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## Tracing Value-Added and Double Counting in Gross Exports: Comment<sup>†</sup>

By BART LOS, MARCEL P. TIMMER, AND GAAITZEN J. DE VRIES\*

*In a recent contribution to the AER, Koopman, Wang, and Wei (2014) proposed a decomposition of a country's gross exports into value-added components and double-counted terms. It is motivated by complex manipulation of basic accounting identities. In this comment we provide an alternative framework based on "hypothetical extraction." This parsimonious approach provides a clear definition of domestic value added in exports and has a natural extension into decompositions of bilateral export flows. (JEL E01, E16, F14, F23, L14)*

To trace a country's participation in global supply chains, new empirical measures of trade are needed. In a recent contribution to the *AER*, Koopman, Wang, and Wei (2014)—henceforth, KWW—proposed an accounting framework that decomposes a country's gross exports into nine terms that are grouped into domestic value added, foreign value added, and double-counted terms. The latter terms arise due to two-way trade in intermediate inputs. Their framework is grounded in manipulation of basic accounting identities regarding inputs, outputs, and value added. While mathematically correct, the derivation and interpretation of the terms appears to be overly complex. In this comment we provide an alternative measurement framework for this decomposition, based on "hypothetical extraction," a parsimonious mathematical technique based on an input-output representation of the global economy. This approach has a clear economic intuition and can be easily taken to the data. It compares actual GDP in a country with hypothetical GDP in case there are no production activities related to exporting. The difference is defined as domestic value added in exports. We show how domestic value added thus defined can be derived in a multi-country setting, and that it equals KWW's measure of domestic value added. We further show how domestic value added can be decomposed into other terms suggested by the KWW decomposition, encompassing the concept of "value-added exports" introduced by Johnson and Noguera (2012). In Section II we

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prove that domestic value added in exports can be calculated on the basis of data from national input-output tables only, justifying the approach of Hummels, Ishii, and Yi (2001). This finding is useful for empirical analyses as national tables are more abundantly available than global tables. In Section III we show that our “hypothetical extraction” approach provides a natural extension into decompositions of bilateral export flows. This underlines its potential to serve as a stepping stone for further research of value-added flows in international trade.

### I. A Decomposition of Gross Exports Based on “Hypothetical Extraction”

In the spirit of KWW, we will decompose the value of gross exports of a country into four main elements. These four elements are combinations of the nine terms given in the KWW decomposition and as such are not new. But in contrast to KWW their derivation is more straightforward and intuitive as we make use of a simple, yet powerful mathematical technique called “hypothetical extraction,” initially developed by Paelinck, de Caebel, and Degueldre (1965) and Strassert (1968).<sup>1</sup> We introduce a new variant of this approach to answer the question of how much domestic value added is included in a country’s exports. This is done by computing value added in a hypothetical world economy, which resembles the input-output structure of the actual world economy but with some trade flows set to zero. Basically, in this hypothetical economy some trade linkages between countries are “extracted.” By comparing value added in the actual and the hypothetical world economy, a country’s value added associated with the extracted linkages can be measured. We then define domestic value added (DVA from now onwards) in exports of a country  $s$  as the difference between actual and hypothetical GDP in  $s$ . We show in a second step how by judicious choice of extracted linkages, this domestic value added can be further decomposed into meaningful terms, which all have a counterpart in the KWW decomposition.

Following the notation of KWW as closely as possible, we partition the global input-output table such that we have a country  $s$  and a region  $r$  containing all other countries in the world, and construct a matrix  $\mathbf{A}$  as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{A}_{sr} \\ \mathbf{A}_{rs} & \mathbf{A}_{rr} \end{bmatrix}.$$

$\mathbf{A}$  contains the input coefficients  $a_{ij}$ , which give the value units of intermediate goods from industry  $i$  required to produce one value unit of gross output in industry  $j$ .  $\mathbf{A}_{ss}$  represents the domestically purchased requirements of industries in country  $s$ , while  $\mathbf{A}_{sr}$  gives the requirements by industries in  $r$  of products bought from industries in  $s$ .<sup>2</sup> For the final demand block, we can write similarly

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{y}_{sr} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} \end{bmatrix},$$

<sup>1</sup> See Miller and Lahr (2001) for an overview of subsequent extensions.

<sup>2</sup> The numbers of rows and columns in each of the matrix-blocks equal the numbers of industries in  $s$  and  $r$ . Note that these do not have to be equal as multiple countries may be included in region  $r$ .

in which the vectors  $\mathbf{y}_{ss}$  and  $\mathbf{y}_{sr}$  represent the values of flows from industries in country  $s$  to all domestic final users and to final users in  $r$ .<sup>3</sup>

Ratios of value added to gross output in industries in country  $s$  are contained in a row vector  $\mathbf{v}_s$ . The length of this vector equals the numbers of industries in  $s$  and  $r$ , with value-added ratios for industries in  $s$  as first elements ( $\tilde{\mathbf{v}}_s$ ) and zeros elsewhere:  $\mathbf{v}_s = [\tilde{\mathbf{v}}_s \ \mathbf{0}]$ . Using Leontief's insight, actual value added in country  $s$  ( $GDP_s$ ) equals

$$(1) \quad GDP_s = \mathbf{v}_s(\mathbf{I} - \mathbf{A})^{-1} \mathbf{Y}\mathbf{i},$$

in which  $\mathbf{i}$  is a column vector where all elements are unity, implying that it sums the two elements in each of the rows of the matrix  $\mathbf{Y}$ . The element  $(\mathbf{I} - \mathbf{A})^{-1}$  is the well-known Leontief inverse, in which  $\mathbf{I}$  is the identity matrix of appropriate dimensions.<sup>4</sup>

What amount of domestic value added should be attributed to exports from  $s$ ? To measure this we create a hypothetical world in which  $s$  does not export anything to  $r$ , while leaving the rest of the economic structure of the world unaffected.<sup>5</sup> That is, blocks  $\mathbf{A}_{sr}$  and  $\mathbf{y}_{sr}$  are set to zero. Hence, we define the matrices  $\mathbf{A}^*$  and  $\mathbf{Y}^*$  as  $\mathbf{A}^* = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{0} \\ \mathbf{A}_{rs} & \mathbf{A}_{rr} \end{bmatrix}$  and  $\mathbf{Y}^* = \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{0} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} \end{bmatrix}$ . Hypothetical GDP in  $s$  in this situation can be obtained by post-multiplying the hypothetical Leontief inverse with hypothetical final demand as

$$(2) \quad GDP_s^* = \mathbf{v}_s(\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{Y}^*\mathbf{i}.$$

Following the logic of hypothetical extraction, domestic value added in exports (DVA) from country  $s$  is derived as the difference in GDP in the actual and hypothetical situation,

$$(3) \quad DVA_s = GDP_s - GDP_s^*.$$

In the Appendix, we show that our measure of DVA in exports is equal to the one given by KWW in their equation (37), derived by manipulation of basic accounting identities. While in the end leading to the same result, our approach has a clean interpretation from an input-output perspective and provides a clear framework for further analysis as shown in the remainder of this comment.

Following the logic of our approach, we can further decompose DVA in exports into three elements that have a counterpart in the KWW decomposition. We first split it into a part that is absorbed by final users abroad,  $DVA(A)$ , and a part that

<sup>3</sup>Final use includes all final demand categories such as household consumption, public consumption, and gross fixed capital formation.

<sup>4</sup>When multiplied with final demand, the Leontief inverse calculates the gross output in the industries producing the final products, but also output in industries producing the intermediate inputs required for this. This interpretation is only valid if the Leontief inverse is post-multiplied with exogenously given final demand levels, as in (1).

<sup>5</sup>As rightfully pointed out by a referee, this is a partial equilibrium exercise with stringent assumptions such as zero substitution elasticities of inputs in production. A fully specified modeling exercise would require a general equilibrium setting with varying unilateral costs of exporting from A to B, in which one might determine the elasticity of A's GDP to exports from A to B.

is returned and absorbed at home. The former is known as “value-added exports,” a concept introduced by Johnson and Noguera (2012).<sup>6</sup> To calculate  $DVA(A)$ , one should only extract a part of the final demand matrix and keep the original intermediate input matrix as is. Hypothetical final demand in this case ( $\mathbf{Y}^{**}$ ) is specified as if there is no demand for final products in country  $r$ , neither for domestically sourced products nor for products purchased abroad. Thus we set both  $\mathbf{y}_{rr}$  and  $\mathbf{y}_{sr}$  to zero such that  $\mathbf{Y}^{**} = \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{0} \\ \mathbf{y}_{rs} & \mathbf{0} \end{bmatrix}$ . Next,  $DVA(A)$  of  $s$  can be derived by subtracting hypothetical GDP from actual GDP, as in (3). Hypothetical GDP is computed by post-multiplying the actual Leontief inverse with hypothetical final demand,

$$(4) \quad DVA(A)_s = GDP_s - \mathbf{v}_s(\mathbf{I} - \mathbf{A})^{-1} \mathbf{Y}^{**} \mathbf{i}.$$

$DVA(A)$  measures domestic value added induced by foreign final demand and is smaller than  $DVA$ , as it excludes the domestic value added reflected back to the home country, defined residually as  $DVA(R)_s = DVA_s - DVA(A)_s$ . The difference between  $DVA$  and  $DVA(A)$  can be sizable and both measures are useful in analyses of international trade.<sup>7</sup>

Following KWW, we can further decompose  $DVA(A)$  into a part that is related to exports of final products  $DVA(A, Fin)$ , and a part that is related to exports of intermediate products  $DVA(A, Int)$ . The first can be derived through hypothetical extraction by only extracting  $\mathbf{y}_{sr}$  from final demand and keeping the original intermediate input coefficients matrix,

$$(5) \quad DVA(A, Fin)_s = GDP_s - \mathbf{v}_s(\mathbf{I} - \mathbf{A})^{-1} \mathbf{Y}^* \mathbf{i}.$$

$DVA$  in exports of intermediates absorbed abroad can be derived residually as  $DVA(A, Int)_s = DVA(A)_s - DVA(A, Fin)_s$ . For countries mainly operating in upstream parts of global production networks, such as natural resource exporters, this term will be large.

Based on our definitions, we can provide a complete decomposition of exports using hypothetical extraction. Let  $E_s$  be the gross value of exports<sup>8</sup> from  $s$  then

$$(6) \quad E_s = [DVA(A, Fin)_s + DVA(A, Int)_s + DVA(R)_s] + RES_s,$$

where the first three elements on the right-hand side are defined above and sum to  $DVA$ . Each element has a clear correspondence to the nine terms in KWW’s decomposition given in their equation (36). They correspond to term 1, the sum of terms 2 and 3, and the sum of terms 4 and 5, respectively. The last element on the right-hand side of (6) is a residual that remains after subtracting  $DVA$  from gross exports and is therefore equal to the sum of remaining terms in the KWW decomposition (terms 6

<sup>6</sup>The name of this concept is potentially confusing and we propose to refer to it as “domestic value added absorbed abroad,” such that it is clearly a subset of the “domestic value added in exports” defined here.

<sup>7</sup>Johnson (2014) provides an overview of various issues in international macroeconomics that should be studied from a “value-added exports” perspective.

<sup>8</sup> $E_s$  is a scalar defined as the summation of exports of final products  $\mathbf{y}_{sr}$  and intermediate products  $\mathbf{A}_{sr} \mathbf{x}_r$ .

through 9).<sup>9</sup> KWW propose a further decomposition of the residual into what they call “foreign value added” and “pure double counted terms.” With some modification, foreign value added can be defined in this approach as well, but only properly in the context of a complete decomposition of global GDP. This needs a more extensive treatment and is left for further research.

In the next section, we will prove the useful result that to calculate DVA in exports one does not need to use global input-output tables. Instead it can be done on the basis of national tables only.

## II. Measuring Domestic Value Added in Exports Based on National Input-Output Data

In this section we will prove that DVA in a country’s exports can be computed based on information from *national* input-output tables only. This result was not made explicit in KWW, but is important for applied research, as national information is much more abundant than the *global* input-output tables needed to perform the decomposition of KWW. In contrast to global tables, national tables neither contain data on the origin of imports, nor on the destination of exports.

### THEOREM:

To calculate the domestic value added in exports of country  $s$ , only information on domestic transactions is needed, namely domestic inter-industry transactions and domestic final demand for domestic goods.

### PROOF:

To calculate DVA in exports we start with the definition of  $GDP_s^*$  as given in equation (2). Because  $\mathbf{A}^*$  is a partitioned matrix with the elements of  $\mathbf{A}_{sr}$  set equal to zero, we can rewrite the Leontief inverse  $(\mathbf{I} - \mathbf{A}^*)^{-1}$  in terms of sub-matrices of  $\mathbf{A}$  as follows:<sup>10</sup>

$$(7) \quad (\mathbf{I} - \mathbf{A}^*)^{-1} = \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{ss})^{-1} & \mathbf{0} \\ (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} & (\mathbf{I} - \mathbf{A}_{rr})^{-1} \end{bmatrix}.$$

Post-multiplying with  $\mathbf{Y}^* \mathbf{i}$  and pre-multiplying the result with value-added coefficients as in equation (2) gives us an alternative expression for  $GDP_s^*$ :

$$(8) \quad GDP_s^* = [\tilde{\mathbf{v}}_s \ \mathbf{0}] \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{ss})^{-1} & \mathbf{0} \\ (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} & (\mathbf{I} - \mathbf{A}_{rr})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{0} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} \end{bmatrix} \mathbf{i}.$$

<sup>9</sup>The equivalence has been established by comparing the results of the KWW decomposition with our decomposition results using the World Input-Output Database (Timmer et al. 2015). KWW decomposition results are taken from the file WIODtotdecomp.xlsx that can be found in the folder “WTO-WIOD” in the online Data Appendix material, provided on the AER website.

<sup>10</sup>It is readily checked that we obtain the identity matrix  $\mathbf{I}$  by multiplying the right-hand side of (7) by  $(\mathbf{I} - \mathbf{A}^*)$ , which implies that  $(\mathbf{I} - \mathbf{A}^*)^{-1}$  is the (unique) Leontief inverse. See Miller and Lahr (2001, p. 435) for more general expressions of bi-regional Leontief inverses.

And after simplification:

$$(9) \quad GDP_s^* = \tilde{\mathbf{v}}_s (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{y}_{ss}$$

The right-hand side of equation (9) contains data that can be taken from national input-output data for country  $s$ , with  $\tilde{\mathbf{v}}_s$  a row vector of value-added coefficients for the industries in  $s$ , the matrix  $\mathbf{A}_{ss}$  with domestic intermediate input coefficients, and  $\mathbf{y}_{ss}$  domestic final demand for domestic products. As before,  $DVA_s = GDP_s - GDP_s^*$ , where  $GDP_s$  is simply the sum over industry value added. ■

This proves that DVA in a country's gross exports can be easily calculated on the basis of information in national tables. This result aligns with KWW following their definition of DVA given in equation (37). It also provides an ex post rationalization of the measure of vertical specialization (VS) introduced by Hummels, Ishii, and Yi (2001). VS is defined as the imported input content of exports calculated from national input-output tables (their equation 3), and we find that it is equal to 1 minus DVA in exports.<sup>11</sup> Hence DVA in exports, expressed as a share of gross exports is a useful (inverse) measure of the degree of a country's vertical specialization in trade.

Global input-output tables are still required to decompose DVA in exports further as in equation (6), for example, to measure "DVA absorbed abroad" introduced by Johnson and Noguera (2012). This is also true for any analysis of DVA in bilateral export flows as outlined next.

### III. Extending the Decomposition to Bilateral Export Flows

An additional advantage of using the hypothetical extraction approach to decompose gross exports is that it can be naturally extended to a bilateral setting, to analyze the geographic orientation of vertical specialization.<sup>12</sup> The domestic value-added content of exports from country  $s$  to country  $r$  can be derived as the difference between actual GDP in  $s$  and the parts of GDP in  $s$  that can be attributed to all transactions that remain when exports from  $s$  to  $r$  are set to zero. We will illustrate this logic for a three-country case, which can be naturally extended to an  $n$ -country context.

In the three-country case we are interested in the domestic value added of country  $s$  in its exports to  $r$ , while it also trades with country  $t$ . We use notation consistent with the two-country case. Actual intermediate inputs coefficients ( $\mathbf{A}$ ) and actual deliveries to final demand ( $\mathbf{Y}$ ) are given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{A}_{sr} & \mathbf{A}_{st} \\ \mathbf{A}_{rs} & \mathbf{A}_{rr} & \mathbf{A}_{rt} \\ \mathbf{A}_{ts} & \mathbf{A}_{tr} & \mathbf{A}_{tt} \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{y}_{sr} & \mathbf{y}_{st} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} & \mathbf{y}_{rt} \\ \mathbf{y}_{ts} & \mathbf{y}_{tr} & \mathbf{y}_{tt} \end{bmatrix}.$$

<sup>11</sup> This finding goes against KWW, who claim that VS is equal to their terms 7 through 9 in equation (36), thus excluding term 6 (see KWW, p. 483). Empirically, term 6 appears to be small (see KWW, Table 3).

<sup>12</sup> Bilateral measures of value added in trade are provided in Wang, Wei, and Zhu (2013) building upon the original KWW decomposition, by Hummels, Ishii, and Yi (2001) for their VS measure and by Johnson and Noguera (2012) for value added absorbed abroad.

Export flows from  $s$  to  $r$  are extracted, which implies that the hypothetical intermediate input coefficients and deliveries to final demand are given by

$$\mathbf{A}^* = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{0} & \mathbf{A}_{sr} \\ \mathbf{A}_{rs} & \mathbf{A}_{rr} & \mathbf{A}_{rt} \\ \mathbf{A}_{ts} & \mathbf{A}_{tr} & \mathbf{A}_{tt} \end{bmatrix} \text{ and } \mathbf{Y}^* = \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{0} & \mathbf{y}_{sr} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} & \mathbf{y}_{rt} \\ \mathbf{y}_{ts} & \mathbf{y}_{tr} & \mathbf{y}_{tt} \end{bmatrix}.$$

The domestic value added per unit of gross output vector for all industries in  $s$  is again represented by a row vector  $\mathbf{v}_s = [\tilde{\mathbf{v}}_s \ \mathbf{0} \ \mathbf{0}]$ . The logic of hypothetical extraction leads to the following expression for the domestic value added in exports from  $s$  to  $r$ :

$$(10) \quad DVA_{sr} = GDP_s - [\mathbf{v}_s (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{Y}^* \mathbf{i}],$$

where the second term represents value added generated that cannot be attributed to exports from  $s$  to  $r$ , while exports to  $t$  still generate value added.

#### IV. Concluding Remarks

KWW (2014) proposed a decomposition of exports to analyze the flow of value added between countries as embedded in international trade, taking two-way trade in intermediates into account. We provide a simple and intuitive underpinning of this decomposition using the “hypothetical extraction” method. It calculates which part of GDP in a country can be attributed to production related to exporting, which we define as domestic value added in exports. Expressed as a share of gross exports it is a useful (inverse) measure of the degree of a country’s vertical specialization in trade. We prove that this measure can be calculated by using national input-output data only. We also show how our approach illuminates the concept of “domestic value added absorbed abroad” introduced by Johnson and Noguera (2012) as part of DVA in exports. In addition, our approach provides natural extensions into bilateral measures, underlining its potential. With the recent availability of global input-output tables (Timmer et al. 2015), this opens up new opportunities to analyze the intricate flows of value added between countries as embedded in international trade.

#### APPENDIX

Koopman, Wang, and Wei (2014) derive their decomposition of gross exports through manipulation of basic accounting identities. In this Appendix we show how such an approach leads to a measure of domestic value added similar as that obtained by our hypothetical extraction approach.<sup>13</sup>

The basic input-output accounting identity states that all output is either used as intermediate input or for final demand. In a two-country context, this can be written as

$$\begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{A}_{sr} \\ \mathbf{A}_{rs} & \mathbf{A}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} + \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{y}_{sr} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} \end{bmatrix} \mathbf{i},$$

<sup>13</sup>This Appendix builds upon insightful comments given by an anonymous referee.



in which  $\mathbf{x}_s$  stands for the vector of gross output levels of industries in  $s$ . Separating out exports of intermediate and final products from  $s$  to  $r$ , we can write

$$(A1) \quad \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{ss} & \mathbf{0} \\ \mathbf{A}_{rs} & \mathbf{A}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} + \begin{bmatrix} \mathbf{y}_{ss} & \mathbf{0} \\ \mathbf{y}_{rs} & \mathbf{y}_{rr} \end{bmatrix} \mathbf{i} + \begin{bmatrix} \mathbf{e}_s \\ \mathbf{0} \end{bmatrix}.$$

In this equation, the vector  $\mathbf{e}_s$  represents the values of exports by each of the industries in  $s$ . This includes exports of intermediates as well as final products ( $\mathbf{e}_s = \mathbf{A}_{sr}\mathbf{x}_r + \mathbf{y}_{sr}$ ). Using notation introduced in the main text, (A1) can be expressed as

$$\begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} = \mathbf{A}^* \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} + \mathbf{Y}^* \mathbf{i} + \begin{bmatrix} \mathbf{e}_s \\ \mathbf{0} \end{bmatrix}.$$

We can solve this equation for  $\mathbf{x}_s$  and  $\mathbf{x}_r$  by rewriting it as

$$\begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} = (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{Y}^* \mathbf{i} + (\mathbf{I} - \mathbf{A}^*)^{-1} \begin{bmatrix} \mathbf{e}_s \\ \mathbf{0} \end{bmatrix}.$$

When we pre-multiply this expression by a row vector  $\mathbf{v}_s$  consisting of value-added shares in gross output for country  $s$  and zeros elsewhere, we arrive at GDP of  $s$ :

$$(A2) \quad GDP_s = \mathbf{v}_s \begin{bmatrix} \mathbf{x}_s \\ \mathbf{x}_r \end{bmatrix} = \mathbf{v}_s (\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{Y}^* \mathbf{i} + \mathbf{v}_s (\mathbf{I} - \mathbf{A}^*)^{-1} \begin{bmatrix} \mathbf{e}_s \\ \mathbf{0} \end{bmatrix}.$$

This equation provides a decomposition of GDP in  $s$  and KWW define domestic value added (DVA) in exports of  $s$  as the last element on the right-hand side of (A2):

$$DVA_s = \mathbf{v}_s (\mathbf{I} - \mathbf{A}^*)^{-1} \begin{bmatrix} \mathbf{e}_s \\ \mathbf{0} \end{bmatrix},$$

which can be simplified as<sup>14</sup>

$$(A3) \quad DVA_s = \tilde{\mathbf{v}}_s (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{e}_s.$$

This is the measure of DVA in exports as defined by KWW in their equation (37). While mathematically correct, it does not have a clear interpretation. One might be led to think that the multiplication of an export vector with the Leontief inverse,  $(\mathbf{I} - \mathbf{A}^*)^{-1}$ , can be interpreted as the gross output associated with the production of exports. This only holds true however when exports exclusively contain exogenous elements of final demand (see Miller and Blair 2009, pp. 261–264). This is not the case here as exports also include exports of intermediates ( $\mathbf{A}_{sr}\mathbf{x}_r$ ). This problem of interpretation does not arise with the hypothetical extraction approach

<sup>14</sup>By employing (7) and realizing that because  $\mathbf{v}_s = [\tilde{\mathbf{v}}_s \ \mathbf{0}]$ , only the upper part of  $(\mathbf{I} - \mathbf{A}^*)^{-1}$  remains after multiplication.

as used in this comment. As shown in equations (1–3) we measure DVA in exports solely in terms of Leontief inverses and final demand  $\mathbf{Y}$ :

$$DVA_s = GDP_s - GDP_s^* = \mathbf{v}_s(\mathbf{I} - \mathbf{A})^{-1} \mathbf{Y} \mathbf{i} - \mathbf{v}_s(\mathbf{I} - \mathbf{A}^*)^{-1} \mathbf{Y}^* \mathbf{i}.$$

Nevertheless in this particular case, the two approaches deliver the same measure of DVA in exports. This can be seen by subtracting  $GDP_s^*$  defined in (2) from  $GDP_s$  as written in (A2):

$$DVA_s = GDP_s - GDP_s^* = \mathbf{v}_s(\mathbf{I} - \mathbf{A}^*)^{-1} \begin{bmatrix} \mathbf{e}_s \\ \mathbf{0} \end{bmatrix}.$$

While in the end leading to the same result as KWW, our approach has a clean interpretation from an input-output perspective. As such it provides a firm foundation and stepping stone for further analysis of value added in trade.

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