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The, B; Diercks, RL; Stewart, RE; van Ooijen, PMA; van Horn, [No Value]; van Horn, J.R.

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Digital correction of magnification in pelvic x rays for preoperative planning of hip joint replacements: Theoretical development and clinical results of a new protocol

B. Thea and R. L. Diercks  
Department of Orthopedic Surgery, University Medical Center Groningen, Hanzeplein 1, 9700 RB, Groningen, The Netherlands

R. E. Stewart  
Department of Biostatistics, Northern Center of Healthcare, Groningen, The Netherlands

P. M. A. van Ooijen  
Department of Radiology, University Medical Center Groningen, Groningen, The Netherlands

J. R. van Horn  
Department of Orthopedic Surgery, University Medical Center Groningen, Hanzeplein 1, 9700 RB, Groningen, The Netherlands

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The introduction of digital radiological facilities leads to the necessity of digital preoperative planning, which is an essential part of joint replacement surgery. To avoid errors in the preparation and execution of hip surgery, reliable correction of the magnification of the projected hip is a prerequisite. So far, no validated method exists to accomplish this. We present validated geometrical models of the x-ray projection of spheres, relevant for the calibration procedure to correct for the radiographic magnification. With help of these models a new calibration protocol was developed. The validity and precision of this procedure was determined in clinical practice. Magnification factors could be predicted with a maximal margin of error of 1.5%. The new calibration protocol is valid and reliable. The clinical tests revealed that correction of magnification has a 95% margin of error of −3% to +3%. Future research might clarify if a strict calibration protocol, as presented in this study, results in more accurate preoperative planning of hip joint replacements. © 2005 American Association of Physicists in Medicine. [DOI: 10.1118/1.1984293]

I. INTRODUCTION

Preoperative planning for hip joint replacement is considered an essential part of the total surgical procedure. It forces the surgeon to think three-dimensionally and improves surgical precision. In addition, it shortens the operation time and greatly reduces the incidence of complications.1–4 Possible complications are fractures of the femur due to use of too large components, great leg length differences and disturbance of biomechanical properties of the hip joint, leading to excessive joint contact forces and limping. It is not without problems, however, to make preoperative plans on analogue plain radiographs.5,6 The main reason is that the magnification factor of the projected hip joint on the x ray was not determined with sufficient precision. Most orthopedic surgeons assumed a standard magnification. The surgeons who tried to obtain a better estimate of the magnification used objects with known dimensions (like coins or prosthetic femoral heads) for calibration, but faced two problems: Accurate measurements on analogue radiographs were not possible with standard equipment, and the templates used in preoperative planning were only available in a very limited range of magnifications. Therefore, analogue planning was never a reliable method for deciding which component size had to be used.5,6 The ability to do so would add to the mentioned clinical advantages and provide a tool to control the stock of implants, having the potential of substantial cost reductions for hospitals and prosthesis manufacturers.

New methods of digital planning on digital plain pelvic radiographs have the potential to accomplish this. In order to correct for the magnification factor digitally, a spherical object with known diameter, is placed between the legs of the patient when making the plain pelvic radiograph. Knowledge of the diameter of the object allows the computer to calculate the magnification factor, or to use it for calibration of measurements and preoperative planning procedures.

The most important problem to overcome is correct positioning of the calibration object when making the radiograph. The calibration object should be positioned with the same distance to the x-ray source and plate as the patient’s own hip joint. This results in a preoperative pelvic radiograph including the projection of the calibration object. If it is accurately positioned, extrapolation of the calculated magnification to the hip joint is valid. For hip joint replacements in particular, this poses a difficult problem, since the position of the hip joint can only be estimated indirectly.

If the calibration object and the patient’s own joint are both located in the same plane parallel to the x-ray plate, but their distance to the center of the image is not the same, their magnification will differ and the calibration will not be optimal. A difference in magnification will also occur if they are...
The purpose of our study was to first model the projection of the calibration object, then to quantify the errors in correction of magnification when the calibration object and hip joint are not positioned similarly with regard to the x-ray source and plate, and finally to use this information to develop a clinical calibration protocol for preoperative planning of total hip arthroplasties.

II. MATERIALS AND METHODS

All measurements in this study follow the same principle. They are intended to measure the diameter of the x-ray projection of an object with known dimensions. This object is in fact a cobalt chromium prosthetic femoral head which is used in hip joint replacements. The part of the head, which articulates with the acetabular component when used in total hip arthroplasty, is part of a sphere with a diameter of 28 mm. It is commonly used as an object for digital calibration of pelvic x rays. This object is adjustable in height and connected to a metal ruler which is fixated in upright position to a base (see Fig. 1). The object is placed between the legs of the patient when making a plain pelvic radiograph. Because the real diameter of the spherical part of the object is known, the magnification factor can be calculated and used for calibration of measurements and preoperative planning of hip joint replacements (see Fig. 2).

Following clinical practice, both calibration and measurement of diameters are performed by means of three-point procedures: Digital markings are manually placed at three points on the outline of the calibration object. For calibration, the computer received input from the user about the real diameter of the calibration object (28 mm). Following this input, the surgeon is asked to place the three points on the outline of the calibration object. The computer constructs a circle with use of this input and uses it to calibrate the digital picture.

When measuring the diameter of the projected femoral head, the computer also uses three manually placed markings on the outline of the projection to construct a second circle. The computer then calculates the diameter of the second circle using the previous calibration as a reference.

A. Inter- and intra-observer precision of three-point measurements

Two main experimental setups were used for the measurements: In the first experiment magnification due to vertical
shift was investigated and only one calibration device was used. The calibration object was positioned on three different heights: 70, 120, and 170 mm. For each height three radiographs were made after manual repositioning of the x-ray source by radiological personnel. This resulted in a total of nine radiographs. Three different observers measured the projected diameter of the calibration object on each radiograph. This series of measurements was performed three times by the same observers. This resulted in a total of 81 measurements, with three distinct sources of possible variance per height group: Variance of measurements within observers, variance between observers, and variance due to repositioning.

To study the precision of the measurements, the differences due to the different heights were canceled out by stratification. They are of interest when investigating the validity of the models (Sec. II B 2 of this paper), and not when investigating the precision of the measurements.

In the second experiment the influence of horizontal shift on magnification was investigated. Two calibration devices were used for each radiograph. The distance between the centers of both calibration objects was either 60, 120, or 180 mm. For each distance three radiographs were made after manual repositioning of the x-ray source by radiological personnel. This resulted in nine radiographs. Three observers measured the projected diameter of the two calibration objects on each radiograph. This series of measurements was performed three times by the same observers. This resulted in a total of 162 measurements with three distinct sources of possible variance per distance group: Variance of measurements within observers, variance between observers and variance due to repositioning.

To study the precision of the measurements, the differences due to the different distances were canceled out by stratification. They are of interest when investigating the validity of the models (Sec. II C 2 of this paper), and not when investigating the precision of the measurements.

The standard deviation of the measured diameters was determined for each experimental setup. The relative contribution of each potential sources of variance (variance between observers, variance within observers and variance due to repositioning) was quantified using statistical analysis of variance techniques.

B. Mathematical model for vertical shift

1. Construction of the model

To model the changes in projection of a sphere due to vertical displacement (towards or away from the x-ray source) the following was considered: A spherical object is located between the apex of a cone and a plane. The spherical object fits exactly inside the cone. Furthermore the cone is directed in such a way that the axis is perpendicular to the plane.

The characteristics of the intersection are determined by the radius of the sphere \( r \), the distance between the apex of the cone and the center of the sphere \( y \), and the distance between the apex of the cone and the plane \( h \) (see Fig. 3).

\[
\begin{align*}

diameter &= 2\sqrt{r h} \\
Sqrt &= S\text{quare root. Details of the mathematical derivation can be found in the Appendix.)}
\end{align*}
\]

2. Validation

To validate the constructed model, the first experimental setup using only one calibration device was used as described in Sec. II A. The calibration object was positioned on three different heights: 70, 120, and 170 mm. The mean measured diameter of the projection at the different heights was used to compare with the predicted diameters by the model.

The model was then used to mimic a range of situations in which the patient’s femoral head and calibration object have different distances to the x-ray plate.

C. Mathematical model for horizontal shift

1. Construction of the model

To model the changes in projection of a sphere due to horizontal displacement (parallel to the x-ray plate) the following was considered: A spherical object is located between the apex of a cone and a plane. The spherical object fits exactly inside the cone. The starting point is the situation in which the cone is directed in such a way that the axis is perpendicular to the plane. Then the sphere is displaced parallel to the x-ray plate and the cone is redirected so that the axis of the cone still runs through the center of the sphere. The cone also becomes narrower because the sphere should...
still exactly fit inside the cone, but is now more distant from its apex. The characteristics of the intersection after horizontal displacement are determined by the radius of the sphere \( r \), the vertical distance between the apex of the cone and the plane \( h \), the vertical distance between the apex of the cone and the center of the sphere before displacement \( y \), and the amount of horizontal displacement \( x \) (see Fig. 4).

The effect of mere horizontal displacement can now be modeled as the change in characteristics of the intersection, when changing the position of the sphere parallel to the x-ray plate. This results in the following formula to calculate the length of the minor and major axis of the elliptical projection:

\[
\begin{align*}
\text{Minor axis} &= 2r\sqrt{h^2 - r^2} \\
\text{Major axis} &= 2r\sqrt{(x^2 + y^2 - r^2)} / (y^2 - r^2)
\end{align*}
\]

(Sqrt= square root. Details of the mathematical derivation can be found in the Appendix.)

2. Validation

To validate this model, the second experimental setup using two calibration devices was used as described in Sec. II A. The distance between the centers of both calibration objects was either 60, 120, or 180 mm. The x-ray source was centered on one of the two calibration objects. The diameters of the projection of both calibration objects were measured and compared with the predicted values. The model was then used to mimic a range of situations in which the patients femoral head (H) and calibration object (C) have different distances (1 and 2) from the center (O) of the radiograph (see Fig. 5).

D. Clinical tests

A calibration protocol was developed. To make the preoperative pelvic radiograph the patient was in a supine position. The legs of the patient were internally rotated up to 20 deg. As a consequence of this manipulation, a palpable bony structure, the greater trochanter is at level with the hip joint. This means that the radiographic imaging plate is equally distant from both the hip joint and the greater trochanter. The position of the greater trochanter was then determined by palpation and a 28 mm calibration object was positioned accordingly when making the pelvic radiograph. When the femur could not be internally rotated sufficiently the palpable part of the greater trochanter would be too close to the radiographic plate. In that case the calibration object was placed one to two centimeters higher than the palpable bony reference to compensate for this. When constructing a preoperative plan the most recent pelvic radiograph with calibration object was retrieved from the digital radiological archive. Three points were placed on the outline of the projected calibration object. Then the computer was given the input that the diameter of the circle which was constructed by the computer, using the three points on the outline, was 28 mm. This completed the calibration of the digital image.

In a clinical study we used this calibration protocol on 25 consecutive patients who were admitted for a hip joint replacement but already had a total hip prosthesis on the contralateral side. Preoperatively, the position of the prosthetic femoral head on the contralateral side was considered to be the best estimate of the future position of the prosthetic femoral head on the ipsilateral side. In the most desirable situation determination of the magnification factor with use
of the 28 mm calibration object leads to the same result as with use of the 28 mm femoral head of the implanted prosthesis. To achieve this the distance from the calibration object and the femoral head to the x-ray source should be equal as well as their distance to the x-ray plate. Calibration was performed on the femoral head (1) of the total hip prosthesis (which now served as the calibration object) and the diameter of the calibration object (2) (which now served as the object of which the diameter was to be determined) was measured (see Fig. 6). In the most desirable situation this would lead to a measured diameter of 28 mm.

### III. RESULTS

#### A. Inter- and intra-observer precision

The different contributions in absolute numbers and percentages are given in Table I. Remarkable is the relatively large contribution of repositioning to the total measurement variance. Overall this seems to be the largest source of variance, followed by variance between observers (inter-observer variance) and variance within observers (intra-observer variance).

In absolute numbers the variance due to repositioning is the only source which seems to have clinical relevance. The part of the standard deviations which it explains vary from 0.07 to 0.77 mm. Between observer variance explained a maximum of 0.08 mm of the total standard deviation, while this was 0.007 mm for within observer variance.

Our data did not show an association between precision and height or vertical displacement of the calibration object.

#### TABLE I. Precision of measurements.

<table>
<thead>
<tr>
<th>Source</th>
<th>Vertical shift</th>
<th>Horizontal shift object 1</th>
<th>Horizontal shift object 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70 mm</td>
<td>120 mm</td>
<td>170 mm</td>
</tr>
<tr>
<td></td>
<td>% mm</td>
<td>% mm</td>
<td>% mm</td>
</tr>
<tr>
<td>between</td>
<td>24.4 0.053</td>
<td>31.6 0.083</td>
<td>39.5 0.060</td>
</tr>
<tr>
<td>within</td>
<td>0.2 &lt;0.001</td>
<td>0.8 0.002</td>
<td>1.2 0.002</td>
</tr>
<tr>
<td>repos</td>
<td>40.1 0.088</td>
<td>27.2 0.071</td>
<td>46.5 0.071</td>
</tr>
<tr>
<td>unexpl</td>
<td>35.3 0.077</td>
<td>40.5 0.106</td>
<td>12.9 0.020</td>
</tr>
<tr>
<td>total</td>
<td>100.0 0.219</td>
<td>100.0 0.261</td>
<td>100.0 0.153</td>
</tr>
<tr>
<td>Source</td>
<td>60 mm</td>
<td>120 mm</td>
<td>180 mm</td>
</tr>
<tr>
<td></td>
<td>% mm</td>
<td>% mm</td>
<td>% mm</td>
</tr>
<tr>
<td>between</td>
<td>6.9 0.040</td>
<td>3.5 0.021</td>
<td>4.6 0.025</td>
</tr>
<tr>
<td>within</td>
<td>0.5 0.003</td>
<td>0.2 0.001</td>
<td>1.4 0.007</td>
</tr>
<tr>
<td>repos</td>
<td>87.9 0.516</td>
<td>93.6 0.565</td>
<td>74.2 0.396</td>
</tr>
<tr>
<td>unexpl</td>
<td>4.7 0.028</td>
<td>2.7 0.016</td>
<td>19.8 0.106</td>
</tr>
<tr>
<td>total</td>
<td>100.0 0.587</td>
<td>100.0 0.604</td>
<td>100.0 0.534</td>
</tr>
<tr>
<td>Source</td>
<td>60 mm</td>
<td>120 mm</td>
<td>180 mm</td>
</tr>
<tr>
<td></td>
<td>% mm</td>
<td>% mm</td>
<td>% mm</td>
</tr>
<tr>
<td>between</td>
<td>2.2 0.010</td>
<td>0.6 0.005</td>
<td>5.5 0.010</td>
</tr>
<tr>
<td>within</td>
<td>0.2 &lt;0.001</td>
<td>0.3 0.002</td>
<td>3.2 0.006</td>
</tr>
<tr>
<td>repos</td>
<td>91.6 0.404</td>
<td>97.8 0.767</td>
<td>64.0 0.116</td>
</tr>
<tr>
<td>unexpl</td>
<td>6.0 0.026</td>
<td>1.3 0.010</td>
<td>27.3 0.049</td>
</tr>
<tr>
<td>total</td>
<td>100.0 0.441</td>
<td>100.0 0.785</td>
<td>100.0 0.181</td>
</tr>
</tbody>
</table>

Note: Source=source of variance; between=between observer variance; within=within observer variance; repos=variance due to repositioning; unexpl=variance not explained by between observer, within observer or repositioning variance. For the experiment with vertical shift three different heights were used: 70, 120, and 170 mm. For the experiment with horizontal shift three different distances between the centers of the two calibration objects were used: 60, 120, and 180 mm. The numbers in the “%” columns represent the percentage of the variance explained by the different sources of variance. The numbers in the “mm” columns represent the absolute amount in millimeters of the standard deviations explained by the different sources of variance.
B. Vertical shift

The values of the measured and predicted projection sizes for the three different heights are given in Table II. When using the model to predict measurement errors in situations in which the patients femoral head and calibration object have different distances to the x-ray plate, each centimeter of vertical shift roughly corresponds with a 1% increase or decrease of magnification (see Fig. 7).

C. Horizontal shift

The values of the measured and predicted projection sizes for the three different heights are given in Table II. The differences between predicted and measured values are larger in the data of the horizontally shifted objects than for vertical displacement. When using the model to predict measurement errors in situations in which the patients femoral head and calibration object have different distances from the center of the radiograph, the maximum error in determination of the magnification factor of plain pelvic radiographs is approximately 3.5% (see Fig. 8). This is the case when the calibration object is exactly in the center of the radiograph, while the hip joint is projected in the corner of the image.

D. Clinical tests

The measured diameter of the calibration object was on average 99.85% of the diameter of the femoral head of the prosthesis preoperatively. The paired differences between the preoperative measurement and the postoperative measurement had a standard deviation of 1.53%. The standard error of the mean was 0.31%.

IV. DISCUSSION

A fast growing number of hospitals have digital radiological facilities (PACS—Picture Archiving and Communication System) nowadays. The precise implications on costs and changing usage have yet to become clear. Still, even without being able to oversee all consequences, it offers great advantages for manufacturing, storing, retrieving, and

![Fig. 7. The figure shows the predicted magnification difference of two identical spherical objects with a diameter of 28 mm, when they are at different vertical distances (heights). Curves are plotted for three hypothetical situations: When the lowest object is resting on the table (●), when the lowest object is at a height of 5 cm (○), and when the lowest object is at a height of 10 cm (▲).](image)
analyzing images.\textsuperscript{12–16} It allows the orthopaedic surgeon to construct digital preoperative plans on plain pelvic x rays for total hip arthroplasties. This demands accurate calibration to correct the magnification factor of the projection of the hip joint. This means accurate positioning of a calibration object, ideally at the same distance from the x ray film and source as the patient's own joint.\textsuperscript{6,17}

Gorski \textit{et al.} describe a method to determine the magnification factor, using a lead plate which was adjustable in height.\textsuperscript{18} The fundamentals of their protocol were not different from ours. However, the exact procedure they use, remains unclear and they do not measure the error associated with their technique.

It is common practice to place a metal femoral head between the legs of the patient as a calibration object, but no previous study has studied the validity and precision of this procedure. In this study we constructed and validated a model for projection of the most commonly used calibration object, which has a spherical shape to minimize the effects of radiographic image distortion. We modeled the projection as the intersection of a cone with a plane. The intersection will be elliptical if the object is not located exactly in the center of the x-ray beam. However, most major preoperative planning software packages make use of circles to determine the magnification factor, and we have accounted for this potential source of nonrandom error in the study design, as well as for sources of random error.

This study has provided data on the different sources of random error, which the clinician will have to bear in mind when calibrating digital radiographs. Although the contribution to measurement variability of inter-observer, intra-observer, and x-ray source repositioning variances could be quantified, it was not possible to quantify the variance due to \textit{patient} repositioning in this experimental setup. The error due to repositioning the x-ray source appears to be the most important—much more important than interobserver differences—and has induced errors of up to 2.7\% in our experiments. Together with the variances in patient positioning, this could explain why total hip arthroplasties during follow-up appear to have quite some variance in magnification. The use of digital edge detection could diminish the error caused by intra-observer and inter-observer variability, but has no effect on the greatest source of error: x-ray source positioning.

When using calibration objects, the differences due to x-ray source positioning variances are canceled out: For example, if the hip joint is projected “too large” because the x-ray source is positioned lower than in the standard setup, the calibration object is also projected equally larger. Unfortunately, differences due to variance in \textit{patient} positioning cannot be compensated for by any method. The magnitude of nonrandom error, due to the use of circles instead of ellipses for the calibration procedure, was also quantified. Although the magnitude of these errors (up to 1.5\%) are small in comparison with the repositioning errors, they are substantial and provide an argument to abandon the classical way of calibrating radiographs.

The models provided us with a tool to estimate the magnitude of errors we could expect when the calibration object was not in the same frontal or anteroposterior plane. This enabled us to develop a calibration protocol which we implemented in clinical practice. The models showed that differences between hip joint and calibration object regarding the distance to the center of the image result in an error less than 1\% as long as the difference is no more than 5 cm (which is a reasonable assumption in clinical practice).

The models made clear that horizontal malpositioning (difference in distance to the center of the radiograph) is—if not extreme—not responsible for large errors. However, it will certainly add to an already existing error caused by vertical malpositioning if the malpositioning is both in the frontal plane and in the anteroposterior direction. The models made clear that we had to be very cautious with the latter, and we considered how to handle this source of error. Using

![Fig. 8. The figure shows the predicted magnification difference of two identical spherical objects with a diameter of 28 mm, when they are at different horizontal distances from the center of the radiograph. Curves are plotted for six hypothetical situations, in which the most centrally located sphere of the two has a fixed distance (0, 5, 10, 15, 20, or 25 cm) from the center.](image-url)
no more than a plain pelvic radiograph, the position of the hip joint out of the plane of the radiograph can only be measured indirectly using bony landmarks like the greater trochanter, or estimated with use of anatomical data of the average femoral anteversion in this specific population. The greater trochanter is most readily palpable when the femoral anteversion (the angular difference between axis of femoral neck and the transcondylar axis of the knee) is neutralized by internal rotation of the leg. This way, the anteroposterior position of the hip joint can be estimated and used to position the calibration object. When making the plain pelvic radiograph, the same positioning of the patient is used.

On average, the degree of internal rotation should be as much as 20 deg (which is not always possible in this population with osteoarthritic hip joints), as recommended by Blackley et al. How much internal rotation exactly is needed to neutralize the femoral anteversion can only be estimated, using data obtained in previous studies. Several studies measuring femoral anteversion angles provide useful information. Measured average femoral anteversion varies from 10 deg±6.5 deg in cadavers and 9.8 deg±8.5 deg in a study using 200 reconstructed skeletons. However, this concerns data of a population with normal hip joints, while our population of interest suffers from osteoarthritis of the hip joints.

Osteoarthritic hip joints are known to have more femoral anteversion than normal hip joints. The measured average femoral anteversion varies from 17 deg to 20 deg ±9 deg. For both normal and osteoarthritic hip joints there is no difference between males and females. Bilateral differences can be expected to be quite large—up to 11.8 deg in 95% of a population with a normal distribution, so this information cannot be used for more accurate positioning of the calibration object.

When the lever arm of the palpable part of the greater trochanter to the center of rotation of the hip joint is known, it is possible to tell how big the error in correction of magnification will be with undercorrections or overcorrections of anteversion. Using the extensive data of Maruyama et al. we can estimate an average lever arm of approximately 8 cm. This rough estimate resulted from the data of the average medial offset of the femoral head in a derotated femur, the average shaft-neck angles, and the assumption that the distance between the anatomical axis of the femoral shaft and the palpable part of the greater trochanter is equal to the diameter of the femoral neck.

When using acetabular components, which are available in sizes with 2 mm variations in diameter, an error of 3%–4% would lead to a projection error as big as the difference between two consecutive sizes. With a lever arm of 8 cm, one may undercorrect or overcorrect the femoral anteversion up to 7.2 deg without introducing a structural error above 1%, which should be possible if the soft tissue layer allows easy palpation of the greater trochanter.

The clinical data we obtained after implementation of our calibration protocol were interesting in two ways. First of all, the positioning of the calibration object resulted on average in a close match with the magnification of the femoral head of the total hip prosthesis. The assumption that on average the object would be placed lower than the hip joint, because of insufficient endorotation in painful hips was, therefore, proven to be incorrect. Placing the calibration object one to two centimeters higher than the trochanter in patients with limited endorotation has probably compensated for this. The magnification of the calibration object and the contralateral total hip arthroplasty in situ was equal, with a standard error of the mean of 0.31%. This means that the mean difference between magnification of the calibration object on the preoperative radiograph and magnification of the prosthetic femoral head in situ on the postoperative measurements lies within a 95% confidence interval of −0.60% to +0.60%. The standard deviation of the differences was 1.53%. Thus the 95% reference range of the difference in magnification between the calibration object and the hip joint which has to be operated is −3.00% to +3.00%. Using the mathematical models we can translate this in a range of malpositioning in height from −3 to +3 cm, which reflects the margin of error in correction of magnification with the protocol. A possible explanation for this range of errors is that the greater trochanter is not always easily palpable, especially in obese patients. Another possible factor might be the variance in patient positioning. These data concerning the expected errors in calibration clarify that, despite the potential advantages over manual planning, digital preoperative planning brings on its own set of problems and demands great attention to the process of calibration.

In conclusion, it was possible to predict magnification factors for different positions of the calibration object with a maximal margin of error of 1.5%. A strict calibration protocol is necessary to create acceptable conditions for digital preoperative planning of total hip arthroplasties on plain pelvic x rays. We have developed and implemented an accurate and reliable calibration procedure. These first clinical results show that the orthopedic surgeon should expect errors in correction of magnification to be in the range of −3% to +3% using our protocol.

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**APPENDIX: MODELS FOR VERTICAL AND HORIZONTAL SHIFT**

The sphere can be taken as a Dandelin sphere (see Fig. 9); the top and bottom of the sphere will be the foci of ellipses similar to the projection in planes passing through those points, so the projections of those points on the image will in fact be the foci.

The formulas are worked out for the semiaxes of the ellipse, given Fig. 4, which shows two views of the setup with the source at A and the image at PQ.

We define the following lines:
- $x=$ horizontal shift DC
- $y=$ vertical distance from source AD
- $h=$ distance from source to plate AB
- $r=$ radius of sphere CS

$E$ is the point of intersection of segments AP and CD. Then triangles ADE and CRE are similar, with:

$$AD = y; \quad CR = r; \quad DE = u; \quad RE = v$$

So $u/v = r/y$, and $v = ur/y$.

But by Pythagoras $r^2 + v^2 = (x - u)^2$

so $r^2 + (ur/y)^2 = (x - u)^2$

Expanding, and multiplying by $y^2$, we have:

$$r^2y^2 + u^2r^2 = x^2y^2 - 2uxy^2 + ur^2y^2$$

Treating this as a quadratic in the unknown $u$,

$$(y^2r^2 - u^2)r^2 - (2xy^2)u + (x^2 - r^2)y^2 = 0$$

By the quadratic formula,

$$u = \frac{2x y^2 + - \sqrt{(4x^2y^2 - 4(y^2 - r^2)(x^2 - r^2)y^2)}}{2(y^2 - r^2)}$$

Now replacing $E$ with $E'$, the intersection of AQ and CD (extended), the same quadratic equation is found, so that the two solutions in fact give the horizontal distances from D to both $E$ and $E'$. And so the distance from $E$ to $E'$ is the difference:

$$u' - u = \frac{2ry \sqrt{y^2 + x^2 - r^2}}{y^2 - r^2}$$

But the major axis PQ satisfies $PQ/EE' = h/y$, so the major
The semiaxis is:

\[
PQ/2 = \frac{rh}{\sqrt{x^2 + y^2 - r^2}}
\]

For the minor semiaxis \( b = B'P' \), consider similar triangles \( A'R'C \) and \( A'B'P' \), which give the proportion \( r/b = y/A'P' \). And since \( A'P' = \sqrt{b^2 + h^2} \), we get:

\[
\begin{align*}
r \sqrt{(b^2 + h^2)} & = yb \\
r^2(b^2 + h^2) & = y^2b^2 \\
r^22h^2 & = (y^2 - r^2)b^2 \\
b = rh/\sqrt{(y^2 - r^2)}
\end{align*}
\]

Since the short axis is not dependent on the horizontal shift \( x \), the diameter of the projected circle when the sphere is located directly below the x-ray source can be calculated by the formula for the minor axis.

In conclusion:

After horizontal shift:

- Major axis = \( 2rh/\sqrt{(y^2 - r^2)} \)
- Minor axis = \( 2rh/\sqrt{(y^2 - r^2)} \)

After vertical shift:

- Diameter of circular projection = \( 2rh/\sqrt{(y^2 - r^2)} \)

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6Telephone: +3150-3610068; Fax: +3150-3611737. Electronic mail: b.the@orth.umcg.nl