Discussion on: “Stabilization of the Experimental Cart-Pendulum System with Proven Domain of Attraction”

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The article presents an experimental implementation of an Immersion & Invariance (I&I), [2], controller design to an experimental cart-pendulum system. The I&I controller was developed in [1], and in the paper under discussion the domain of attraction is proven to be the open upper half plane. The article serves merely as an experimental proof of principle for the I&I method, where in addition to the I&I controller, extra control actions are taken in order to deal with practical issues such as friction, and the fact that the velocities are not measured.

It is interesting to compare these results with another controller implementation on the same set up. In [5] an IDA-PBC (Interconnection and Damping Assignment Passivity Based Control) controller was developed and implemented on the same set up. Even though it is argued in the introduction of the article under discussion that the controller of [5] is rather complicated in comparison to the I&I control, the level of complexity or simplicity of the I&I controller given by expression (19) is not discussed later on in the article. For that, we can consider the IDA-PBC design of [5]. The controller expression is derived by construction of a new mass inertia matrix \( M_d \), interconnection matrix \( J_2 \), and damping matrix \( K_r \), along with a new energy function \( H_d \), and the overall expression for the control law is obtained by substituting these new matrices and functions into the control law given by

\[
\begin{align*}
    u &= (G^T G)^{-1} G^T \left( \frac{\partial H}{\partial q} - M_d M^{-1} \frac{\partial H_d}{\partial q} 
    + J_2 M_d^{-1} p \right) - K_r G^T \frac{\partial H_d}{\partial p},
\end{align*}
\]

where \( G = (0 \ 1)^T \), \( H \) is the total energy (Hamiltonian) of the uncontrolled system, and \( M \) the mass inertia matrix of the uncontrolled system. \( H, H_d, M, M_d, J_2 \) depend on the state \( x = (q, p) \), where \( q = (q_1, q_2) \) are the angular displacement of the pendulum, and displacement of the cart, and \( p \) the corresponding momenta. Indeed, the overall expression of the IDA-PBC controller can become more complex than the I&I controller expression of equation (19) of the paper under discussion. However, for both IDA-PBC and I&I physical considerations can be used to design the controller, and comparing the complexity of the controller design from that point of view is not straightforward.

1. **Region of Attraction**

Like for I&I also for IDA-PBC a PDE needs to be solved, which is done in [5]. Also similar to the I&I design, the IDA-PBC (or total energy shaping) controller in [5] is developed for the system without friction. The region of attraction of the IDA-PBC controlled system is the same as for the I&I controlled system, namely the upper half plane. However, in [5] it is shown that this would require the cart to move beyond the length of the rail, and thus the constraints of the setup do not allow for stabilization from the full upper half plane. This is also the case for the I&I implementation of the paper under discussion. Hence, in both cases first a swing up is accomplished by a bang-bang control, and then the control is switched to either the I&I or the IDA-PBC controller at 0.4 rad.

2. **Friction Compensation**

The I&I controller is developed for the cart-pendulum system without friction. However, it is known that the friction...
of the cart with the rail cannot be neglected. No theoretical analysis of the stabilization properties of the I&I controller applied to the system with friction is given, but a simulation shows that with a friction model that is reasonable accurate, still stability is achieved. In [5] first an IDA-PBC controller is developed for the system without friction, and then the system with friction between the cart and the rail is studied. It is clarified that the pendulum system on the cart does not fulfill the dissipation condition, [3], to design an IDA-PBC controller for the system with friction. For that reason, [5] implements a friction compensation before applying IDA-PBC. Surprisingly, this is also done for the I&I controller implementation, due to limitations of the system, in particular the energy that can be provided by the actuator. The friction model that is used is the same in both cases, however, the parameters are quite different. It could be interesting to clarify this, and make a comparison between both controller implementations with the same friction compensation parameters. For both the I&I and IDA-PBC controller implementations oscillatory behaviour of the cart is seen. In the I&I case an additional integrator loop is introduced to suppress these oscillations. In other studies, such as in [4] on a different cart-pendulum setup, different solutions to deal with this are proposed.

3. Concluding Remarks

Finally, some tests with perturbations are performed on the I&I controlled set-up, and it is seen to perform reasonably well. In [5] tests are performed by not compensating fully for the friction, and for certain control parameters, the controlled system still performs reasonably well. Again, it could be interesting to perform similar tests for both implementations. In [5] a comparison is made with a linear \( H_{\infty} \)-controller implementation, and clearly the nonlinear IDA-PBC implementation outperforms the linear one. The same is expected to hold for the I&I implementation.

The cart-pendulum system is a benchmark for many controller implementations, as mentioned in the introduction of the article under discussion, where a number of references are given. The I&I implementation is successfully implemented on the experimental setup and appears to perform well. It is of interest to compare the performance of the different controller implementations reported in the literature dealing with the cart-pendulum benchmark system.

References