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Buoyancy-driven flow in a peat moss layer as a mechanism for solute transport

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Transport of nutrients, CO₂, methane, and oxygen plays an important ecological role at the surface of wetland ecosystems. A possibly important transport mechanism in a water-saturated peat moss layer (usually Sphagnum cuspidatum) is nocturnal buoyancy flow, the downward flow of relatively cold surface water, and the upward flow of warm water induced by nocturnal cooling. Mathematical stability analysis showed that buoyancy flow occurs in a cooling porous layer if the system’s Rayleigh number (Ra) exceeds 25. The temperature difference of 10 K between day and night, a typical Ra value for a peat moss layer is 80, which leads to quickly developing buoyancy cells. Numerical simulation demonstrated that fluid flow leads to a considerable mixing of water. Temperature measurements in a cylindrical peat sample of 50-cm height and 35-cm diameter were in agreement with the theoretical results. The nocturnal flow and the associated mixing of the water represent a mechanism for solute transport in water-saturated parts of peat land and in other types of terrestrializing vegetation. This mechanism may be particularly important in continental wetlands, where Ra values in summer are often much larger than the threshold for fluid flow.

The upper part of a living mire consists of a sponge-like layer of predominantly moss species, the acrotelm (1), with a porosity above 95%. The green and brownish plants near the surface (Fig. 1) intercept light and fix CO₂. Further down, porosity above 95%. The green and brownish plants near the surface (Fig. 1) intercept light and fix CO₂. Further down, the porosity becomes 80, which leads to fast developing buoyancy cells. Numerical simulation demonstrated that fluid flow leads to a considerable mixing of water. Temperature measurements in a cylindrical peat sample of 50-cm height and 35-cm diameter were in agreement with the theoretical results. The nocturnal flow and the associated mixing of the water represent a mechanism for solute transport in water-saturated parts of peat land and in other types of terrestrializing vegetation. This mechanism may be particularly important in continental wetlands, where Ra values in summer are often much larger than the threshold for fluid flow.

In this paper, we investigate a mechanism for fluid flow in a water-saturated peat moss layer, which does not depend on capillarity or an external hydraulic pressure. During the night, the surface cools, leading to relatively cold water on top of warm water, and if the temperature drop is sufficiently large, the cold water sinks and the warm water rises. This type of flow is called buoyancy flow, and it implies convective transport of the heat and solutes carried with the water. Buoyancy flow often occurs as “cells” consisting of adjacent regions with upward and downward flow. We studied the phenomenon in a peat moss layer by means of a mathematical model, numerical simulation, and laboratory measurements.

Model Equations and Stability

The Mathematical Model. The model describes the heat flow in a water-saturated porous layer that undergoes periodic and sudden temperature changes at its surface.

The imposed surface temperature involves the model parameters ΔT, which is the temperature difference between “day” and “night,” and t₀, which is the duration of each of the two periods. Four parameters describe geometrical and physical properties of the layer: the thickness H, the thermal expansion coefficient of the fluid α, the thermal diffusivity of the layer D_eff, and the hydraulic conductivity K. Table 1 lists parameter values for the water-saturated peat moss layer used in the experiment.

The equations for heat transport and fluid flow, together with the boundary conditions, are given in Appendix B. Here, we briefly discuss the physical content of the model equations in dimensionless form. Dimensionless temperatures \( T \) are expressed in units \( \Delta T \) and lie between 0 and 1. Similarly, the time interval \( t₀ \) is used as the unit of time, and the distance \( \sqrt{D_{eff} t₀} \), as the unit of length. This length scale (~0.078 m) characterizes the distance over which a daily “temperature wave” penetrates by conductive heat transport. The dimensionless thickness \( h \) of the layer becomes

\[
h = \frac{H}{\sqrt{D_{eff} t₀}}.
\]

The dimensionless equation for heat transport,

\[
\frac{\partial T}{\partial t} = - \text{div}(Ra \, q \, T - \text{grad} \, T),
\]

describes the temperature change \( \frac{\partial T}{\partial t} \) as the result of convective transport \( Ra \, q \, T \) (containing the fluid velocity \( q \)) and conductive transport \(- \text{grad} \, T\). The constant \( Ra \) is the Rayleigh number, which is defined by

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\[ Ra = \frac{K_\alpha}{\rho c_p} \frac{\Delta T}{\sqrt{\frac{t_0}{D_{\text{eff}}}}} \]  

This dimensionless group of model parameters plays a crucial role in the presence or absence of buoyancy flow.

Water movement is described by using Darcy’s law for flow in a porous medium. The gravity term in this law contains the density of water, which is written as a linear function of the temperature (Eq. 9 in Appendix B). In this way, the buoyancy effect enters the model. Darcy’s law in dimensionless form then reads

\[ \tilde{q} = \text{grad} \, p - T \text{grad} \, e = 0, \]

where the unit vector \( \tilde{e} \) points upward. Finally, the Boussinesq approximation (9) implies that

\[ \text{div} \, \tilde{q} = 0, \]

which states that the density changes are so small that direct effects of the fluid expansion on the flow can be neglected. In the mathematical model, we also disregard the effect of temperature on the physical properties of the layer listed in Table 1.

A Ground-State Solution. A ground-state solution of the model Eqs. 2–5 is defined as a solution without fluid flow (\( \tilde{q} = 0 \)). Such a solution is characterized by a 1D temperature profile \( T(z, t) \). The no horizontal temperature differences, and heat transport takes place by conduction only. Fig. 2 shows the ground-state solution for a “block wave” in a thick layer (\( h = \infty \)). Because there is no fluid flow, there is a perfect symmetry between diurnal heating and nocturnal cooling.

During the cooling phase, a ground state is not necessarily stable, however. Cold surface water may start to fall, and warm water from below may start to rise. If the Rayleigh number of the system is sufficiently large, this process amplifies itself and leads to the formation of convection cells.

The Mathematical Stability Analysis. This analysis involves small perturbations of a ground-state solution (e.g., the solution in Fig. 2), which are either amplified or fade away (see Appendix C). The results in Fig. 3 show that a system with a large Rayleigh number, say 230, becomes unstable at a delay of 0.001 (which is 0.001 \( t_0 \approx 43 \) s) after a sudden temperature drop at its surface. The expected size of the buoyancy cells is \( \approx 0.2 \) (see the dashed curves in Fig. 3), which is \( \approx 16 \) mm (\( 0.2 \times \sqrt{D_{\text{eff}} t_0} \)). In systems with a smaller Rayleigh number, the time delay to the onset of flow is larger, and the cell size is larger as well.

The curves in Fig. 3 can be understood from the penetration of a “cooling front” into the layer. After a sudden temperature drop, the size of the top layer affected by the cooling increases with approximately \( \sqrt{t} \). Initially, a thin cold layer is easily stabilized by conduction and requires a large Rayleigh number to become unstable. If it is unstable, the buoyancy cells are small, at least at the onset of the flow. In the case of a smaller Rayleigh number, the cold top layer has to grow longer before buoyancy flow sets in. For a very thick layer (\( h = \infty \)), the Rayleigh number must exceed 18 to get fluid flow before the end of the night. Fluid flow in a finite layer requires a larger Rayleigh number. For \( h = 2 \) (\( 16 \) cm), the threshold is 32 (see Fig. 3), and for \( h = 1 \), the threshold is 44 (curves not shown). For \( Ra \) values below these thresholds, the cooling front reaches the bottom of the layer before the flow sets in, and cooling continues without fluid flow.

For the parameter values in Table 1, we find \( Ra \approx 83 \), which is sufficiently large. For this \( Ra \), Fig. 3 predicts a time delay to the onset of flow of 0.015, which is \( \approx 11 \) min (0.015 \( \times 43200 = 648 \) s).

**Table 1. Parameter values for the water-saturated peat moss layer used in the experiment**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta T )</td>
<td>10</td>
<td>Kelvin</td>
<td>Surface temperature difference between day and night&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>43,200</td>
<td>Second</td>
<td>Half a day</td>
</tr>
<tr>
<td>( H )</td>
<td>0.15</td>
<td>Meter</td>
<td>Typical acrotelm thickness is 0.1–0.4 m&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( 15 \times 10^{-5} )</td>
<td>K&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>For water between 10°C and 20°C</td>
</tr>
<tr>
<td>( D_{\text{eff}} )</td>
<td>( 1.4 \times 10^{-7} )</td>
<td>m&lt;sup&gt;2&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>Quotient between thermal conductivity and thermal capacity of water at 10°C</td>
</tr>
<tr>
<td>( K )</td>
<td>0.1</td>
<td>m&lt;sup&gt;2&lt;/sup&gt;s&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>Based on measured values&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup>See Discussion.

<sup>b</sup>See ref. 4.

<sup>c</sup>See ref. 8.

<sup>d</sup>See Appendix A.

The corresponding Rayleigh number (see Eq. 3) is 83. Note that \( \alpha \) is expressed as a relative density change per unit temperature.

**Numerical Simulation**

The stability analysis above provides us with conditions for convective flow, but it does not predict the further development of the convection cells or their final fate. Therefore, we carried out numerical calculations based on the 2D equivalent of Eqs. 2, 4, and 5. The methods used are listed briefly in Supporting Text, which is published as supporting information on the PNAS website.

The arrows in Fig. 4 show a simulated flow field \( \tilde{q}(x, z, t) \) at several stages of its development. These arrows are only local velocities, however, and to follow the moving water, we tracked the position of notional “colored dust particles” carried around by the fluid and with initial positions indicated in Fig. 4A. The grey background of the four images depicts the simulated temperature distribution. A movie of the simulation is available.
as Movie 1, which is published as supporting information on the PNAS web site.

Using $Ra\sqrt{\Delta\text{eff}/t_0}$ as velocity scale (with $\Delta\text{eff}$ and $t_0$ from Table 1), we can describe the results as follows. Fig. 4A shows a “daytime” situation without flow at $\sim$8 h before the temperature drop. Buoyancy flow started within a few minutes after the temperature drop, which is in accordance with Fig. 3 for $Ra = 100$. The cells rapidly grew in height, and after 2 h they almost reached the bottom of the layer. After 4.4 h (Fig. 4B), the fluid velocity reached a maximum of 2.5 mm s$^{-1}$ in the middle of the regions with downward flow. Then the flow gradually slowed down, and toward the end of the night, the buoyancy cells almost disappeared (Fig. 4C).

The decay of the cells is caused by the absence of a heat source at the bottom of the layer. When the entire layer has cooled down (see the grey background of Fig. 4C), there is no temperature difference anymore to keep the flow going.

The visual impression given by the simulation is that the buoyancy cells turn around once every night. The continuously changing flow field also leads to mixing, however. This is illustrated by Fig. 4D, showing the system at precisely 4 d after the situation of Fig. 4B. Flow field and temperatures are the same, but a considerable mixing of the dust particles has taken place.

Analysis of Measured Temperatures

Laboratory measurements have been carried out in a climate chamber by using a cylindrical sample of 50-cm height and 35-cm diameter. During the sampling in the field, the spongy top layer inevitably got disturbed, but after a few weeks in a growth chamber, the peat moss had restored itself. Radial heat transport was minimized by 10 cm of insulating foam around the cylinder.

We applied a temperature difference of 10 K between “days” and “nights” of 12 h. Temperatures were measured with a vertical array of thermocouples installed at eight depths between 1 and 99 mm. Details are provided in Supporting Text. Fig. 5A–C show daily temperature cycles measured at three consecutive horizontal positions of the thermocouple array. At each position, temperature recording was started 1 or 2 d after the installation of the thermocouples.

The temperatures in Fig. 5D–F have been calculated with the above-mentioned 2D model for the three locations indicated in Fig. 4. The convective flow in the simulated system indeed caused the expected asymmetry between diurnal heating and nocturnal cooling (see Fig. 2). The differences between the cooling patterns in Fig. 5D–F are caused by the differences in direction and size of the nocturnal flow in the model system.

The asymmetry between heating and cooling is also visible in the measured cycles in Fig. 5A–C. The cooling curves lie much closer together than the heating ones, and there are also signs of fluid flow (marked by arrows). In Fig. 5B and C, for instance, the temperatures between 25 and 99 mm practically coincide between 1 and 3 h after the temperature drop. This suggests upward flow of relatively warm water, similar to the simulated upward flow causing the temperatures of Fig. 5F.

In the simulated system, the buoyancy cells gradually decay after $\sim$4 h (see Fig. 4). Both in the measured and simulated temperature profiles, we indeed see that the cooling curves “normalize” toward the end of the night. They describe again a neat temperature gradient without the bumps and coinciding temperatures of the first half of the night.

As a consequence of the fluid flow is that nocturnal cooling takes place more efficiently than diurnal heating. Hence, starting at
Discussion

A simplification made in the model is the use of an insulated lower boundary. Below a real peat moss layer, there is a denser zone (the catotelon) in which conductive heat flow will occur. This dense zone will not have much effect on the onset of flow, however, which is caused by the instability of a cooling top layer. Furthermore, conductive heat flow from below cannot keep the same temperature everywhere, the nocturnal heat loss from the layer will be larger than the diurnal heat gain, which means that we have a heat pump. The average temperature of the layer will decrease until nocturnal loss and diurnal gain balance each other.

We indeed find this decrease. For each of the measured curves in Fig. 5A–C a 24-h average temperature has been calculated. Fig. 6 shows the deviation of these averages from the average surface temperature, together with curves for the simulated system. The points for Fig. 5A and B clearly show a decrease with depth, which is somewhat larger than the decrease in the simulated system. The points for Fig. 5C (open circles) reach a plateau at about −0.7 K. The reason is probably that daily averages were still decreasing in the lower part of the layer.

We finally mention a noticeable small effect for upward flow locations. The predicted average temperature just below the surface slightly exceeds the surface average (see the curve for upward flow in Fig. 6).

Quantitative deviations between observed and calculated temperatures in Fig. 5 may be caused by the absence of dispersion in the model, the heterogeneity of the peat moss layer, the more gradual surface temperature changes in the experiment, and the limitations of Darcy’s law. There is a good qualitative agreement between the measured and calculated temperature patterns, however, and the decrease of the average temperature with depth fits surprisingly well (Fig. 6).

From the consistency among the mathematical stability analysis, the numerical calculations, and the cooling patterns observed, we conclude that buoyancy flow in a peat moss layer will occur provided the Rayleigh number of the system is sufficiently large.

The layer thickness \( H \) is of little importance for the stability of the system as long as the dimensionless thickness \( h \) exceeds a value of about 2 (see Fig. 3). The reason is the fast decrease of the amplitude of temperature waves with depth (Fig. 2). There is little difference between a layer with \( h = 2 \) (~16 cm) and a semi-infinite system.

Temperature changes in the field will usually be more gradual than the sudden transitions in Figs. 2 and 3. A similar analysis for a sinusoidal temperature wave is shown in Fig. 8 in Appendix C. The Rayleigh number required for fluid flow is 25, which is somewhat higher than the threshold of 18 for a block wave. Numerical simulation for \( h = 1.92 \) again shows significant mixing for \( Ra = 100 \) (with an average fluid displacement 14 cm per day; see Fig. 4). For a Rayleigh number of 50, however, the displacement is <1 cm per day, and there will be little mixing. Movies of these simulations are available as Movies 2 and 3, which are published as supporting information on the PNAS web site.

Significant mixing apparently requires Rayleigh numbers well above the threshold for instability. Rayleigh numbers in the field will vary with the values of three parameters: the hydraulic conductivity \( K \) of the layer; the thermal expansion coefficient \( \alpha \), which increases almost linearly with the (average) temperature in degrees Celsius (see appendix F of ref. 8); and the temperature drop \( \Delta T \).

The hydraulic conductivity of 0.1 m s\(^{-1}\) in Table 1 is a conservative estimate for the peat moss layer in our experiment (see Appendix A). We have used this value, which is also typical for coarse-grained materials like gravel (10), to calculate Ray-
leigh numbers from the daily minimum and maximum temperatures available from the National Oceanic and Atmospheric Administration. For ~1,400 weather stations at latitudes above +50° and altitudes below 1,000 m, we calculated the fraction of days in June and July with Ra above 100. A graph of this probability as a function of latitude is available as Fig. 9, which is published as supporting information on the PNAS web site (see also Supporting Text on the calculation of Ra). For coastal stations in Europe and Alaska, the probability is <10°C, which implies that buoyancy flow will be rare. For many continental weather stations in Alaska, Canada, and Russia, however, summer Rayleigh numbers are above 100 at 30–60% of the days and above 140 at 20–40% of the days.

The relation between a difference in air temperature and a difference in water surface temperature is not straightforward, due to effects of radiation, both by day and night. The daily temperature differences in huge continental areas, however, seem large enough to cause frequent nocturnal buoyancy flow.

The main ecological consequence of this flow is solute transport and mixing of the water. This mixing will be enhanced by dispersion and diffusion and, at the time scale of a few days, the peat moss layer can be considered well mixed.

Reeve and colleagues (11, 12) describe the “shallow-flow” and “ground-water” flow hypotheses of peatland hydrology. Clearly, the dominant transport process at a landscape scale is the flow of water, which is driven by rainfall and evaporation. The buoyancy flow described here operates at the much smaller scale of a water-saturated acrotelm, but there it may play an important role. Besides nutrients and oxygen, also dissolved organic compounds and CO2 formed by aerobic decay and methane oxidation (13–15) will be carried by the water.

Because buoyancy flow takes place only at night, the solute concentrations involved will show periodic behavior. Periodic behavior of the pH has indeed been observed in the hollows of a mire (16, 17), but resolving the roles of fluid flow and peat moss physiology will probably require new field measurements.

We studied nocturnal buoyancy flow in saturated peat moss, having in mind the hollows of a mire. In the vast continental wetland areas of the world, many other types of terrestrializing vegetation occur in which nocturnal buoyancy flow may play a significant ecological role, either as a mixing mechanism or perhaps even as a heat pump.

Appendices

A. Hydraulic Conductivity. We measured the hydraulic conductivity using the constant head method (10). We carefully packed green peat moss into a transparent column with a length of 100 cm and a diameter of 9 cm. The hydraulic head over a distance of 61 cm was measured by means of open piezometer tubes. We plotted the measured flux density as a function of the hydraulic head and fitted a parabolic curve (y = ax – bx2) to the data. The slope of this curve at the origin (the fitted a value) has been taken as the hydraulic conductivity. For the maximum velocity of 0.25 cm s–1 occurring in the simulated buoyancy cells (Fig. 4), the observed nonlinearity corresponds to a decrease in apparent conductivity of ~10%, which seems unlikely to cause important effects.

We measured conductivities of 25.6 ± 1.8 cm s–1 (after packing), 17.3 ± 1.5 cm s–1 (3 d later), and 16.6 ± 3.5 cm s–1 (after another 11 d). The dry matter content of the column (determined afterwards) of 5.2 liter–1 is within the range of 3–10 liter–1 reported on page 39 of ref. 18 for a green and growing peat moss layer. A graph of the measurements can be found as Fig. 10, which is published as supporting information on the PNAS web site.

Because the conductivity of a peat moss layer obviously does not exist, we have put the order of magnitude of 10 cm s–1 in Table 1.

Note that Poiseuille flow through the tube is more than four orders of magnitude larger than the flow rates observed, which implies that the peat moss represented a considerable resistance.

B. Scaling the Model Equations. We consider a horizontal porous layer of thickness H, which is saturated with water. The layer has a porosity or (close to unity in our case) and the properties mentioned in Table 1. The temperature, the velocity of the water, and the pressure are denoted by T = T(x, y, z, t), q = q(x, y, z, t), and p = p(x, y, z, t), respectively. The equation for heat transport is given by

\[ \phi \frac{\partial T}{\partial t} + \text{div}(\dot{q}T - D_{\text{eff}} \text{grad} T) = 0. \]  

For the velocity \( \dot{q} \), we have Darcy’s law,

\[ \dot{q} = -\frac{\kappa}{\mu} (\text{grad} p + \rho(T)g\hat{e}_z), \]

where \( \mu \) denotes the viscosity, \( \kappa \) the permeability; \( \rho(T) \), the temperature-dependent density; and \( g \), the gravity constant. Furthermore, we have the fluid mass balance, which has the form

\[ \phi \frac{\partial p}{\partial t} + \text{div}(\dot{q} \rho) = 0. \]

The properties \( \phi, D_{\text{eff}}, \kappa, \) and \( \mu \) in refs. 6 and 7 are assumed to be independent of the temperature \( T \). Buoyancy effects enter the Darcy equation by substituting the equation of state

\[ \rho(T) = \rho - \alpha p(T - \tilde{T}), \]

where \( \alpha \) is the relative density change per unit temperature; \( \tilde{T} \), the offset temperature; and \( \rho \), the density of water at \( \tilde{T} \). Although this temperature dependency causes the buoyancy effects, the actual volumetric changes are disregarded, so that Eq. 8 reduces to \( \text{div} \dot{q} = 0 \) (the Boussinesq approximation).

The boundary conditions for a 2D setting are given in Fig. 7. The ground states analyzed are given in Appendix C and differ with respect to the surface temperature \( P(t) \) and the type of layer (finite or semi-infinite).

The coordinates are scaled by means of the process length \( \sqrt{D_{\text{eff}} t_0} \). The deviation of the temperature from the offset temperature \( \tilde{T} \) is scaled by \( \Delta T \), the velocity, by \( K_\alpha \Delta T \) (using \( K = \frac{\rho g e}{\mu} \)), the pressure, by \( (p + \rho g e)/(\Delta T \rho ge) \sqrt{D_{\text{eff}} t_0} \); and finally the time, by \( \phi \tilde{T} \). Note that the velocity scale can also be written as \( Ra \sqrt{D_{\text{eff}} t_0} \). The derivation of the dimensionless Eqs. 2 and 4 is then straightforward.

C. Linear Stability Analysis. There have been many studies of the linear stability (19) of flows in fluid layers and (fluid-saturated)
porous layers subject to time periodic boundary conditions (20–24). For a problem similar to ours, the results of linear stability analysis, the energy method, and a numerical model are close together (23).

Following Van Duijn et al. (25), we sketch the linear method we used. Infinitely small perturbations, vanishing at the boundaries of the flow domain, are superimposed on the ground-state solution. The perturbed ground state is substituted in the model equations, and higher-order terms are neglected. We further assume that the perturbations are periodic in the horizontal plane with wavenumber $a$, and that they grow exponentially in time. The equations lead then to a fourth-order eigenvalue problem. Solving this problem yields, for each moment in time, a Rayleigh number $R_0(a, t)$, at which the perturbations are “just not amplified” (neutrally stable). For each $t$, the $R_0(a, t)$ has a minimum $R_0^{\text{crit}}(t)$ for the least stable perturbations with cell size $\pi/\sqrt{a^2}$ (24). These are the quantities shown in Figs. 3 and 8.

The curves in Fig. 3 refer to ground states for a square wave (with day and night of equal length $t_0$) at the surface of a semi-infinite layer (eq. II.20 in ref. 26), and a finite layer (eq. III.21 in ref. 26). For a sinusoidal temperature wave in a half-infinite layer, the ground state given by eq. II.8 in ref. 26 leads to the stability graph in Fig. 8.

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