Chapter 5

Classroom Formative Assessment and its Relationship with Teachers’ Knowledge of Mathematical Errors and Learning Trajectories

This chapter is based on the following article that has been submitted for publication:
Abstract

Classroom formative assessment (CFA) is considered to be a promising way to enhance primary school students’ mathematics achievement. In this study, we used a questionnaire to investigate how teachers’ use of three CFA elements (goal-directed instruction, assessment, and instructional feedback) are related to each other. Additionally, we explored whether teachers’ knowledge of mathematical errors and learning trajectories influence the use of these elements. A sample of 137 teachers completely filled in this questionnaire. The results showed that most of the participating teachers almost always used the CFA elements during mathematics lessons and had adequate knowledge of mathematical errors and learning trajectories. There were moderate positive relations between the CFA elements suggesting a coherent use of CFA. The teachers’ knowledge of errors was weakly related to goal-directed instruction and their knowledge of learning trajectories was weakly related to the teachers’ provision of instructional feedback.
5.1 Introduction

Recent reports have shown that in many countries primary school students’ mathematics performance is not up to par or is declining (OECD, 2014, pp. 50-56; OECD, 2016, pp. 181-184). To reverse this trend and improve students’ mathematical abilities policy makers, researchers and teachers tend to focus on elements of effective teaching such as goal-directed instruction, assessment, adaptive teaching, grouping strategies, feedback, and reinforcement (Good, Wiley, & Flores, 2009; Reynolds et al., 2014; Scheerens, 2016). As a combination of goal-directed instruction, assessment and feedback, classroom formative assessment (CFA) is considered to be promising in enhancing student performance. CFA is considered to be an iterative cycle (see Figure 5.1) of gathering and analysing information about the students’ understanding of a learning goal to provide instructional feedback that moves students forward in their learning process during a lesson cycle (Black & Wiliam, 2009; Conderman & Hedin, 2012; Supovitz, Ebby, & Sirinides, 2013; Wiliam & Thompson, 2008).

Where is the student going:
Setting a learning goal and providing instruction accordingly.

How should the student reach the learning goal:
Providing (instructional) feedback.

Where is the student at:
Assessing the students’ mastery of the learning goal.

Figure 5.1. Three elements of formative assessment.

CFA starts with goal-directed instruction. This implies that a teacher sets a goal for each lesson and thereby focuses the instruction, seeing that a clear goal describes which knowledge and skills will be taught. As the intention of instruction is that students will eventually master the learning goal, this mastery should be assessed during the same lesson. Therefore,
goal-directed instruction is also a precondition for effective assessment to take place. It determines the precise knowledge and skills that need to be assessed (Ashford & De Stobbeleir, 2013; Locke & Latham, 2006; Marzano, 2006). The assessment provides information about gaps in what the students should master (learning goal) and what the students in practice actually do master. Thus, setting a clear goal allows a teacher to draw conclusions about the extent to which the students have mastered it (Moon, 2005). This information is essential for a teacher to give effective instructional feedback that focuses on closing the students’ knowledge gap, for instance by pacing the instruction of providing small group instruction (Hattie & Timperley, 2007; Moon, 2005). Ideally, the use of the three elements is repeated until the learning goal is mastered. The provision of instructional feedback can thus lead to the teacher making decisions about, for instance, maintaining the existing learning goal until mastery is reached or setting a new one for the next lesson (Supovitz et al., 2013). However, to our knowledge, the assumption that the CFA cycle is used in such a coherent manner has never been tested. It is unknown whether teachers actually use these elements in a structured way.

Nonetheless, applying the CFA cycle in an iterative manner is considered to be effective in enhancing student performance. This should particularly be the case for a subject like mathematics, where all subject matter is strongly interdependent: A student needs previously learned mathematical knowledge and skills to master new topics. It thus seems that a teacher should apply the CFA cycle during the lesson in order to provide timely instructional feedback that prevents students from practicing with faulty mathematical knowledge and procedures allowing for an uninterrupted learning cycle (Irons, 2008; Marzano, Pickering, & Pollock, 2001).

Despite its assumed benefits, CFA has, unfortunately, been reported to be difficult to implement (cf. Even, 2005 and Chapter 3 of this dissertation). It is suggested that teachers’ mathematical knowledge can present itself as a barrier that impedes the effective implementation of CFA (Even, 2005; Heritage, Kim, Vendlinski, & Herman, 2009; Hodgen & Marshall, 2005). Recently, attention has shifted to teachers’ knowledge of mathematical errors and learning trajectories as important prerequisites for the use of effective CFA (Furtak, Morisson, & Kroog, 2014; Scheider &
Gowan, 2013; Supovitz et al., 2013). Although research has shown that mathematical knowledge for teaching in general is positively related to quality of instruction (Hill et al., 2008) and has a positive effect on student achievement (Hill, Rowan, & Ball, 2005), it has never been investigated to what extent these particular types of mathematical knowledge influence teachers’ use of CFA as a specific part of effective instruction.

In this study, we tried to find out to what extent the three CFA elements (goal-directed instruction, assessment, and instructional feedback) are used in a coherent manner by Dutch primary mathematics teachers. Furthermore, we investigated how teachers’ knowledge of mathematical errors and learning trajectories affect the teachers’ use of the CFA elements. In the upcoming sections we will elaborate on the CFA elements, the teachers’ knowledge of mathematical errors and learning trajectories, and their presumed relationships. Hereafter, we will describe our study design, used instruments and statistical analyses.

### 5.2 Theoretical Framework

#### 5.2.1 Classroom formative assessment

##### 5.2.1.1 Goal-directed instruction

As we mentioned in the introduction, CFA is an iterative cycle consisting of goal-directed instruction, assessment and instructional feedback (Black & Wiliam, 2009; Conderman & Hedin, 2012; Supovitz et al., 2013; Wiliam & Thompson, 2008). As such, CFA aims at closing the gap between a learner’s current status and his/her learning goals (Sadler, 1989). Effective CFA starts with goal-directed instruction. In this study, goal-directed instruction consists of setting a goal for instruction, preparing the lesson with this goal in mind and focusing the instruction on this goal. Before the lesson a teacher should formulate attainable and assessable goals (Brookhart, 2011). These goals describe what should be the results of the following instruction (Andrade, 2013) and are essential for the stages of assessment and providing instructional feedback. Goal-directed instruction at the beginning of the cycle allows for assessment of possible gaps in student knowledge and for providing appropriate feedback aimed at closing these gaps.
To be able to assess students’ mastery of the learning goal, a teacher should formulate ‘assessable’ learning goals. Although this might sound like stating the obvious, many goals in mathematics textbooks are formulated in ways that make it difficult to do that very thing. For instance, in a mathematics textbook the learning goal might only describe the content that will be taught, such as ‘Adding up till 100’. However, such a ‘goal’ does not shed any light on what kind of knowledge or skills students should be demonstrating to be able to assess their mastery of the goal. A learning goal formulated as ‘The student can add up till 100 and cross tens by jumping on a number line’ provides much more information about the skills that a student should be able to show and is therefore easier to assess. In addition to formulating one attainable and assessable learning goal, a teacher should prepare mathematics tasks for the students that concur with the learning goal and by which they can show their mastery of the learning goal (Hollingsworth & Ybarra, 2009). Once the lesson starts, the teacher should communicate with the students about the goal for them to understand what is expected and what will be assessed (Hollingsworth & Ybarra, 2009; Wiliam & Thompson, 2008).

5.2.1.2 Assessment

Teachers can assess the students’ understanding of the learning goal during different lesson episodes. For instance, several studies (e.g. Suurtamm, Koch, & Arden, 2010; Veldhuis, Van Den Heuvel-Panhuizen, Vermeulen, & Eggen, 2013) have focused on the extent to which teachers use different assessments, such as classroom discussions, questioning, or games during instruction to gain insight into the class’s understanding of the learning goal. The teacher can use this information to make instructional decisions, such as pacing the instruction or select appropriate scaffolds to support the instruction. However, a teacher can also assess each individual student’s understanding (e.g. by observing the students or checking seat work) of the learning goal after the instruction. This information can be used to provide timely instructional feedback (for example small group instruction) to those students who need it. The use of different kinds of assessments at different moments during the lesson is considered to be particularly effective in mathematics education. Unlike end-of-unit-tests, assessments during the lesson trigger students to express their mathematical
thinking and are therefore most insightful for teachers (Suurtamm et al. 2010; Wiliam, 2007). As a result, the information gathered during these assessments could serve as a solid basis on which to build the instructional feedback.

5.2.1.3 Instructional Feedback

The assessments that the teacher applies during a lesson should lead to the provision of instructional feedback aimed at closing gaps in students’ knowledge and skills or – in some cases – helping students progress in their learning if they have shown to have already mastered the learning goal. In the literature many different definitions for and interpretations of the term ‘feedback’ are used. For instance, Shute (2008) reserves the term feedback for “information communicated to the learner that is intended to modify his or her thinking or behaviour for the purpose of improving learning” (p. 154). However, this definition leaves room for multiple interpretations. Information provided to students to change their thinking or behaviour can vary from feedback that provides a limited amount of information to the learner to feedback consisting of instructional help aimed at enhancing students’ knowledge and skills. Grades or number of errors are examples of feedback with little information, while providing the correct answer in combination with an explanation or instructing students how to perform a task are examples of feedback consisting of instructional help (cf. Ruiz-Primo & Li, 2013 for a more elaborate discussion). In this study, we use the term ‘instructional feedback’ for feedback that consists of instructional help to move the students’ learning progress forward. The assessment that precedes the instructional feedback provides the teacher with information about the students’ current level of understanding of the learning goal. This information is of the utmost importance for providing effective instructional feedback (Hattie & Timperley, 2007; Moon, 2005). For example, a teacher might be observing a student who is practising with the table of six. The student makes multiple mistakes and the teacher asks him to explain what he is doing. The student shows how he tries to solve the problem ‘7 x 6 = ’ by jumping on a number line (abstract representation of the calculation), but the student cannot figure out how many times he should jump on the number line or what number the ‘jumps’ should consist of. Based on this information the teacher might find it worthwhile to provide feedback by using a more
concrete representation of the calculation in the form of seven baskets with six apples in them and relating it to the number line. Without the detailed information gathered during the assessment, the teacher might have provided the wrong kind of instructional feedback to this student. This implies that instructional feedback is highly dependent on the preceding assessment.

The instructional feedback should be provided as soon as possible, seeing that students benefit from timely instructional feedback, such as re-teaching or small group instruction (Irons, 2008; Marzano et al., 2001). Specifically for difficult, conceptual or procedural tasks – which are common tasks in mathematics education – feedback should be provided immediately (Shute, 2008). In addition to the timing of the feedback, the feedback should be presented in manageable units and be specific and clear. Especially for low-achieving students the feedback should be explicit and immediate, while the feedback can be more facilitative and sometimes delayed for high-achieving students. This implies that the provided feedback should be based on the learner’s instructional needs (Shute, 2008).

5.2.2 Teachers’ knowledge of mathematical errors

Teachers’ knowledge of students’ mathematical errors and learning trajectories, which are considered to be specific elements of teachers’ mathematical knowledge for teaching (Hill et al., 2008), can be regarded as being most important for effective use of CFA. Teachers’ knowledge of mathematical errors raises a teachers’ awareness of possible difficulties that students experience with a particular learning goal. It allows the teacher to focus his or her assessment as the teacher understands what kind of errors and their patterns he/she should be aware of. This information helps a teacher to determine the content of the instructional feedback that should be provided to correct these misconceptions (Mastropieri & Scruggs, 2002; Schneider & Gowan, 2013; Shuhua & Zhongwe, 2012). Thus, teachers’ knowledge of errors will have an influence on teachers’ use of both assessment and instructional feedback.

5.2.3 Teachers’ knowledge of learning trajectories

In recent years, attention has been drawn to the importance of knowledge of learning trajectories in applying CFA (Furtak et al., 2014; Scheider & Gowan, 2013; Supovitz et al., 2013). In both the Netherlands and
abroad, learning trajectories covering different domains, such as number reasoning, adding and subtracting, multiplying and dividing, geometry and measurement, have been developed (cf. Boswinkel, Buijs, & Klein Tank, 2014; Daro, Mosher, & Corcoran, 2011; Noteboom, Aartsen, & Lit, 2017). These learning trajectories often describe the development of mathematics concepts (in relation to other concepts and domains), common misconceptions, examples of lessons and examples of tasks that can be used as assessments. This information can be used to not only understand which prior knowledge students need to possess in order to master a particular learning goal, but also to gain insight into the kind of knowledge and skills that build upon the learning goal being taught. Therefore, knowledge of learning trajectories enables teachers on the one hand to assess where in the student’s development problems arise and on the other hand to subsequently provide more precise instructional feedback that helps the student forward (Furtak et al., 2014; Ginsburg, 2009; Marzano, 2006). For example, a teacher has set a goal for instruction about adding up to twenty with two one-digit numbers. The teacher is aware that students need to understand how to break down numbers till nine before he or she can correctly complete these tasks (e.g. for the task ‘7 + 5’, the student needs to know that ‘5’ can be broken down into ‘3’ and ‘2’ in order to compute the broken down task ‘7 + 3 + 2 = 12’). The teacher also knows that students usually experience difficulties with this particular topic because they do not understand how to break down numbers till nine and use this for the bridge-to-ten strategy. This knowledge will focus the teachers’ assessment practice and allows the teacher to provide students who face this problem with instructional feedback about breaking down numbers till nine in combination with the bridge-to-ten strategy and to select appropriate scaffolds such as a double-ten frame. If a teacher lacks this kind of knowledge he or she has limited ways to adapt his or her instructional feedback to the students’ needs. Thus, learning trajectories form a framework that supports teachers in assessing students’ mastery of learning goals and providing feedback that adheres to the students’ needs (Ebby, Sirinides, Supovitz, & Oettinger, 2013).
5.3 Research Questions

By means of an online survey, we gathered information about the relations between the participating teachers’ use of the three CFA elements: ‘goal-directed instruction’, ‘assessment’, and ‘instructional feedback’. We also assessed the teachers’ knowledge of mathematical errors and their knowledge of learning trajectories to be able to investigate their influence on the teachers’ CFA practice. The study is descriptive in nature and aims to answer the following research questions:

1. To what extent are the teachers’ use of the different elements of CFA related to each other?
2. To what extent do the teachers’ knowledge of mathematical errors and learning trajectories influence their use of the CFA elements?

Based on the above-described literature we hypothesized a model for relations between the teachers’ knowledge of mathematical errors and learning trajectories and the CFA elements that can be seen in Figure 5.2.

Figure 5.2: Model for the use of CFA.
The model consists of two parts. The left part contains the teachers’ knowledge of mathematical errors and their knowledge of learning trajectories. We expect that there will be a correlation between the two, as they are both parts of teachers’ mathematical knowledge for teaching. The two factors will directly influence two of the CFA elements that are depicted on the right side of the model, namely ‘assessment’, and ‘instructional feedback’. We hypothesize that teachers who have insight into mathematical errors and learning trajectories will be more inclined to assess their students’ mastery of those goals, as they will be more able to assess what kind of errors the students make and what kind of help should be provided. Additionally, teachers with knowledge of mathematical errors and learning trajectories will provide timely instructional feedback more frequently. These teachers will have the necessary knowledge to provide effective feedback readily available during the lesson, whilst teachers who do not possess this knowledge will perhaps be more inclined to delay the feedback in order to prepare it better. Finally, we expect that goal-directed instruction will enable teachers to assess each student’s mastery. We also expect that the assessment will lead to timely instructional feedback to the students who need it. Once a teacher establishes whether the instructional feedback has led to mastery of the learning goal, he/she will probably decide on the learning goal for the next lesson. Based on the instructional feedback the teacher can, for example, choose to adjust the learning goal provided in the mathematics curriculum materials or set a new goal for instruction. In any case this will lead to the start of a new CFA cycle. We thus presume that teachers who frequently provide instructional feedback will also be more inclined to set specific goals for instruction and provide instruction accordingly.

5.4 Study Design

5.4.1 Participants

At the beginning of the study 400 Dutch primary school principals received an e-mail containing a request to fill in a questionnaire and a link to the website. The school principal was asked to forward the e-mail to the teachers of grade 1 to 6. In total, 14 of the e-mail requests could not be delivered and bounced back. Therefore, another 14 school principals were send the e-mail with the request to forward it to their teachers. After a few
weeks, the response turned out to be quite low, despite several reminders that were sent to the schools. Consequently, we send another 100 schools an e-mail and used our network to request schools to participate in the study. This resulted an effective sample of approximately 500 schools (7.9%) out of a possible number of 6362 schools. Considering that the schools on average had six teachers (one per grade) who could fill in the questionnaire, this meant that around 3000 teachers were asked to reply. Out of these 3000 teachers 137 teachers filled in the complete questionnaire. This response was approximately 4.6 percent.

The teachers were asked to provide information about their age, gender, years of teaching experience, class size and the percentages of low and high achieving students in their class. Out of the 137 respondents 89.1 percent were female teachers. Most teachers (54.7%) were between 31 and 50 years old and had on average 16.58 years of teaching experience ($SD = 9.45$). A class size of between 21 and 30 students was most common among the teachers (63.5%). Many teachers (73.7%) indicated that between 0 and 20% of their students showed low performance on mathematics and 57.6% of the teachers indicated that between 11 and 30% of their students showed high mathematics achievements. The teachers were not asked to provide information about their schools (e.g. denomination or location).

Although the sample group composition concerning gender, age and class size seems to reflect the population of Dutch primary school teachers (Dekker, 2016; CBS, 2016; CBS, Dutch Ministry of Education, Culture and Science, & DUO, 2016), we cannot be sure that this is in fact the case due to our small number of respondents. For instance, our small sample size may have been influenced by self-selection bias. Perhaps our 137 teachers were already interested in CFA and chose to thus fill in the questionnaire, whilst others chose to end the questionnaire prematurely because of, for example, its length. For the purpose of this study, the small sample size (and with it its representativeness) will be less of a concern, as we are not aiming to describe teachers’ use of CFA, but are mainly interested in estimating the strength of the relationships between the different CFA elements and the teachers’ knowledge of mathematical errors and learning goals.
5.4.2 Instruments

An online questionnaire (see Appendix D) was used to collect data about the primary school teachers. The questionnaire contained questions about the teachers' background, their use of the CFA elements, and their knowledge of mathematical errors and learning trajectories. The questionnaire was tested three times. During the first and second time a total of six persons filled in the complete questionnaire. These respondents were asked to highlight questions that they found to be unclear and gave their interpretation of the question. If this interpretation was not in accordance with the researcher’s intention, the researcher and respondent discussed how to paraphrase the question. Hereafter, a group of 20 persons filled in the questions about mathematical errors and learning trajectories. These respondents were also asked about the clarity of the questions. Their responses were used to test the reliability of the scales. After each pilot several changes were made to the questionnaire. In the following sections all scales that were used in the questionnaire are discussed.

5.4.2.1 Teachers’ use of CFA elements in mathematics education

The questions about the practical use of the elements were divided into three scales: ‘goal-directed instruction’, ‘assessment’, and ‘instructional feedback’. The items for the scales were based on our review of the literature as described in our theoretical framework.

The scale for ‘goal-directed instruction’ consisted of 13 items. Examples of items about goal-directed instruction are 'Before the start of the lesson I prepare an instruction that focuses on one specific goal’ and ‘At the beginning of the lesson I communicate the learning goal to my students’. Teachers could reply to these statements by selecting one of the following answers: Never (score 1), rarely (score 2), sometimes (score 3), almost always (score 4), always (score 5). An average score was computed to signify the use of goal-directed instruction. The internal consistency of the scale was good with Cronbach’s $\alpha=.81$.

Eleven items determined the frequency with which teachers assessed the mastery of the learning goal of the students. Examples of these items are ‘I check my students’ understanding of the learning goal by observing them while they are working together on tasks’ and ‘During my instruction I check my students’ understanding of the learning goal via questioning’.
Teachers could reply to these statements by selecting one of the following answers: Never (score 1), rarely (score 2), sometimes (score 3), almost always (score 4), always (score 5). A final scale score was computed based on the average score of the respondents. This scale had an internal consistency of Cronbach’s $\alpha = .79$, which is considered to be acceptable.

The teachers’ provision of instructional feedback was measured by twelve items, such as ‘I adjust my instruction by breaking it down in small and specific steps’ and ‘Immediately after the instruction I provide small group instruction’. Teachers could reply to these statements by selecting one of the following answers: Never (score 1), rarely (score 2), sometimes (score 3), almost always (score 4), always (score 5). Again, the average score was computed as a final score. The internal consistency was good with Cronbach’s $\alpha = .80$.

5.4.2.2 Teachers’ knowledge of mathematical errors

The scale of teachers’ knowledge of errors initially consisted of 14 items. These items represented particular classifications of student error patterns (Ashlock, 2010; Burrows, 1976; Geary, 2006; Kraemer, 2009):

- Conceptual error: The student made a mistake, because he/she does not understand the underlying concept of the mathematical task;
- Procedural error: The student made a mistake in the procedure that he/she used to complete the task;
- Place value error: The student made a mistake, because, whilst completing the task, he/she did not take into account that the position of a cipher determines its value;
- Error in the automaticity and memorisation of basic mathematics facts and skills: Incorrect memorisation of facts, such as addition and subtraction up to 20 or basic multiplications.

To prevent the teachers from making mistakes in identifying types of errors merely because of terminology, they were provided with an explanation of the categories on each page of the survey that concerned the mathematical errors. Hereupon, the teachers were asked to qualify 14 student errors according to the categories that are described above. The teachers could also respond with a question mark, if they felt they were not able to identify the
type of error. This option was made available to avoid that teachers would randomly fill in an answer.

For each item one category was most probable. A teacher would receive a score of 1 when he/she identified the correct error category. In some cases a second category would be awarded a score of 0.5, as this category could be a possible explanation for the error made, albeit a less plausible one. For example, the teachers were presented the following error:

- Adira (fifth grade): 368 + 6 = 373

This error could best be identified as an error due to an incorrect memorisation of basic mathematical facts, as students in the fifth grade should have memorised that 8 + 6 = 14 and not 13. Teachers who filled in this answer received a score of 1. Another possible, yet less plausible, error would be a procedural error. This would imply that a fifth grade student would still count to solve this problem. When starting to count this student included the first addend (368, 369, 370, 371, 372, 373) resulting in the incorrect answer. Teachers who gave this response would receive a score of 0.5.

When analysing the reliability of this scale, we found that four items had a corrected item-total correlation below .10, which are considered to discriminate poorly between different abilities (cf. Nunnally & Bernstein, 1994). Therefore, we deleted these items. This resulted in a scale of 10 items with an acceptable internal consistency of Cronbach’s $\alpha$. The number of correctly diagnosed items was computed to function as a final score.

5.4.2.3 Teachers’ knowledge of learning trajectories

In order to determine the teachers’ knowledge of mathematical learning trajectories, the teachers were asked to complete three arrangement questions. For every question the teachers had to arrange five mathematical tasks within a mathematical domain (multiplications, fractions and number sense) in such a way that they would fit the mathematical learning trajectory. For every item the teachers would receive a score of 1 if he/she put the mathematical task at the right place or one place lower/higher with regard to the ideal order. If the teacher placed the mathematical task anywhere else he/she would receive a score of 0. A score for each mathematical domain was then formulated by adding up the scores for each mathematical task. Finally, we computed an index score running from 0 to 3 for the teachers’
knowledge of learning trajectories by adding up the teachers’ scores for the three mathematical domains.

5.4.3 Data Analysis

The central questions of this study concern the relations between the CFA elements and the impact of the teachers’ knowledge of errors and learning trajectories on their’ use of the CFA elements in mathematics education. Descriptive statistics were applied concerning these variables to provide an impression of the teachers’ background and practice, not to show their representativeness. Correlations between the CFA scale scores were analysed to assess the relation between the elements of CFA. To determine the impact of the teachers’ knowledge of errors and learning trajectories on their use of the CFA elements correlations between these scale scores were analysed too.

Based on our theoretical hypothetical model as represented in Figure 5.2 constituted the foundation for the path analysis examined by the computer program LISREL VIII (Jöreskog & Sörbom, 1996). LISREL VIII uses the specifications of our proposed model and tries to find out to what extent this model is a good reproduction of (or ‘fit’ for) the correlation matrix. The extent to which the model deviates from the original correlation matrix can be determined by looking at the $\chi^2$-test for goodness of fit, and the root mean square error of approximation (RMSEA). The $\chi^2$-test tests to what extent the correlation matrix as estimated from the proposed model and the observed correlation matrix are different from one another. This means, that a low $\chi^2$-statistic indicates a good fit. With regard to RMSEA a perfect model fit would be indicated by a RMSEA of 0, while values close to or lower than 0.05 indicate an acceptable fit. In other words, the lower value of RMSEA the better the fit of the model is. One can change the model by changing the causal paths between the variables. Preconditions for changes in our model were that all changes to the model should be based on theoretical considerations, modification indices in LISREL VIII and t-values of path coefficients.

To account for measurement error in the variables we included error variances in the measurement part of the path model. The error variances were based on the Cronbach’s alpha coefficients described above.
5.5 Results

5.5.1 Description of teachers’ CFA practice

Table 5.1 contains descriptive data on the teachers’ use of the CFA elements. Because of our small sample size and the possibility of bias due to self-selection, this data should not be interpreted as being representative of the population of Dutch teachers’ use of the CFA elements and their knowledge of mathematical errors and learning trajectories.

The descriptive data shows that most of the teachers that participated in the study almost always set specific goals for their mathematics instruction. The mean scores and standard deviations of ‘assessment’ and ‘instructional feedback’ indicate that many respondents regularly, though not always, assess their students’ mastery and provide their students with instructional feedback during the mathematics lesson.

Table 5.1
Mean Scores and Standard Deviations for CFA during Mathematics Lessons. All Scales Run from 1 to 5, with 1 Referring to ‘Never’ and 5 Referring to ‘Always’.

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>Scale</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal-directed instruction</td>
<td>137</td>
<td>1 – 5</td>
<td>4.10</td>
<td>0.42</td>
</tr>
<tr>
<td>Assessment</td>
<td>137</td>
<td>1 – 5</td>
<td>3.82</td>
<td>0.51</td>
</tr>
<tr>
<td>Instructional feedback</td>
<td>137</td>
<td>1 – 5</td>
<td>3.91</td>
<td>0.45</td>
</tr>
</tbody>
</table>

5.5.2 Description of teachers’ knowledge of errors and learning trajectories

Table 5.2 shows the descriptive data for the teachers’ knowledge of errors and their knowledge of learning trajectories. The participating teachers’ knowledge of students’ mathematical errors appears to be reasonably well developed. On average the teachers in this study were able to classify more than six out of ten mathematical errors. The teachers’ knowledge of the learning trajectories was of a sufficient level. The sequencing of mathematics tasks in number sense according to the learning trajectory was the least difficult for the teachers. Out of all the teachers who
filled in this question, 75.9% was able to put these five tasks in a correct order. It is important to remark that there was quite some variance for both variables, which indicates that not all teachers were able to identify mathematical errors and align the learning trajectories to a sufficient level.

Table 5.2
_Descriptive Data about the Teachers’ Knowledge of Errors and of Learning Trajectories._

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>Scale</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge of mathematical errors</td>
<td>137</td>
<td>0 – 10</td>
<td>6.77</td>
<td>1.73</td>
</tr>
<tr>
<td>Knowledge of mathematical learning trajectories</td>
<td>137</td>
<td>0 – 3</td>
<td>2.04</td>
<td>0.81</td>
</tr>
</tbody>
</table>

5.5.3 _Relationships between the teachers’ use of the CFA elements and their knowledge of mathematical errors and learning trajectories_

Table 5.3 contains correlations between the three CFA elements and the teachers’ knowledge of mathematical errors and learning trajectories. Analysis of these correlations shows that there are significant correlations between ‘goal-directed instruction’ and ‘assessment’, between ‘assessment’ and ‘instructional feedback’, and between ‘goal-directed instruction’ and ‘instructional feedback’. Table 5.3 also shows that ‘teachers’ knowledge of errors’ relates to ‘goal-directed instruction’, albeit to a small extent. ‘Teachers’ knowledge of learning trajectories’ appears to be related to both ‘goal-directed instruction’ and ‘instructional feedback’, but not to ‘assessment’.
Next, we specified our theoretical model in LISREL and tested whether it fitted the correlation data. It turned out that this model had a reasonable fit to the data with a $\chi^2$-statistic of 5.38 ($df = 2, p = 0.067$) and a RMSEA of 0.112. However, four paths were found to be non-significant (see Figure 5.3).
Figure 5.3: Model for the use of CFA including path coefficients and error variances.
We tried to improve the fit of the model in four steps. Based on the modification indices in LISREL VIII we first added a path between the teachers' knowledge of errors and goal-directed instruction. This led to a significant improvement of fit with $\chi^2_{\text{diff}}(1) = 5.08$ and $p < .05$. However, this model contained some insignificant paths, which we removed from our model one by one. This resulted in a final model as depicted in Figure 5.4. All paths are unidirectional, with their direction indicated by arrows. All path coefficients are significant. The strength of the causal paths between variables depends on the standardized path coefficients that indicate the increase in the dependent variable caused by one standard unit on the independent variable scale (Jöreskog & Sörbom, 1996). For example, an increase in teachers’ knowledge of errors leads to an increase in the teachers’ use of setting goals for instruction with $\cdot .22$. The path model fits the data well with $\chi^2 = 1.55$, $df = 4$, $p = 0.817$, RMSEA = 0.000. There is no significant difference ($\chi^2_{\text{diff}}(3) = 1.25$ and $p > .50$) between our second and our final model, providing an additional argument – taking into account that there were insignificant paths in our second model – to opt for the more parsimonious model.

Figure 5.4 shows that, in this study, the teachers’ knowledge of errors influences the frequency by which teachers provide goal-directed instruction. The figure also reveals that teachers’ knowledge of learning trajectories affects the teachers’ use of instructional feedback to a small extent. Goal-directed instruction is only moderately positively related to assessment. This finding suggests that the teachers in our sample do not necessarily base their assessment of each student’s mastery on the goals they set for instruction. There is a rather strong causal path between teachers’ use of assessments to instructional feedback. This means that teachers who assess their students’ mastery also tend to provide instructional feedback during the lesson. Finally, there is a moderately positive relation between the teachers’ provision of instructional feedback and goal-directed instruction. This implies that teachers who provide frequently provide instructional feedback are also more inclined to focus their instruction on specific learning goals by, for example, adjusting a learning goal based on the students’ prior knowledge. However, it also indicates that some teachers do not base their goals for instruction on the instructional feedback they have provided before. This may be caused by the teachers’ use of mathematics curricula, as most
mention one or two learning goals per lesson whilst assuming that all students have mastered the previous learning goals.

In our theoretical framework we explained how we expected there to be a relationship between the teachers’ knowledge of errors and learning trajectories and their use of assessment. However, our final model suggests that there is merely an indirect relationship between teachers’ knowledge of errors and assessment through goal-directed instruction and no direct relationship between teachers’ knowledge of learning trajectories and assessment.
Figure 5.4. The final path model depicting the influence of the teachers’ knowledge of mathematical errors and learning trajectories on the CFA elements and the relations between the CFA elements.

" p < .05 (2-sided)
5.6 Conclusion

In this study, we have tried to find out to what extent the participating teachers’ use of the CFA elements ‘goal-directed instruction’, ‘assessment’ and ‘instructional feedback’ are related to each other. Additionally, we have investigated to what extent the teachers’ knowledge of mathematical errors and learning trajectories affect their use of the CFA elements.

The descriptive analyses of the teachers’ answers suggest that most teachers who filled in the questionnaire frequently (‘almost always’) provide goal-directed instruction. These high scores may imply that the participating teachers follow up on the instructions in the curriculum materials, as these materials contain learning goals and suggestions for instruction (Dutch Inspectorate of Education, 2010; Nicol & Crespo, 2005; Stylianides, 2008). The teachers assess their students’ mastery and provide them with instructional feedback less frequently than that they provide goal-directed instruction. The teachers’ ability to identify mathematical errors and their knowledge of learning trajectories was in both cases sufficiently well developed. Be that as it may, the variance for both variables indicates that not all teachers had a sufficient understanding of the mathematical errors and the learning trajectories. Other research confirms this finding suggesting that teachers are capable of analysing the students’ errors, but that this is nonetheless a challenging endeavour (Heritage et al., 2009; McGuire, 2013; Schneider & Gowan, 2013).

As we expected with regard to the relations between the three CFA elements, in this study, the teachers who provide goal-directed instruction also tend to assess their students’ mastery of these goals more often. This relation was, however, only moderately positive. This might mean that, although teachers do assess their students’ mastery often, the assessment might not always be based on the goals they have set for instruction. This finding is in line with other research stating that some teachers tend to assess their students’ understanding without setting clear goals and criteria for success (cf. Antoniou & James, 2013). Our analyses also show that there is a rather strong relation between the teachers’ use of assessments and timely instructional feedback. This is not surprising as the assessment should provide the teacher with information about gaps in the students’ current level of understanding of the learning goal. This information is considered to be a prerequisite for providing effective instructional feedback (Hattie &
Finally, we found that there is only a moderate positive relationship between the teachers’ provision of instructional feedback and goal-directed instruction. This indicates that not all teachers base their goals for instruction on the instructional feedback they have provided before. This is perhaps a result of the teachers’ use of the curriculum materials, which specify learning goals for each lesson that teachers seem to follow up on regardless of their findings during the lessons.

As regards the relations between the teachers’ knowledge of mathematical errors and learning trajectories and the teachers’ use of the CFA elements we found some unexpected results. Firstly, there was a weak positive relationship between the teachers’ knowledge of errors and goal-directed instruction. Possibly, in some cases teachers’ knowledge of errors helps them to focus their instruction and set appropriate goals by taking into account possible errors and addressing them during instruction (Hill et al., 2008; Riccomini, 2005). Secondly, contrary to our expectations, the teachers’ knowledge of errors and learning trajectories appeared to be unrelated to the teachers’ use of assessment. In addition, the teachers’ knowledge of learning trajectories was only slightly positively related to the frequency of timely instructional feedback. In our study, these factors do not seem to have a lot of predictive value for the frequency by which the teachers use the CFA elements. An explanation might be that the curriculum materials provide teachers with suggestions for instructional feedback when students experience difficulties. Therefore, the teachers might not feel the need to know precisely which errors students make, while some knowledge of mathematical learning trajectories may suffice in being able to select appropriate scaffolds for instructional feedback from the curriculum materials. Another explanation for these unexpected results might be that knowledge of errors and learning trajectories perhaps say more about the quality by which the CFA elements are used than about their mere presence. Teachers do not have to have knowledge of mathematical errors or learning trajectories to believe that they are assessing the work of their students and giving appropriate feedback. Therefore, it might be necessary to pay attention to the knowledge of teachers as a quality measure, but not as a restrictive condition for CFA to take place.

During this study, we were confronted with several issues that might have influenced the results of our study. Firstly, we have tried to find out whether the teachers in our study used the CFA elements in a coherent
manner. We found that the elements related rather strongly to one another. However, as we used a cross-sectional design, we do not know whether teachers who provide goal-directed instruction indeed follow this up with assessment. Nor do we know for a fact whether the teachers who assess their students’ mastery, will use this information to provide instructional feedback. We were merely able to determine the plausibility of these relations by means of our path model.

Secondly, our sample of 137 respondents was small and perhaps biased by self-selection. It is possible that the 137 participating teachers were more interested in CFA than the teachers who chose to not fill in the survey. This may have resulted in more positive outcomes as regards teachers’ use of the CFA elements and perhaps even their knowledge of mathematical errors and learning trajectories. Thus, our sample makes it difficult to extend our conclusions to the whole population of teachers. Additionally, this study was conducted in Dutch primary education. We cannot be sure that our results will also be found if the study is conducted elsewhere.

Nonetheless, the results of this study in combination with the growing interest for teachers’ use of CFA and the influence of teachers’ mathematical knowledge for teaching on this practice may serve as a starting point for further research with larger samples and in other countries. It would be particularly interesting to find out whether the model found in this study can be retraced in practice.