms slope $m_0$. The pertinent inversion of relation (7) would lead to the ill-behaved hypothetical "Lambertian" slope distribution

$$P_L(m) \propto (1 - m^2/k^2 \xi^2)^{-1}, \quad m < k\xi. \quad (12)$$

![Fig. 1. Radiation efficiency $C_\nu$ of $K$ correlated scatterers for various values of index $\nu = \frac{1}{2}, \ldots, 5$ as function of $kl$, where $l$ denotes the effective-source-plane correlation length.](image-url)
We conclude that well-behaved facet models with constant $\xi$ and $m_0$ for strictly Lambertian scatterers do not exist. However, a Lambertian reflectance can be approximately obtained at least for not too large observation angles. We notice that this result is in agreement with classical studies\(^{(11,12)}\) based on the geometric optics version\(^{(7)}\) of the facet model.

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