Nonlinear Subgrid-Scale Models for Large-Eddy Simulation of Rotating Turbulent Flows

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Abstract

We study the construction of subgrid-scale models for large-eddy simulation of rotating turbulent flows. In particular, we aim to find an improved description of the Reynolds stresses in such flows. We do so by considering subgrid-scale models that, apart from the usual eddy viscosity term, contain a nonlinear model term that is not dissipative. We show that the addition of the nonlinear model term [indeed] leads to an improved prediction of the Reynolds stresses in large-eddy simulations of spanwise-rotating plane-channel flow at $Re_\tau = 395$ and $Ro^+ = 100$.]

1 Introduction

We consider large-eddy simulation of incompressible rotating turbulent flows. In large-eddy simulation one seeks to predict the large-scale behavior of turbulent flows without resolving all the relevant flow details. This is commonly done by supplementing the Navier–Stokes equations with an additional forcing term, a subgrid-scale model, aimed at representing the unresolved flow physics.

Rotating turbulent flows form a challenging test case for large-eddy simulation due to the presence of the Coriolis force. The Coriolis force conserves the total kinetic energy, while also redistributing it. In particular, the Coriolis force transports kinetic energy from small to large scales of motion, leading to the formation of large-scale anisotropic structures [8]. Many subgrid-scale models for large-eddy simulation are, however, (primarily) designed to parametrize the dissipative nature of turbulent flows, ignoring transport processes.
We therefore consider subgrid-scale models consisting of two model terms. The first term is an eddy viscosity term that is linear in the rate-of-strain tensor and that is used to represent the dissipative behavior of turbulent flows. The second term is nonlinear in the local velocity gradient and is aimed at parametrizing nondissipative processes, such as those due to rotation. We study the behavior of these nonlinear subgrid-scale models in large-eddy simulations of spanwise-rotating plane-channel flow.

The structure of this paper is as follows. Nonlinear subgrid-scale models for large-eddy simulation are introduced in Section 2. Then, Sect. 3 is devoted to studying the behavior of these subgrid-scale models in large-eddy simulations of spanwise-rotating plane-channel flow. Finally, conclusions are drawn in Sect. 4.

2 Nonlinear subgrid-scale models

The behavior of incompressible rotating turbulent flows is governed by the incompressible Navier–Stokes equations in a rotating frame of reference. In large-eddy simulation these equations are supplemented with an additional forcing term, providing us with

$$\frac{\partial u_i}{\partial x_i} = 0, \quad \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + 2 \nu \frac{\partial}{\partial x_j} S_{ij} - 2 \varepsilon_{ijk} \Omega_k u_j - \frac{\partial}{\partial x_j} \tau_{ij}^{\text{mod}}. \quad (1)$$

Here, the $x_i$-component of the velocity field is indicated by $u_i$, while $p$ represents the modified pressure including the centrifugal force. The density and kinematic viscosity are labeled $\rho$ and $\nu$, respectively. The rate-of-strain and rate-of-rotation tensors are defined according to

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad \Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right), \quad (2)$$

while $\Omega_i$ represents the rotation rate of the frame of reference about the $x_i$-axis. Without loss of generality we will assume that the axis of rotation is the $x_3$-axis, i.e., $\Omega_i = \delta_{i3} \Omega$. The Einstein summation convention is assumed for repeated indices. Note that we consider large-eddy simulation without explicit filtering. Hence, no bars or tildes indicating a filtering operation appear in the above equations.

We model the deviatoric part of the subgrid-scale stress tensor with the following nonlinear model,

$$\tau_{ij}^{\text{mod,dev}} = -2 \nu e S + \mu e (S \Omega - \Omega S). \quad (3)$$

The first term on the right-hand side of (3), the usual eddy viscosity term, is used to parametrize dissipative processes in turbulent flows. The second term, that is nonlinear in the velocity gradient, is added because it is perpendicular to the rate-of-strain tensor. Therefore, it does not directly contribute to the subgrid dissipation and it represents energy transport. This term can be said to be the stable part of
the gradient model [2, 5]. As this term contains the rate-of-rotation tensor, it has “a particular potential for [the simulation of] rotating flows” [6].

We define the eddy viscosity, $\nu_e$, and the transport coefficient, $\mu_e$, by

$$\nu_e = (C_\nu \delta)^2 \frac{1}{2} |S| f_{VS}^2,$$

$$\mu_e = C_\mu \delta^2 f_{VS}^4.$$  

Here, $C_\nu$ and $C_\mu$ are the model constants, $\delta$ represents the subgrid characteristic length scale and the magnitude of the rate of strain is defined as $|S| = \sqrt{\text{tr}(S^2)}$. The nondimensionalized vortex stretching magnitude,

$$f_{VS} = \frac{|\Omega \times S|}{|S| |\Omega|},$$

is used to enforce the proper near-wall scaling behavior of the modeled stresses and to make sure that the model vanishes in two-component flows [7]. The vorticity vector is given by $\omega_i = -\epsilon_{ijk} \Omega_{jk}$.

### 3 Numerical results

We studied the vortex-stretching-based nonlinear subgrid-scale model of (3) in large-eddy simulations of spanwise-rotating plane-channel flow. These simulations were performed using an incompressible Navier-Stokes solver employing a kinetic-energy-conserving spatial discretization of finite-volume type [9]. As such, the kinetic energy in the simulations is by construction conserved by convection, by the Coriolis force and by the nonlinear term of the subgrid-scale model.

#### 3.1 Spanwise-rotating plane-channel flow

We performed direct and large-eddy simulations of spanwise-rotating plane-channel flow. Spanwise-rotating plane-channel flow can be characterized using the friction Reynolds and rotation numbers,

$$Re_\tau = \frac{u_\tau d}{v}, \quad Ro^+ = \frac{2\Omega d}{u_\tau},$$

where $u_\tau$ is the friction velocity corresponding to the wall shear stress.

The rotation number, $Ro^+$, determines the behavior of the flow in a spanwise-rotating plane-channel flow. For small rotation numbers the flow is mostly turbulent, although a small region close to one of the walls, the stable side of the channel, turns laminar. As the rotation number increases, the portion of the channel in which the
flow becomes laminar grows, until, for significant rotation numbers \((\text{Ro}^+ > 250)\), the flow fully laminarizes. The mean velocity profile of a spanwise-rotating plane-channel flow exhibits a characteristic linear slope (proportional to \(\text{Ro}^+\)) corresponding to the turbulent or unstable part of the flow, while a parabolic profile emerges on the laminar or stable side. Laminarization is further characterized by the decay of the Reynolds stresses. Refer to the work by Grundestam et al. [4] for more information about spanwise-rotating plane-channel flow.

We performed direct and large-eddy simulations of spanwise-rotating plane-channel flow with \(Re_x \approx 395\) and a moderate rotation number, \(\text{Ro}^+ = 100\). In these simulations the flow domain had dimensions \(2\pi d \times 2d \times \pi d\) and was taken periodic in the streamwise \((x_1)\) and spanwise \((x_3)\) directions. Large-eddy simulations and direct numerical simulations were, respectively, performed on \(32^3\) and \(128 \times 256 \times 128\) grids that were stretched in the wall-normal direction but were uniform otherwise.

The large-eddy simulations made use of the vortex-stretching-based eddy viscosity model ((3) with \(C_\mu = 0\)) and the vortex-stretching-based nonlinear subgrid-scale model of (3). The value of the eddy viscosity constant, \(C_\nu\), was estimated by requiring that the average dissipation due to the model matches the average dissipation of the Smagorinsky model [7]. This provided us with \(C_\nu = 0.59\). The value of the transport coefficient of the nonlinear model, \(C_\mu\), was subsequently tuned so as to obtain the best prediction of the Reynolds stresses. The subgrid characteristic length scale was defined using the local grid size, \(\delta = (\Delta x_1 \Delta x_2 \Delta x_3)^{1/3}\) [3]. Results from direct numerical simulations and large-eddy simulations without a subgrid-scale model serve as reference data.

Figure 1 shows the mean velocity profile and the behavior of the Reynolds shear stress and spanwise Reynolds stress as obtained from the simulations. Since we consider traceless subgrid-scale models, only the deviatoric part of the Reynolds stresses is considered. These results are further compensated by the average contribution from the subgrid-scale model [10].

The typical features of the flow in a spanwise-rotating plane-channel are clearly visible: the mean velocity profile exhibits a linear slope on the turbulent side of the channel, while the Reynolds shear stress attains small values on the laminar side of the channel. Contrary to what could be expected, the stresses are not exactly zero on the laminar side of the channel, which is most likely due to the occurrence of turbulent bursts [1].

The large-eddy simulations with the vortex-stretching-based subgrid-scale models show a slight improvement in the prediction of the peak height and the slope of the mean velocity profile when compared to the no-model result. Corresponding behavior can be observed in the Reynolds shear stress. These results indicate that the vortex-stretching-based eddy viscosity and nonlinear subgrid-scale models behave well.

The power of these subgrid-scale models, however, becomes clear when considering the deviatoric part of the streamwise Reynolds stresses. From that result it can clearly be seen that large-eddy simulations without a subgrid-scale model fail to predict the streamwise Reynolds stresses. [Thus, one indeed needs a subgrid-scale
model for the proper prediction of these flows, even at friction Reynolds numbers as low as $Re_\tau \approx 395$. Large-eddy simulations with the vortex-stretching-based eddy viscosity model improve the prediction a lot. Further improvements in the prediction of the streamwise Reynolds stress are obtained when including the nonlinear model term, as can most clearly be seen on the turbulent side of the channel ($0 \leq x_2/d \leq 1$). Similar observations can be made for the prediction of the wall-normal and spanwise Reynolds stresses (not shown). Thus, the addition of the nonlinear model term leads to an improved prediction of the diagonal Reynolds stresses, while not adversely affecting the mean velocity profile.

4 Conclusions

Acknowledgements

References

Fig. 1  a Mean velocity profile, b Reynolds shear stress compensated by the model contribution and c deviatoric part of the streamwise Reynolds stress compensated by the model contribution, as obtained from large-eddy simulations (LES) of spanwise-rotating plane-channel flow at $Re_z \approx 395$ and $Ro^+ = 100$ on a $32^3$ grid. Simulations were performed without a subgrid-scale model (dotted line, circles), with the vortex-stretching-based eddy viscosity (VS EV) model ((3) with $C_\nu \approx 0.59$ and $C_\mu = 0$) (dashed line, squares), and with the vortex-stretching-based nonlinear (VS NL) subgrid-scale model ((3) with $C_\nu \approx 0.59$ and $C_\mu = 5$) (solid line, triangles). Results from direct numerical simulations (DNS) on a $128 \times 256 \times 128$ grid are shown as reference (thick solid line).