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Capillary condensation and quantum vacuum effects on the pull-in voltage of electrostatic switches with self-affine rough plates

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In this work, we study the influence of capillary forces in combination with electrostatic and quantum vacuum generated forces on the pull-in voltage of microswitches having self-affine rough surfaces. This type of roughness is described by the rms roughness amplitude \( w \), the in-plane correlation length \( \xi \), and the roughness exponent \( H \) that quantifies the degree of surface irregularity. At short length scales \( \langle \xi \rangle \), it is shown that an attractive capillary force decreases more the effective pull-in voltage when the plate surfaces are rougher. The latter corresponds to smaller roughness exponents \( H \) and/or larger long wavelength roughness ratios \( w/\xi \). Notably, the capillary contribution increases the sensitivity of the effective pull-in voltage on the roughness exponent \( H \). This behavior takes place for values of \( H \) close to its experimental accuracy. © 2006 American Institute of Physics.

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I. INTRODUCTION

An important component in the design of microelectromechanical systems (MEMSs)/nanoelectromechanical systems (NEMSs) such as nanotweezers, nanoscale actuators, etc., is microswitches.\(^1\)\(^-\)\(^16\) A typical switch is constructed from two conducting electrodes. One electrode is usually fixed, and the other one is able to move but it remains suspended by a mechanical spring. By applying a voltage difference between the two electrodes, the mobile one moves towards the ground electrode because of the electrostatic force. At a certain voltage, the mobile electrode becomes unstable and collapses or “pulls in” onto the fixed ground electrode.\(^3\) For this system a two degree of freedom pull-in model is presented in Ref. 4 for direct calculation of pull-in characteristics for electrostatic actuators. In addition, residual stress and fringing-field effects have great influence on the behavior and failure of radio frequency switches.\(^5\)\(^,\)\(^6\)

Furthermore, when the proximity between the plates of switches becomes of the order of nanometers up to a few microns, a regime is entered in which forces that are quantum mechanical in nature, namely, van der Waals and Casimir forces, become operative.\(^17\)\(^-\)\(^19\) These forces may also be responsible for stiction by causing mechanical elements in close proximity to adhere together, and therefore profoundly change the actuation dynamics of switches.\(^10\) In any case, the Casimir force has been considered to be an exotic quantum phenomenon that results from the perturbation of zero point vacuum fluctuations by the presence of conducting plates.\(^17\)\(^-\)\(^19\) Because of its relatively short range, it is now starting to attract technological importance for the design and operation of MEMSs/NEMSs.\(^10\)\(^-\)\(^15\) Recent studies for switches with rough plates have shown that random self-affine roughness, which often occurs during nonequilibrium film deposition, strongly influences pull-in parameters and phase maps of microswitches in the presence of electrostatic and Casimir forces.\(^20\)

In addition, since MEMSs/NEMSs are fabricated on silicon substrates by deposition and selective etching of multiple layers of structural and sacrificial films,\(^16\) if the resulting surface is hydrophilic, strong capillary forces develop. This takes place when the microstructures are pulled out of water or when two surfaces approach each other in a humid environment. These interfacial forces can cause catastrophic adhesion between moving components. Drying techniques can eliminate release-related adhesion, and it has become possible to fully release extremely compliant microstructures.\(^16\) However, these techniques do not address the problem of in-use adhesion.\(^16\) In this case, liquids that wet or have small contact angles on surfaces condense from vapour into small cracks and pores in a phenomenon known as capillary condensation.\(^18\) As a result when the plate surfaces of microswitches come in close proximity attractive capillary forces can start acting on them.

Therefore, capillary forces can influence the operation of microswitches, which is further complicated by the presence of rough surfaces. This will be the focus of the present study, where we will examine the influence of random self-affine roughness on the effective pull-in voltage of microswitches. In this case, the elastic restoring force of the moving plate will be counterbalanced by attractive capillary, Casimir, and electrostatic forces.

II. FORCES IN MICROSWITCHES WITH PARALLEL ROUGH PLATES

Here, we consider a parallel plate configuration where the electrostatic, Casimir, and capillary forces are pulling the plates together and opposing the elastic restoring force (Fig. 1). The initial plate distance is \( d \), the average flat plate surface area is \( A_p \), the plate spring constant is \( k \) and its mass \( m \), the applied voltage in between the plates is \( V \), and the electrical permittivity of the condensed liquid between the plates
is $\epsilon$. Moreover, we also assume the same roughness for both plates, which is characterized by single valued random roughness fluctuations $h(R)$ of the in-plane position $R = (x, y)$.

### A. Elastic and electrostatic forces

The elastic restoring force is given by\textsuperscript{15,20}

$$ F_k = -k(d-r). \quad (1) $$

The electrostatic force without accounting for fringing fields for a plate separation $r (\leq d)$ is given by $F_e = (\epsilon A_r / 2) \times (V^2 / r^2)$.\textsuperscript{16,20} where $A_r$ is the surface area of a rough plate surface. For a Gaussian height distribution we have $A_r = R_s A_f$ with $R_s = \int_0^{\infty} e^{-x^2} (1+\rho^2 x dx)$.\textsuperscript{21} Upon substitution, the latter yields the electrostatic force

$$ F_e \approx \frac{\epsilon A_f V^2}{2} \int_0^{r^2} e^{-1/r^2} \rho^2 d\rho dx; $$

with $\rho = \sqrt{\langle (\nabla h)^2 \rangle}$ is the finite plasmon frequency and wavelength $\lambda_P$ the plasma wave-length $\lambda_P (\sim 100 \text{ nm})$ where the finite conductivity corrections are important. Therefore, these corrections are significant at different length scales, and they can be multiplied for theory estimations with accuracy above $1\%$.\textsuperscript{25}

$$ \rho = \sqrt{\langle (\nabla h)^2 \rangle} \approx \sqrt{\int_0^{\infty} q^2 \langle (h(q))^2 \rangle d^2 q / (2\pi)^2}, \quad (2) $$

and

$$ F_e(T,r) = F_{cT}(T) \left( 1 + \frac{2C_f}{r} \right), \quad (3) $$

with

$$ C_r = \begin{cases} \frac{0.4492}{15} \int_0^{Q_{f_r}} q q^2 (h(q))^2 d^2 q / (2\pi)^2, & \text{if } r < \lambda_P, \\ \frac{1}{5} \int_0^{Q_{f_r}} q^2 (h(q))^2 d^2 q / (2\pi)^2 + \frac{7}{15} \frac{\lambda_P}{r} \int_0^{Q_{f_r}} q q^2 (h(q))^2 d^2 q / (2\pi)^2, & \text{if } r > \lambda_P, \end{cases} \quad (4) $$

and

$$ F_{cT}(T,d) = \begin{cases} 1 + \frac{720}{\pi^2} \left( \frac{K_g T d}{\hbar c} \right)^3 \frac{\zeta(3)}{2\pi} - \frac{45}{\pi^2} \left( \frac{K_g T d}{\hbar c} \right)^4, & \text{if } K_g T d < \frac{1}{2} \\ \left( \frac{K_g T d}{\hbar c} \right) \frac{\zeta(3)}{8\pi} - \frac{\pi^2}{720}, & \text{if } K_g T d > \frac{1}{2}. \end{cases} \quad (5) $$

Moreover, we have $Q_{f_r} = 2\pi / \lambda_P$ and $Q_r = 2\pi / r$. $\zeta(3) = 1.202$ is the Riemann zeta function. $F_{cT}(T) = (\pi^2 \hbar c/240r^3) A_r$ is the Casimir force for flat perfectly conducting plates, $c = c/\sqrt{\epsilon}$ the velocity of light in between the plates, and $\omega_P$ and $\lambda_P$ the finite plasmon frequency and wavelength (e.g., $\lambda_P = 100 \text{ nm}$ for Al, etc.) assuming here the plasma dielectric function $\epsilon(\omega) = 1 - (\omega_P / \omega)^2$ (Ref. 23) for the plates. The finite temperature correction is considered as a multiplying factor $F^{\pi,24}$ At room temperature ($T = 300 \text{ K}$) the thermal wavelength $\lambda_T (\sim 7 \text{ m})$ is much larger than the plasma wavelength $\lambda_P$. We consider the situation when two hydrophilic rough surfaces approach each other in a humid environment. Liquids that wet or have small contact angles $\theta$ on surfaces\textsuperscript{26} condense from vapor into small cracks and pores in a phe-

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**FIG. 1.** Schematic of parallel plate switch with meniscus formation that exerts the additional capillary force.
nomenon known as capillary condensation.\textsuperscript{16,18} At equilibrium, the meniscus curvature is related to the relative vapor pressure $P/P_{\text{sat}}$ by the Kelvin equation $(R_1^{-1}+R_2^{-1})^{-1}=R_{\text{Kel}}=(\gamma U_m/RT)\left[ \log (P/P_{\text{sat}}) \right]^{-1}$ (assuming absolute values of the meniscus curvature).\textsuperscript{16,18} $\gamma$ is the liquid vapor pressure (e.g., $\gamma=73$ mJ/m$^2$ for water), $R_{\text{Kel}}$ is the Kelvin radius, $R_1$ is the out-of-plane meniscus radius so that $d=2R_1 \cos \theta$, $R_2$ is the in-plane radius or curvature with $R_2 \ll R_1$, and $U_m$ is the molar volume of the liquid. When two hydrophobic plate surfaces approach each other in a humid environment, the liquid undergoes capillary condensation if the surface separation becomes $d_o=2R_{\text{Kel}} \cos \theta$. The attractive capillary force for separation $r\geq d_o$ is given by\textsuperscript{16}

$$F_{\text{cap}} = A r_0^4 \frac{4 \gamma R_{\text{Kel}}}{r^2} \cos^2 \theta,$$

with $A_r$ the wetted rough surface area. The latter is assumed to have dimensions larger than the characteristic lateral roughness wavelength. The average contact angle $\theta$ on a rough surface is given by (see also Appendix)\textsuperscript{20-29}

$$\theta = \sqrt{2}\left( \theta_o + \sqrt{2} \pi^{1/2} \rho \right)^{-2} + \left\{ \cos^{-1} \left[ \int_{0}^{\infty} du (1 + \rho^2 u) e^{-\nu u} \cos \theta_o \right] \right\}^{-1/2},$$

where $\theta_o$ is the theoretical contact angle for a smooth surface which is determined by Young’s law.\textsuperscript{20}

### III. RESULTS AND DISCUSSION

The plate motion is described by the second law of Newton or $m(d^2u/dt^2)=[F_u]-[F_{\text{cap}}]-[F_{\text{Cas}}]-[F_{\text{damp}}]$. If we consider the change of variables $u=\lambda /d$, $M=m/k_2 T$, and $T=t/T$ (T a characteristic time), the equation of motion takes the form (with $0<u<1$)

$$M \frac{d^2u}{dt^2} = f(u, a, b) = 1 - u - \frac{\beta}{2u^2} - \frac{\alpha}{4u^3} W(u),$$

$$\alpha = \frac{\pi^2 h}{kd} A R, \quad \beta = \left[ e A R V^2 + 8A R \gamma R_{\text{Kel}} \cos^2 \theta \right] \frac{1}{kd},$$

and $W(u) = u^{-4}\left[1+(2C_1/d)F_T(T, du)\right]$. In order to obtain the pull-in potential we set $f(u, a, b)=0$ and $df/du=0$. Thus, we obtain $1-u-(\beta/2u^2)-(\alpha/240)w(u)=0$ and $(\beta/2u^2)-(\alpha/240)W-1=0 (W=dW/du)$. The solution of these equations yields in combination with Eq. (9) the effective pull-in potential

$$V_{\text{pr}} = V_o \left[ Q(u) - \frac{8 \gamma A R_{\text{Kel}}}{kd} \cos^2 \theta \right]^{1/2},$$

where $Q(u) = (2u^2/R_c)(1-u+WW^{-1})(1+2u^{-1}WW^{-1})^{-1}$ and $V_o = \sqrt{kd}/eA R$.

Further calculations of the pull-in voltage require knowledge of the roughness spectrum $\langle \mid h(q) \mid^2 \rangle$. Indeed, a wide variety of surfaces and interfaces that appear in thin films grown under nonequilibrium conditions possesses the so-called self-affine roughness.\textsuperscript{30} In this case the roughness spectrum $\langle \mid h(q) \mid^2 \rangle$ shows a power law scaling;\textsuperscript{30} $\langle \mid h(q) \mid^2 \rangle \propto q^{-2H}$ if $q \xi \ll 1$, and $\langle \mid h(q) \mid^2 \rangle \propto \text{const}$ if $q \xi \gg 1$. This is satisfied by the analytic model for $\langle \mid h(q) \mid^2 \rangle$,\textsuperscript{31}

$$\langle \mid h(q) \mid^2 \rangle = 2 \pi \frac{w^2 \xi^2}{(1+a q^2 \xi^2)^{1+H}},$$

with $a=1/2h[(1+(a q^2 \xi^2)^{-H})]$. If $0<H<1$ and $a=1/2 \ln(1+(a q^2 \xi^2)^2)$ if $H=0$. Small values of the roughness exponent $H\sim 0$ characterize jagged or irregular surfaces while large values $H\sim 1$ characterize surfaces with smooth hills and valleys.\textsuperscript{30} For other correlation models see also Refs. 30–32. From the Fourier transform of $\langle \mid h(q) \mid^2 \rangle$ we obtain the height difference correlation function $g(r)=\langle \mid h(r) - h(0) \mid^2 \rangle \propto \langle \mid h(q) \mid^2 \rangle \exp(-K^{-1} r^2)$, where $K$ is the Fourier transform of $\langle \mid h(q) \mid^2 \rangle$. The latter scales for $r<s$ as $g(r) \propto r^{2H}$ and yields $g(r)/r^2 \rightarrow 0$ if $r \rightarrow +\infty$ and only if $H<1$. This ensures bounded roughness fluctuations besides the restriction of a finite correlation length $\xi$. In addition, Eq. (11) yields for the average local surface slope $\rho$ the analytic form $\rho = (w/\sqrt{2a}) \propto \sqrt{(1-H)^{-1}[1+(a q^2 \xi^2)^2]^{1-H}-1}-2a$.\textsuperscript{22} The latter shows that the average local slope is proportional to the rms amplitude ($\rho \propto w$).

Figure 2 shows calculations of the pull-in voltage versus pull-in gap $u$ with increasing plate dimensions that lead also to an increasing capillary contribution ($\sim A_r$). The pull-in voltage is strongly affected by the capillary term for lower gap separations ($u<0.5$). The influence of the capillary term will be further amplified with increasing relative humidity. In Eq. (10) the direct dependence of the pull-in voltage on the humidity conditions is obtained if we substitute $R_{\text{Kel}}=(\gamma U_m/RT)\left[ \log (P/P_{\text{sat}}) \right]^{-1}$.

$$V_{\text{pr}} = V_o \left[ Q(u) - \frac{8 \gamma A R_{\text{Kel}}}{kd} \cos^2 \theta \right]^{1/2},$$

where $Q(u)=2u^2/R_c(1-u+WW^{-1})(1+2u^{-1}WW^{-1})^{-1}$ and $V_o = \sqrt{kd}/eA R$.

In our calculations we have used $R_{\text{Kel}}=1.6$ nm, which is the value for water at $T=293$ K with $\gamma U_{\text{m}}/RT=0.54$ nm and $\lambda=100$ nm, and roughness exponents $H=0.8$. 

FIG. 2. (Color online) Pull-in voltage ratio vs pull-in normalized gap $u$ for roughness correlation length $\xi=200$ nm, various plate dimensions $\Lambda/d$ (with $A_r=\Lambda^2$), $R_{\text{Kel}}=1.6$ nm, $\theta_o=20^\circ$, $e=1$, $w=5$ nm, $d=200$ nm, $\lambda=100$ nm, and roughness exponents $H=0.8$. 

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50% relative humidity ($P/P_{sat}=0.5$). In order to have $R_{Kel} = \lambda_p (=100 \, \text{nm})$, the required relative humidity is $P/P_{sat} = 0.99$. The latter is close to extreme humidity conditions for a microswitch to operate. Furthermore, the sensitive influence of the relative humidity $P/P_{sat}$ on the pull-in voltage ratio is shown in Fig. 3. Clearly with decreasing correlation length $\xi$ (or equivalently for rougher plate surfaces) and increasing relative humidity $P/P_{sat}$ the effective pull-in voltage decreases significantly. If $Q(u) < (8 \gamma A_{Kel} / kd^3) \cos^2 \theta$ the pull-in potential is zero, and spontaneous adhesion of the plates occurs, leading to collapse of the switch. Indeed, for the parameters under consideration, the adhesion condition is satisfied for relative humidity $P/P_{sat} \approx 0.9$, yielding a zero effective pull-in potential.

Figure 4 shows calculations of the pull-in voltage versus lateral roughness correlation length $\xi$ and various roughness exponents $H$. Clearly the roughness exponent $H$ influences the pull-in voltage strongly for sufficiently small correlation lengths before saturation takes place ($\rho \approx 1$ or equivalently almost flat plate surfaces). The high sensitivity to consecutive roughness exponents $H$ ($\approx 1$, which is within its experimental accuracy) is further enhanced by the presence of the capillary term as Fig. 5 shows for two relatively different roughness exponents. If the capillary contribution on $V_{PI}$ is relatively weak so that $Q(u) \gg (8 \gamma A_{Kel} / kd^3) \cos^2 \theta$ and taking into account the fact that $|\cos \theta| \leq 1$, we obtain from Eq. (10) the simpler form $V_{PI} \approx V_o \sqrt{Q(u) - [4 \gamma A_{Kel} / \sqrt{(Q(u)kd^3)]}$. In any case, Figs. 3–5 show that the influence of the capillary term is more pronounced for small correlation lengths $\xi$ and/or smaller roughness exponents $H$. The latter conditions correspond to rougher surfaces and therefore to lower pull-in voltages.

**IV. CONCLUSIONS**

In summary it is shown that the presence of capillary condensation that leads to attractive forces can strongly influence the effective pull-in voltage of microswitches. The capillary contribution lowers the effective pull-in voltage for rougher plate surfaces or equivalently lower exponents ($H < 1$) and/or larger long wavelength roughness ratios $w/\xi$. More precisely, the capillary contribution increases the sensitivity of the effective pull-in voltage on the roughness exponent $H$. This behavior takes place for values of $H$ close to its experimental accuracy, $\approx 0.1$. The capillary condensation increases its effect on the pull-in voltage by increasing humidity and plate roughness.

We should also mention that the assumption of Gaussian height distribution which yields a closed form for the roughness ratio $R_s$ is not very restrictive within the weak roughness approximation ($\rho \approx 1$). This is because the dominant term up to second order in $\rho$ [e.g., $R_s \approx 1 + (\rho^2/2)$] is obtained without this assumption. Finally, in future studies it would be interesting to consider the more general case of capillary condensation with incomplete wetting (unwetted sharp grooves), which is expected to lead to weaker capillary effects.

**APPENDIX**

Following the geometrical model of periodical circular grooves, the contact angle of a liquid that partially wets a random rough surface (Fig. 1) is given by the theoretical
contact angle $\theta_o$ (determined by Young’s law) and modified by the surface local slope: $\theta^G = \theta_o + \tan^{-1}(|\nabla h|)$. For weak roughness ($|\nabla h| \ll 1$) we have $\tan^{-1}|\nabla h| \approx |\nabla h|$. Ensemble average over possible roughness configurations yields $\theta^G = \theta_o + |\nabla h|$. For a Gaussian height-height distribution since $\langle |\nabla h|\rangle = [\langle 1/\pi \rangle (\nabla h)^2]^{1/2}$, the maximum geometric contact angle is given by

$$\theta^G \equiv \theta_o + (2/\pi)^{1/2} \rho.$$  
(A1)

On the other hand, following Wensel’s thermodynamic approach (which applies for radial grooves), the contact angle is given by

$$\theta^W = \cos^{-1}(R_e \cos \theta_o).$$  
(A2)

Since rough surfaces can be thought of as a combination of radial and circular grooves we consider the harmonic average value between $\theta^G$ and $\theta^W$. This is given by $2/\theta^H = (1/\theta^G)^2 + (1/\theta^W)^2$.27,28 This average is considered as a compromise to the fact that the geometric contact angle $\theta^G$ overestimates the roughness effect because the maximum rms local surface slope is considered yielding $\theta^W < \theta^G$.27,28 Note also that since $|\cos \theta^H| \ll 1$, our calculations are restricted to surface area ratios $R_e$ and theoretical angles $\theta_o$ so that $R_e |\cos \theta_o| \ll 1$.