Surface roughness influence on the pull-in voltage of microswitches in presence of thermal and quantum vacuum fluctuations

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Abstract

In this work we investigate the influence of the combined effect from random self-affine roughness, finite conductivity, and finite temperature on the pull-in voltage in microswitches influenced by thermal and quantum vacuum fluctuations through the Casimir force and electrostatic forces. It is shown that for separations within the micron or sub-micron range the roughness influence plays a dominant role, while temperature starts to show its influence well above micron separations. Indeed, increasing the temperature leads to higher pull-in voltages since it leads to an increased Casimir force. The temperature influence is more significant for relatively large roughness exponent $H\sim 1$, while its influence is significantly lower with increasing lateral roughness correlation length $\xi$ or due to long wavelength surface smoothness.

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1. Introduction

When the proximity between material objects, as for example in switches used in micro/nanoelectromechanical (MEMS/NEMS) [1–15], becomes of the order of nanometers up to a few microns, a regime is entered in which forces that are quantum mechanical in nature, namely, van der Waals and Casimir forces, become operative [16]. Historically, the Casimir force has been considered to be an exotic quantum phenomenon that results from the perturbation of zero point vacuum fluctuations by conducting plates [16–18]. Because of its relatively short range now it is starting to take on technological importance in the design and operation of MEMS/NEMS.

Quantum derived forces may also be responsible for stiction, i.e., causing close by elements to adhere together and thus profoundly change actuation dynamics [10]. These forces supplement the electrostatic force in countering Hooke’s law to determine the beam actuation behavior in microswitches. The latter is typically constructed from two conducting electrodes having one fixed and the other moving but suspended by a mechanical spring. Voltage application between the electrodes moves the electrodes towards each other because of the electrostatic force. At a certain voltage, the moving electrode becomes unstable and collapses or pulls-in onto the ground electrode [3,4].

Residual stress and fringing field effects have also shown to have a great influence on the behavior of RF switches, and strongly influence their failure characteristics [5,6].

The influence of van der Waals forces on the pull-in voltage between the plates was studied in Ref. [11] by ignoring its influence on the pull-in gap. These studies were extended in Ref. [12] where the effect of the van der Waals force on the pull-in gap was investigated, and an analytical expression of the pull-in gap and voltage was also shown in Ref. [12]. The dynamical behavior for nanoscale electrostatic actuators was studied by considering the effect of the van der Waals force in Ref. [13]. Furthermore, the
influence of the Casimir force on the pull-in gap and voltage, and phase maps of MEMS switches was also studied in Ref. [10] in the limit of perfectly reflecting flat plates coated with Au. An approximate expression of the pull-in gap with the Casimir force was presented by means of perturbation theory including the case of non-linear electrostatic actuators [14,15]. Studies of the influence of the Casimir force for nanoscale electrostatic actuators with flat electrodes showed that their phase maps exhibit periodic orbits around a Hopf point, and a homoclinic orbit to pass through an unstable saddle point [15]. For self-affine rough metal plates [19] it was also shown assuming separations larger than the plasmon wavelength that the fine roughness details at short and long wavelengths influence the stability and phase maps of microswitches [20]. In many instances the roughness of deposited metal films is termed as self-affine and it is characterized by anisotropic scaling of the out of plane dimension with respect to the in plane dimensions [19].

So far, the former studies for the pull-in parameters with rough plate surfaces [20] were limited to plate separations larger than the plasmon wavelength, while below that length scale finite conductivity effects should be taken into account. In addition, the influence of finite temperature (>0 K) was not taken into account. A typical thermal wavelength is \( \lambda_T = \frac{hc}{2K_B T} \) which at \( T = 300 \text{ K} \) yields \( \lambda_T = 6.55 \mu \text{m} \). Thus, thermal fluctuations for \( T \gg 300 \text{ K} \) are important at micron plate separations and produce their own radiation pressure and a larger Casimir force. The finite conductivity correction concerns the fact that real materials become transparent for electromagnetic waves of frequencies \( \omega > \omega_p \), where \( \omega_p \) is the plasma frequency [21].

2. Theory for switches with parallel rough plates

Here, we consider a parallel plate configuration with the electrostatic force and Casimir force pulling the plates together, while an opposing elastic restoring force is present. The initial plate distance is \( d \), the average flat plate area \( A_f \), the plate spring constant is \( k \) and its mass \( m \), the voltage across the plates is \( V \), and \( \varepsilon_0 \) is the vacuum permittivity.

The restoring force is given by [15]

\[
F_k = -k(d - r) .
\]

The electrostatic force without accounting for fringing fields for a plate separation \( r \leq d \) is given by \( F_e = \left( \varepsilon_0 A_f / 2 \right) (V^2 / r^2) \) [15] where \( A_f \) is the surface area of a rough plate surface. This is by \( F_e = -\partial U_e / \partial r \) with \( U_e = CV^2 / 2 \) the capacitor energy. The rough surface area \( A_f \) is related to the average flat plate area \( A_f \) for a Gaussian height distribution [22] and thus we have for the electrostatic force

\[
F_e = \frac{\varepsilon_0 A_f}{2r^3} \rho^2 R_r , \quad R_r = \int_0^{\infty} e^{-r} \sqrt{1 + \rho_{rms}^2 y^2} \, dy ,
\]

(2)

where \( \rho_{rms} = \sqrt{\langle \nabla h \rangle^2} \) is the average local slope, which is given after Fourier transformation in the form

\[
\rho_{rms}^2 = \int_0^{\infty} q^2 \langle |h(q)|^2 \rangle \, dq / (2\pi)^2 . \tag{23}
\]

\( Q_s = \pi / \rho_0 \) with \( \rho_0 \) an atomic dimension lower roughness cut-off. Moreover, we assumed single valued roughness fluctuations \( h(R) \) of the in-plane position \( R = (x,y) \).

Furthermore, we consider the influence of plate roughness and temperature corrections on the Casimir force. We assume the same roughness for both plates, so that the Casimir energy is given by [21]

\[
E_{ct} = E_{cf} + \frac{1}{2} \left( \frac{2^2 E_{cf}}{\partial r^2} \right) 2 \int P(q) \langle |h(q)|^2 \rangle \, dq / (2\pi)^2 \]

(3)

with \( E_{cf} = -(\pi^2 hc/720r^3) \lambda_p \) the Casimir energy for flat perfectly conducting plates, and \( \langle |h(q)|^2 \rangle \) the roughness spectra \( \langle h \rangle = 0 \) of the plate surfaces. The scattering function \( P(q) \) for a finite plasmon wavelength \( \lambda_p \) (e.g., \( \lambda_p \approx 100 \text{ nm} \) for Al, 130 nm for Au, etc.) is given by the power law expressions [21]

\[
P(q) = \begin{cases} \text{if } d < \lambda_p : 0.4492dq & \text{for } q \gg 2\pi / d, \quad q \gg 2\pi / \lambda_p , \\ \left( 1/3 \right) dq & \text{for } 2\pi / d \ll q \ll 2\pi / \lambda_p , \\ \text{if } d > \lambda_p : (7\lambda_p / 15\pi)q & \text{for } q \gg 2\pi / d , \quad q \gg 2\pi / \lambda_p . \end{cases}
\]

(4)

It is also assumed that the optical response of the metallic plates is described by the dielectric function \( \varepsilon(\omega) = 1 - (\omega / \omega_p)^2 \) where \( \omega_p \) is the plasma frequency.

Since for example at \( T = 300 \text{ K} \) the thermal wavelength is \( \lambda_T \approx 7 \mu \text{m} \), while the finite conductivity corrections act strongly at separations \( \sim \lambda_p \ll 1 \mu \text{m} \) it means that conductivity and thermal corrections to the Casimir force are important in different distance ranges [23,24]. A deviation factor measuring a kind of interplay between finite conductivity taken simultaneously with temperature effects turns out to lie in the 1% range for metals used in experiments performed close to ambient temperature [24]. Thus, the conductivity and temperature corrections may be treated independently and multiplied for theory estimations above the 1% accuracy level [24]. Hence, we shall consider the temperature influence as a multiplying factor \( F^T \) that is given by [23].

\[
F^T(T,d) = \begin{cases} \frac{1 + \frac{720}{T^2}}{1 + \frac{(3)(1/3)}{2\pi}} & \text{if } K_B T d / h c < 1/2 , \\ \left( \frac{K_B T d}{h c} \right)^{\frac{1}{2\pi}} - \frac{1}{2\pi} & \text{if } K_B T d / h c > 1/2 . \end{cases}
\]

(5)

where \( \zeta(3) = 1.202 \) is the Riemann Zeta function. In reality Eq. (5) applies for low temperatures or \( T \ll T_{eff} \) \( (T_{eff} = h c / K_{Bf} (2d)) \) and high temperatures or \( T \gg T_{eff} \) ignoring any finite skin depth effects. The corrections to Eq. (5) are exponentially small or \( \sim \exp(-2\pi T_{eff} / T) \) at low temperatures and \( \sim \exp(-2\pi T / T_{eff}) \) at high temperatures [23]. Thus, the asymptotic regime is even achieved when the temperature is only \( T \approx 2T_{eff} \) [23].

Upon substitution of Eq. (4) into Eq. (3) taking the derivative and multiplying by Eq. (5) we obtain the total Casimir force \( F_{ct}(T,r) = -dE_{ct} / dr \) \( F^T(T,r) \). If we consider
the change of variables so that \( u = r/d, M = m/kT^2, \tau = t/T \) (\( T \) a characteristic time), \( \alpha = \pi^2h\lambda d/kd^3 \) and \( \beta = \lambda dV^2/kd^3 \) [15], the second law of Newton \( m(d^2u/dt^2) = |F_k| - |F_c| = |F_c| \) that describes the plate motion takes the more convenient form (with \( 0 < u < 1 \)).

\[
M \frac{d^2u}{dt^2} = f(u) = 1 - u - \frac{\beta R_e}{2u} - F_c(T, du).
\] (6)

In order to obtain the pull-in potential we set in Eq. (6) \( M(d^2u/dt^2) = 0 \) which yields

\[
f(u) = 1 - u - \frac{\beta R_e}{2u} - F_c(T, du) = 0
\] (7)

and considering also the first derivative of \( f(u) \) equal to zero

\[
\frac{df}{du} = -1 + \frac{\beta R_e}{u^2} \frac{df}{du} = 0.
\] (8)

Solution of Eqs. (7) and (8) yields the parameters \( a = a(u) \) and \( \beta = \beta(u) \) and as a result the pull-in voltage from \( \beta = \lambda dV^2/kd^3 \).

3. Results and discussion

The full Casimir force in Eq. (6) after taking into account temperature corrections is given by

\[
F_c(T, r) \simeq F_c(r) \left( 1 + \frac{2C_i}{r} \right) F^T(T, r)
\] (9)

with the roughness factor \( C_i \) given by

\[
C_i = \begin{cases} 
0.4492 f_{q_{1}, q} g(|h(q)|^2) \frac{d^2q}{(2\pi)^2} 
& \text{if } r < \lambda_p, \\
\int_{Q_{\text{r}}} f_{q_{1}, q} g(|h(q)|^2) \frac{d^2q}{(2\pi)^2} + \frac{4}{1+\frac{2r}{\lambda_p}} \int_{Q_{\text{r}}} f_{q_{1}, q} g(|h(q)|^2) \frac{d^2q}{(2\pi)^2} 
& \text{if } r > \lambda_p,
\end{cases}
\] (10)

where \( Q_{\text{r}} = 2\pi/\lambda_p \) and \( Q_r = 2\pi/r \). Upon substitution of Eq. (9) into Eqs. (7) and (8) we obtain the pull-in potential

\[
V_{\text{PI}} = V_{0}u\sqrt{2} \left[ \frac{1}{R_e} \left( 1 - u + \frac{W}{W} \right) \right]^{-1}
\] (11)

with \( W(u) = u^{-1}(1 + (2C_i/du)F^T(T, du) \). \( V_0 = \sqrt{kd^3/\lambda dV} \) and \( W = dW/du \).

Further calculations with Eqs. (6)–(11) require evaluation of the roughness factor \( C_i \) and therefore of the roughness spectrum \( \langle |h(q)|^2 \rangle \). A wide variety of surfaces and interfaces that appear in thin films grown under non-equilibrium conditions possess the so-called self-affine roughness [19]. In this case the roughness spectrum \( \langle |h(q)|^2 \rangle \) shows a power law scaling [19] \( \langle |h(q)|^2 \rangle \propto q^{-2-2H} \) if \( q \xi \gg 1 \) and \( \langle |h(q)|^2 \rangle \propto \text{constant} \) if \( q \xi \ll 1 \). This is satisfied by the analytical description [25].

\[
\langle |h(q)|^2 \rangle = 2\pi \frac{w^2}{(1 + aw^2)}^{1+H}
\] (12)

with \( a = 1/2H \{ 1 - (1 + aQ^2_{\xi}^2)^{-H} \} \) (0 < \( H < 1 \), \( 0 < H < 1 \), \( a = 1/2(1 + aQ^2_{\xi}^2) \) \( H = 0 \). Small values of the roughness or Hurst exponent \( H \rightarrow 0 \) characterize jagged or irregular surfaces; while large values of \( H \rightarrow 1 \) surfaces with smooth hills and valleys [19]. For other correlation models see also Refs. [19,26]. Eq. (12) yields for the average local slope \( \rho_{\text{rms}} \) the analytic form \( \rho_{\text{rms}} = (w/\sqrt{2\pi}) \times \sqrt{(1-H)^{-1}[(1 + aQ^2_{\xi}^2)^{-H} - 1]} - 2a \) [27], and therefore an analytic expression for \( R_t \) within the weak roughness limit \( \rho_{\text{rms}} < 1 \)

\[
R_t \simeq 1 + \frac{1}{2} \rho_{\text{rms}}^2 + \sum_{n=2}^{\infty} S(n) \rho_{\text{rms}}^{n-2}
\] (13)

with \( S(n) = [1 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot (2w - 3)]^{-1} w^{-1/2} \). Notably, the present roughness model shown by Eq. (12) as well as other roughness models [19,26] are valid within the nanometer range as a wide variety of nanoscale growth studies have shown in the past [19,25,26]. However, they cannot take into account the atomic structure of the solid/vacuum interface. The latter will have influence on the electrical properties (e.g., dielectric function) of the material coating for the switch plates, which has to be taking into account in more realistic system calculations [24].

Here, we should point out that we consider for the scattering function \( P(q) \) the power law regimes from which deviations occur for wave vectors \( q < 10^{-3} \) where \( P(q) \ll 1 \) [20]. On the other hand the roughness spectrum approaches the asymptotic limit \( \langle |h(q)|^2 \rangle \approx (2\pi)w^2z^2 \) for \( q \xi \ll 1 \) (as can also be seen by Eq. (12)) contributing less than the power law approximation as \( q \rightarrow 0 \). Fig. 1 shows calculations of the pull-in voltage vs. pull-in normalized separation \( u \) for various roughness exponents \( H \) and relatively large plate separation \( d = 2000 \text{ nm} \) (>> \( \lambda_p = 100 \text{ nm} \)). The roughness influence plays significant role on the pull-in voltage for separations approximately half the initial separation \( u \approx 0.5 \), while for large separations \( u \sim 1 \) where \( V_{\text{PI}} < 1 \) the roughness influence is negligibly small. More precisely from Eq. (11) it can be seen that the pull-in voltage is

![Fig. 1. Results of calculations of the pull-in voltage \( V_{\text{PI}} \) vs. \( u \) for \( \alpha_0 = 0.3 \text{ nm}, \lambda_p = 100 \text{ nm}, w = 5 \text{ nm}, \xi = 200 \text{ nm}, \) various roughness exponents \( H \) as indicated, plate separations \( d = 2000 \text{ nm} \) and \( T = 300 \text{ K} \).](image-url)
becoming zero $V_{\text{PI}} = 0$ for separations given by the complex equation $1 - u + W/W = 0$.

Figs. 2 and 3 show results of the calculations for the pull-in voltage $V_{\text{PI}}$ for various system temperatures and two different plate separations $d$. For the $d = 5000$ nm, the jump that appears at high temperatures (indicated by the arrow) is from the contribution of the high temperature branch in Eq. (5) for $K_B T (du)/\hbar c > 1/2$. However, with lower temperatures $T$ in Fig. 2 or lower separations $d$ as in Fig. 3, the system is still in the low temperature regime in or $K_B T (du)/\hbar c < 1/2$. Indeed, as is shown in Figs. 2 and 3 with increasing temperature $T$ the pull-in voltage increases for pull-in gap separations $u \sim 0.5$, whereas it decreases for larger separations $u > 0.7$. Fig. 4 shows the direct dependence of the pull-in voltage $V_{\text{PI}}$ as a function of temperature where a maximum appears for $T < T_{\text{eff}}$. In this case we have $T_{\text{eff}} = 458$ K ($u = 0.5$), while the maximum occurs for $T_{\text{max}} \approx 350$ K. The latter is however well above the temperature $T_u = 38$ K where the third- and fourth-order terms in Eq. (5) (low temperature branch) cancel each other or equivalently $[K_B T (du)/\hbar c]^3 (3)/2\pi - (45/\pi^2) [K_B T (du)/\hbar c]^3 = 0$.

If we compare Fig. 1 with Figs. 2–4 it becomes evident that plate roughness has a more pronounced effect on the pull-in voltage than the temperature, assuming that the system oscillates within the sub-micron and micron range. This is also indicated in Fig. 5 where for $u = 0.5$ we plot the pull-in voltage as a function of the roughness exponent $H$ assuming values $H > 0.3$ to stay within the weak roughness limit. The influence of temperature is rather weak for $d = 2000$ nm (Fig. 5a), while with increasing $d$ already at 5000 nm (Fig. 5b) the effect of temperature starts to become significant. The dominant roughness contribution arises from the electrostatic term, which gives for weak roughness the simpler form $V_{\text{PI}} \approx V_0 (1 - \rho_{\text{rms}}^2/4) u \nu\sqrt{2} \times \sqrt{1 - u + W/W} [1 + (2/u) W/W]^{-1}$. The latter explains the sensitive dependence on $H$, which originates from the sensitive dependence of the rms local slope on the roughness exponent $H [27]$.

Moreover, as Fig. 5a and b indicates the influence of the temperature is slightly higher at relatively large roughness exponent $H \sim 1$. The temperature influence is lower with increasing lateral correlation length $\zeta$ in comparison to that of the roughness exponent $H$ as Fig. 6 indicates. Therefore, short wavelength surface roughness as quantified by the roughness exponent can play a dominant role on the pull-in voltage. The latter indicates that the coating technology of MEM’s plates should take care of proper depositions conditions (e.g., deposition rates, substrate temperature and inclination, material, etc.) that lead for example to lower exponents where surfaces are rougher at short wavelengths and appear to suppress more effective thermal effects arising from radiation pressure. Of course higher temperatures will influence other mechanical parameters such as the spring constant $k$ and thus the pull-in potential since $V_0 \propto \sqrt{k}$. Higher temperatures could also alter the plate surface morphology (e.g., by surface diffusion, evaporation of material, etc.) and thus the operating parameters.
Temperature effects on the pull-in voltage in microswitches can be minimized by out of plane profile control. Indeed, three-dimensional finite element analysis (FEA) of radio frequency (RF) MEMS switches has shown that temperature reduction, from 300 to 233 K, causes an increase in pull-in voltage to values that could compromise the switch reliability as a result of charge build-up in the dielectric layer [28]. By proper designing of the membrane out-of-plane profile, it was possible to hold the pull-in voltage (for the operational temperatures) within acceptable values [28]. This design feature of RF MEMS switches could lead to reliable pull-in voltages in applications where large temperature variations take place (e.g., satellites and airplane condition monitoring) [28].

4. Conclusions

Our investigation was focused on the combined influence of random self-affine roughness and temperatures effects on the pull-in voltage in microswitches. It is shown that for separations within the micron or sub-micron range the roughness influence plays a dominant role, while temperature starts to show significant impact well above micron separations. The more significant influence of the temperature takes place at relatively large roughness exponent \( H \sim 1 \), while its influence is significantly lower with increasing lateral roughness correlation length \( \xi \). It also shown the pull-in voltage as a function of temperature shows a maximum for \( T < T_{\text{eff}} \).

Thermo-electro-mechanical (TEM) effects limit the design possibilities, e.g., of micrometer dimensions radio frequency (RF) switches and affects their reliability [28,29]. These effects are a result of joule heating generated at contact areas due to the current flow (characteristics of the contact interfaces, and other parameters characterizing a particular design). The latter significantly raises the switching temperature, which affects its electro-mechanical properties [29], and can also alter the plate surface morphology (e.g., by surface diffusion, evaporation of material, etc.) and thus operating parameters. Although these temperatures are below the materials melting temperatures, new designs are needed with low operating temperatures in order to increase their reliability [28,29]. In addition, vacuum related thermal fluctuations could have some impact on operating parameters at elevated temperatures as our study shows.

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