Mechanical energy in toddler gait

A trade-off between economy and stability?

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Summary

Mechanical energy expenditure was investigated in children who are just learning to walk and compared with adult mechanical energy expenditure during walking. First, we determined whether the inverted pendulum (IP) mechanism of energy exchange was present in toddlers. It seems that new walkers partially make use of this energy saving mechanism, but it is less efficient than in adults. The reduced recovery values (R=40% at optimal speeds in toddlers compared to 70% in adults) can be explained by their low self-selected walking speed in combination with their tossing gait (large vertical oscillations of the body) and by the observation that during as much as 25–50% of the gait cycle kinetic and potential energy are oscillating in-phase.

The second step was to calculate positive external mechanical work ($W_{ext}$). Since the IP mechanism is less efficient in toddlers, more mass-specific positive work has to be performed to lift and accelerate the centre of mass than in adults walking at the same speed, even when differences in body size are taken into account.

The amount of positive internal work ($W_{int,k}$) necessary to move the body segments relative to the centre of mass was the third parameter we calculated. In toddlers $W_{int,k}$ is largely determined by the kinetic energy of the lower limb. Compared to adults, toddlers have to perform less mass-specific work per unit distance to accelerate the body segments since the upper body is kept relatively stiff during walking and there is no arm swing.

Apart from work performed on the centre of mass and work performed to move the body segments relative to the centre of mass, when walking some work is also performed during double contact as both legs are pushing against each other. Two methods were used to calculate this amount of work, both leading to the same conclusions. Mass-specific work during double contact is small in toddlers compared to adults because of their low walking speed.

Finally the total amount of mechanical work performed in toddlers was compared to the work production observed in adults. $W_{tot}$ seems to be the major determinant for total mechanical energy expenditure. At intermediate Froude numbers work production is comparable between adults and toddlers, but at low and high Froude numbers $W_{tot}$ increases due to the steep increases in $W_{ext}$. Despite the fact that mechanical work requirements in toddler gait are underestimated if work during double contact is not taken into account, it is not a major determinant of the energy cost of walking.

Key words: toddler, gait, energetics, inverted pendulum, stability.

Introduction

During walking metabolic energy is consumed, even if the average walking speed is constant and there is no net change in height of the body. Energy is lost at each step and has to be put into the system again. The total amount of positive muscle–tendon work that has to be performed can be divided into two categories: external work and internal work.

External work ($W_{ext}$) is the amount of work performed to lift and accelerate the centre of mass. To minimize $W_{ext}$ adults make use of an imperfect inverted pendulum (IP) mechanism of energy exchange (Fig. 1), which was first formulated by Cavagna and others (Cavagna et al., 1963, 1966, 1976) over 40 years ago. The IP is characterized by an out-of-phase oscillation of potential ($E_p$) and kinetic energy ($E_k$) allowing energy exchange to occur. At preferred walking speed, as much as 70% of the required external mechanical energy can be recovered due to this energy saving mechanism (Cavagna et al., 1977). The other 30% of external mechanical energy is lost from the system and must be supplied by the muscles.

Internal work comprises all the work performed by the muscles and tendons that does not directly lead to a displacement of the centre of mass. In the past only the work necessary to accelerate the body segments relative to the centre...
of mass (classical internal work, $W_{\text{int,k}}$) was measured. Recently both Donelan et al. (2002; individual limbs method) and Bastien et al. (2003; $W_{\text{int,dc}}$) developed a method to determine another component of internal work, the work done during double contact when both legs are working against each other. At this time the propulsive back leg has to overcome the energy absorbed by the braking front leg to maintain a constant walking speed.

Children aged between 3 and 11 years old consume more energy per unit body mass to walk at a given speed than do adults (Schepens et al., 2004). Both $W_{\text{ext}}$ and $W_{\text{int,k}}$ are larger at speeds above 0.5 m s$^{-1}$. $W_{\text{int,dc}}$ reaches a maximum at lower speeds in younger subjects. However, differences between adults and children above the age of 3 years disappear when mechanical work is expressed as a function of the dimensionless froude number that takes into account the differences in body proportions (Schepens et al., 2004). This suggests it is the small stature of children that makes them consume more energy than adults when walking at a given speed. In other words, children above the age of 3 years are dynamically similar to adults, at least with respect to the energetics of walking.

Is this also the case in toddlers who are just learning to walk? They are even smaller and show markedly different body proportions compared to adults. Balance problems and immature control of movement are compromising factors that make their gait pattern different from mature walking. Previous studies on infant walking have revealed differences between adults and toddlers in the spatio–temporal gait parameters, joint kinematics and ground reaction force patterns (e.g. Statham and Murray, 1971; Burnett and Johnson, 1971; Endo and Kimura, 1972; Sutherland et al., 1980; Grimshaw et al., 1998). Considering the toddler as a mechanical system, the observed kinematic and kinetic differences could result in different energy and power requirements of the system.

The aim of this paper is to find out whether toddlers differ from adults in mechanical energy production. The amount of external and internal work was calculated and evaluated over a range of speeds. To find out whether toddlers make use of the IP mechanisms of energy exchange the time profiles of $E_p$ and $E_k$ were considered and percentages of recovered energy were calculated. We were also particularly interested in the work performed during double contact, when the front and back limbs are working against each other. The double support phase accounts for a substantial portion of the gait cycle. We wanted to find out whether this prolonged phase of double support had an effect on the mechanical work requirements.

**Materials and methods**

**Study subjects**

Nine healthy children aged between 12 and 18 months participated in this study. Their walking experience ranged from 2 weeks to 6 months. Walking experience was defined as the time period between the onset of independent walking (ability to perform 2–3 consecutive steps) and the time of

<table>
<thead>
<tr>
<th>Child</th>
<th>Age (months)</th>
<th>Gender</th>
<th>Walking experience (weeks)</th>
<th>Body mass (kg)</th>
<th>Height (m)</th>
<th>Leg length (m)</th>
<th>Number of trials</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>13.5</td>
<td>Female</td>
<td>2</td>
<td>11</td>
<td>0.73</td>
<td>0.31</td>
<td>5</td>
</tr>
<tr>
<td>T2</td>
<td>15</td>
<td>Female</td>
<td>6</td>
<td>8</td>
<td>0.74</td>
<td>0.29</td>
<td>4</td>
</tr>
<tr>
<td>T3</td>
<td>15.5</td>
<td>Male</td>
<td>11</td>
<td>10</td>
<td>0.74</td>
<td>0.32</td>
<td>1</td>
</tr>
<tr>
<td>T4</td>
<td>16.5</td>
<td>Female</td>
<td>11</td>
<td>13</td>
<td>0.76</td>
<td>0.33</td>
<td>4</td>
</tr>
<tr>
<td>T5</td>
<td>15.5</td>
<td>Male</td>
<td>13</td>
<td>11</td>
<td>0.76</td>
<td>0.28</td>
<td>5</td>
</tr>
<tr>
<td>T6</td>
<td>16.5</td>
<td>Female</td>
<td>14</td>
<td>9</td>
<td>0.76</td>
<td>0.30</td>
<td>5</td>
</tr>
<tr>
<td>T7</td>
<td>15.5</td>
<td>Male</td>
<td>15</td>
<td>12</td>
<td>0.77</td>
<td>0.34</td>
<td>5</td>
</tr>
<tr>
<td>T8</td>
<td>18.5</td>
<td>Male</td>
<td>22</td>
<td>12</td>
<td>0.80</td>
<td>0.31</td>
<td>2</td>
</tr>
<tr>
<td>T9</td>
<td>17</td>
<td>Female</td>
<td>27</td>
<td>11</td>
<td>0.80</td>
<td>0.32</td>
<td>1</td>
</tr>
</tbody>
</table>
testing. Detailed information on the study subjects can be found in Table 1.

The ethical review board of the University of Antwerp approved the study protocol. Prior to participation, parents gave their informed consent. All experiments were carried out according to the guidelines stated in the Declaration of Helsinki.

**Experimental set-up**

Data were collected at the HIKE campus of the department of Health Care (Hoger Instituut voor Kinesitherapie en Ergotherapie, Hogeschool Antwerpen, Belgium). The experimental set-up consisted of an instrumented walkway (3 m x 1.5 m) surrounded by six infrared cameras (Mcam 460, 250 Hz; Vicon Motion Systems, Oxford, UK). Two force platforms (AMTI, MA, USA; 0.5 m x 0.4 m, 250 Hz) were built into the walkway to record ground reaction forces under the left and right foot separately.

The Helen-Hayes marker set-up was used for measuring full body kinematics (Fig. 2). The retro-reflective markers (14 mm) were sewn on to a tight-fitting suit to prevent problems with ‘marker plucking’ in young children. Foot markers were attached to socks or soft leather shoes.

The children were encouraged to walk over the platform towards a parent or experimenter at self-selected speeds. We tried to obtain five successful trials for each individual. A successful trial was defined as a trial for which ground reaction force measurements of both feet were available during at least one complete stride and all markers were visible throughout the trial. Sometimes the children became tired and failed to cooperate further before a sufficient number of trials could be obtained. The number of successful trials per individual is given in Table 1. After performing the calculations, the results from the different trials of each individual were averaged to prevent pseudo-replication.

In energetic analysis it is generally required that the average walking speed is fairly constant over the whole trial. Toddlers, however, are constantly accelerating and decelerating, taking a few steps and stopping again. The average net speed change over a trial amounts to 0.10 m·s⁻¹. Because of their low walking speed this accounts for an average variation in speed of 25%.

**Data analysis**

**The body centre of mass (COM)**

The three-dimensional (3-D) velocities of the COM and the vertical oscillations were determined by integrating the resultant of the ground reaction forces underneath both feet. This technique was first formulated by Cavagna (1975) and is well described in literature (e.g. Cavagna et al., 1983; Willems et al., 1995). Therefore it is only briefly discussed here.

If air resistance is neglected, the 3-D accelerations of the COM can be calculated using Equations 1–3. Body weight was determined by dividing the time impulse of the resultant

vertical ground reaction force during an entire gait cycle (period between two contacts of the same foot, expressed from 0 to 100% in Fig. 1) by stride time, thus:

\[
A_x = F_x / M_{tot}, \quad (1)
\]

\[
A_y = F_y / M_{tot}, \quad (2)
\]

and

\[
A_z = (F_z - 9.81 M_{tot}) / M_{tot}, \quad (3)
\]

where \((A_x, A_y, A_z)\) = 3-D linear accelerations of the COM, \(F_x\) = lateral force component, \(F_y\) = fore–aft force component, \(F_z\) = vertical force component and \(M_{tot}\) = body mass.

The 3-D linear velocities of the COM \((V_x, V_y, V_z)\) were determined by numerical integration of the 3-D accelerations \((A_x, A_y, A_z)\). Integration constants were determined so the average \(V_x, V_y\) and \(V_z\) equalled the average 3-D linear velocities measured by the video system. The vertical displacement \((z)\) of the COM was calculated by numerical integration of \(V_z\).

**The inverted pendulum mechanism**

Potential \((E_p)\) and kinetic \((E_k)\) energy fluctuations were calculated according to Equations 4 and 5 and plotted as a function of gait cycle duration:

\[
E_p = M_{tot} \times 9.81 \text{COM}(z), \quad (4)
\]

and

\[
E_k = \frac{1}{2} M_{tot} (V_x^2 + V_y^2 + V_z^2). \quad (5)
\]

To find out how well the IP mechanism is working in toddlers, recovery values \((R)\) were calculated (Equation 6) and plotted.

![Fig. 2. A modified Helen-Hayes marker set-up was used for measuring full body kinematics. Markers were attached to a tight fitting suit to prevent problems with ‘marker plucking’ in young children. Foot markers were attached to socks or soft leather shoes.](image-url)
as a function of walking speed (v) and froude number \( \sqrt{v/(9.81 \times \text{leg length})} \). \( R \) is a measure of the pendulum-like transfer between \( E_p \) and \( E_k \) observed in (mature) walking (Cavagna et al., 1976).

\[
R = \frac{\Delta^+ E_p + \Delta^+ E_k - \Delta^+ E_{tot}}{\Delta^+ E_p + \Delta^+ E_k},
\]

where \( \Delta^+ E_p \) = the sum of the positive increments in \( E_p \) over an integral number of steps, \( \Delta^+ E_k \) = the sum of the positive increments in \( E_k \) over an integral number of steps and \( \Delta^+ E_{tot} \) = the sum of the positive increments in the total mechanical energy curve over an integral number of steps (total mechanical energy is the sum of \( E_p \) and \( E_k \)).

The possibility of energy transfer depends not only on the shape of the \( E_p \) and \( E_k \) curves but also on their relative magnitude and phase relationship. Energy exchange is optimal when both curves show equal amplitudes. Therefore their relative amplitudes (RA) were calculated (Equation 7) and plotted as a function of walking speed and froude number:

\[
RA = \frac{\max(E_p) - \min(E_p)}{\max(E_k) - \min(E_k)}. \tag{7}
\]

Concerning the phase relationship between \( E_p \) and \( E_k \), energy exchange is optimal when they are oscillating exactly 180° out-of-phase. In walking, however, \( E_p \) and \( E_k \) are never exactly in- or out-of-phase. Percentage congruity (Ahn et al., 2004) measures the proportion of the pendulum cycle during which \( E_p \) and \( E_k \) change similarly in direction. In an ideal inverted pendulum, \% congruity would equal zero. \% congruity was determined as the proportion of the gait cycle during which the pendulum, \% congruity would equal zero. \% congruity was determined by adding the positive increments in \( E_{tot} \) over an integral number of steps (total mechanical energy is the sum of \( E_p \) and \( E_k \)).

**External mechanical work**

The amount of positive work performed on the COM \( (W_{ext}) \) was determined by adding the positive increments in \( E_{tot} \) over an integral number of steps. \( W_{ext} \) was plotted as a function of walking speed and froude number.

**Classical internal mechanical work**

To determine the kinetic energies of the body segments, a 12-segment body model (Table 2) was developed based on anthropometrical data for toddlers from Sun and Jensen (1994) (Table 3). For each body segment the \( (x, y, z) \) positions of its segmental centre of mass were calculated based on the filtered (quintic spline) 3-D marker trajectories (Table 2). The 3-D linear velocities of the body segments \( (v_x, v_y, v_z) \) were obtained from the time derivatives of the \( (x, y, z) \) positions of the segmental centres of mass.

To determine the rotational kinetic energy of the body segments, only rotations in the sagittal plane were considered. For each segment a line vector was created between two points: \( a (y_1, z_1) \) and \( b (y_2, z_2) \). Definitions of origins \( (a) \) and endpoints \( (b) \) for each segment are found in Table 2. The angular position of this line vector \( (\alpha_i) \) for each instant in time was calculated using Equation 8:

\[
\alpha_i = \arctan((z_2-z_1)/(y_2-y_1)). \tag{8}
\]

The angular velocity \( (\dot{\alpha}_i) \) of each line segment was then determined by differentiating \( \alpha_i \) with respect to time.

**Table 3. Segmental intertial parameters based on anthropometrical data for toddlers***

<table>
<thead>
<tr>
<th>Segment</th>
<th>Mass (m)</th>
<th>Moment of inertia (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>1.315±0.018x</td>
<td>2.63×10⁻³±6.77×10⁻⁶x</td>
</tr>
<tr>
<td>Trunk</td>
<td>2.336±0.026x</td>
<td>5.53×10⁻³±1.12×10⁻⁴x</td>
</tr>
<tr>
<td>L upper arm</td>
<td>0.101±0.003x</td>
<td>2.54×10⁻⁵±4.93×10⁻⁶x</td>
</tr>
<tr>
<td>R upper arm</td>
<td>0.101±0.003x</td>
<td>2.54×10⁻⁵±4.93×10⁻⁶x</td>
</tr>
<tr>
<td>L lower arm</td>
<td>0.120±0.001x</td>
<td>7.15×10⁻⁵±2.44×10⁻⁶x</td>
</tr>
<tr>
<td>R lower arm</td>
<td>0.120±0.001x</td>
<td>7.15×10⁻⁵±2.44×10⁻⁶x</td>
</tr>
<tr>
<td>L thigh</td>
<td>0.294±0.010x</td>
<td>2.73×10⁻⁵±3.47×10⁻⁵x</td>
</tr>
<tr>
<td>R thigh</td>
<td>0.294±0.010x</td>
<td>2.73×10⁻⁵±3.47×10⁻⁵x</td>
</tr>
<tr>
<td>L shank</td>
<td>0.161±0.004x</td>
<td>7.37×10⁻⁵±1.02×10⁻⁵x</td>
</tr>
<tr>
<td>R shank</td>
<td>0.161±0.004x</td>
<td>7.37×10⁻⁵±1.02×10⁻⁵x</td>
</tr>
<tr>
<td>L foot</td>
<td>0.130–0.012x</td>
<td>1.07×10⁻⁶±1.93×10⁻⁶x</td>
</tr>
<tr>
<td>R foot</td>
<td>0.130–0.012x</td>
<td>1.07×10⁻⁶±1.93×10⁻⁶x</td>
</tr>
</tbody>
</table>

**Table 2. The 12-segment body model**

<table>
<thead>
<tr>
<th>Segment</th>
<th>COM</th>
<th>Origin (a)</th>
<th>End (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>( \frac{1}{4}(\text{LFHD}+\text{RFHD}+\text{LBHD}+\text{RBHD}) )</td>
<td>COM head</td>
<td>( \frac{1}{4}(\text{LSHO}+\text{RSHO}) )</td>
</tr>
<tr>
<td>Trunk</td>
<td>( \frac{1}{4}(\text{LSHO}+\text{RSHO}+\text{LASI}+\text{RASI}) )</td>
<td>LSHO</td>
<td>( \frac{1}{4}(\text{LSHO}+\text{RSHO}) )</td>
</tr>
<tr>
<td>L upper arm</td>
<td>( \frac{1}{2}(\text{LSHO}+\text{LELB}) )</td>
<td>LSHO</td>
<td>( \frac{1}{2}(\text{LSHO}+\text{RSHO}) )</td>
</tr>
<tr>
<td>R upper arm</td>
<td>( \frac{1}{2}(\text{RSHO}+\text{RELB}) )</td>
<td>RELB</td>
<td>( \frac{1}{2}(\text{RSHO}+\text{RASI}) )</td>
</tr>
<tr>
<td>L lower arm</td>
<td>( \frac{1}{2}(\text{LELB}+\text{LWRI}) )</td>
<td>LELB</td>
<td>RELB</td>
</tr>
<tr>
<td>R lower arm</td>
<td>( \frac{1}{2}(\text{RELB}+\text{RWRI}) )</td>
<td>RWRI</td>
<td>LWR</td>
</tr>
<tr>
<td>L thigh</td>
<td>( \frac{1}{2}(\text{LASI}+\text{LKNE}) )</td>
<td>LASI</td>
<td>LKNE</td>
</tr>
<tr>
<td>R thigh</td>
<td>( \frac{1}{2}(\text{RASI}+\text{RKNE}) )</td>
<td>RASI</td>
<td>RKNE</td>
</tr>
<tr>
<td>L shank</td>
<td>( \frac{1}{2}(\text{LKNE}+\text{LANK}) )</td>
<td>LKNE</td>
<td>LANK</td>
</tr>
<tr>
<td>R shank</td>
<td>( \frac{1}{2}(\text{RKNE}+\text{RANK}) )</td>
<td>RKNE</td>
<td>RANK</td>
</tr>
<tr>
<td>L foot</td>
<td>( \frac{1}{2}(\text{LANK}+\text{LHEE}+\text{RTOE}) )</td>
<td>LANK</td>
<td>COM L foot</td>
</tr>
<tr>
<td>R foot</td>
<td>( \frac{1}{2}(\text{RANK}+\text{RHEE}+\text{RTOE}) )</td>
<td>RANK</td>
<td>COM R foot</td>
</tr>
</tbody>
</table>

*Data taken from Sun and Jensen (1994).

x = age in weeks.
To calculate the kinetic energy of the body segments translational and rotational terms were summed (Equation 9):

$$E_{k,int} = \frac{1}{2} \sum m_i (v_{x,i}^2 + v_{y,i}^2) + I_i \dot{\theta}_i^2,$$

where $m_i =$ segmental mass and $I_i =$ the segmental moment of inertia in the sagittal plane.

By adding the positive increments in the internal energy curve over an integral number of steps, positive internal work ($W_{int,k}$) was determined. $W_{int,k}$ was plotted as a function of walking speed and froude number.

**Work during double contact**

Alexander and Jayes (1978) recognized that work is performed during double support, due to the fact that one leg is pushing against the other. Classical methods for calculating $W_{ext}$ and $W_{int,k}$ do not account for this. Recently two models have been proposed to determine the amount of work that has to be produced by the propulsive back leg to overcome the energy absorption by the front leg.

The first model (Fig. 3A) was proposed by Donelan et al. (2002). The model pictures the legs as two rigid struts joined by the centre of mass. During single support, the mass moves over the supporting limb as an inverted pendulum and energy exchange is allowed between $E_p$ and $E_k$. Double support is seen as a transition state during which the COM velocity is redirected from one pendular arc to the next. Instead of calculating the amount of work performed on the COM from the resultant of the ground reaction forces of both limbs, positive work performed by the front limb and back limb during a step is calculated separately (Equations 10–12) and then summed:

$$W_{ILM}^+ = W_{front}^+ + W_{back}^+,$$

$$W_{front}^+ = \int (F_{x,fron}V_x + F_{y,fron}V_y) \, dt,$$

$$W_{back}^+ = \int (F_{x,back}V_x + F_{y,back}V_y) \, dt.$$

This $W_{ILM}$ is the sum of work actually performed on the COM (and equal to the classical $W_{ext}$) and work resulting from the opposite action of both legs on the COM during double support (which does not lead to a change in velocity or height of the centre of mass and therefore is actually internal work).

The second model (Fig. 3B) was proposed by Bastien et al. (2003). The body consists of a COM and two sticks for the legs. The legs are seen as two oscillating actuators performing work on the COM. Since both actuators are performing work on the same structure, they can also perform work on each other. This feature is what distinguishes Bastien’s model from the ILM. Energy exchange is not only possible between $E_p$ and $E_k$ during single support but also during double support, where which the COM moves over the stiff supporting limb. Energy exchange is allowed between $E_p$ and $E_k$. Double support is a transition state during which the COM is redirected from one pendular arc to the next. To maintain a constant walking speed, the propulsive back leg (black lines) has to perform work to overcome the braking action of the front leg (grey lines) on the COM. When using the classical method of calculating work performed on the COM (i.e. by integration of the resultant of forces underneath both feet) simultaneous positive ($W_{y,back}$) and negative ($W_{y,fron}$) work is cancelled out and work during double contact is underestimated. The ILM calculates the amount of positive work performed by the front and back limb separately, consequently opposite work of both limbs during double contact is not cancelled out. (B) Bastien et al. (2003) proposed an alternative method for calculating internal work during double contact ($W_{int,k}$). The legs are seen as two oscillating actuators performing work on the COM. Since both are performing work on the same structure, they can also perform work on each other. Consequently the energy absorbed when the downward movement of the COM is slowed down after foot contact, ($W_e$ is decreasing during the first half of double support) can be used by the propulsive back limb to accelerate the COM forward (transfer of energy from $W_e$ to $W_{back}$). Also, the energy absorbed by the front limb ($W_{fron}$) can be used during the second half of double support to lift the COM against gravity (transfer from $W_{fron}$ to $W_n$, which is increasing during the second half of double support). Allowing for this energy transfer between the front and back limb will reduce the amount of work that has to be performed by the back limb to overcome the braking action of the front limb. $F$, force component.
a transfer of energy is allowed between the front and back limb. Allowing this energy transfer will decrease the amount of work that has to be performed by the back limb to overcome the braking action of the front limb.

Bastien et al. (2003) ignore work resulting from the lateral force component. However, Donelan et al. (2001) showed that the lateral component might be important in the case of a wide base of support. Therefore, in the current study, the instantaneous work curve resulting from the lateral force component was compared to the instantaneous work curves resulting from the fore–aft and vertical force components. Lateral work showed to be negligible, despite the wide base of support, and was ignored for further analysis.

Following Bastien et al. (2003), we first calculated the four components of work from the individual ground reaction force measurements (Equations 13–16) during double support:

\[
W_y,\text{front} = \int F_y,\text{front} V_y \, dt ,
\]

\[
W_z,\text{front} = \int F_z,\text{front} V_z \, dt ,
\]

\[
W_y,\text{back} = \int F_y,\text{back} V_y \, dt ,
\]

\[
W_z,\text{back} = \int F_z,\text{back} V_z \, dt .
\]

The work done by the vertical ground reaction force components of both limbs can be added because they will always simultaneously do positive and/or negative work:

\[
W_v = W_{z,\text{front}} + W_{z,\text{back}} .
\]

Following foot contact the downward movement of the COM is slowed down and energy is absorbed by the front limb (\(W_v\) is decreasing during the first half of double support in Fig. 3B). If energy transfer is allowed between both limbs, the back limb can use this energy to aid in forward propulsion. Thus the work performed by the back limb equals the horizontal work of the back limb minus the amount of energy absorbed by the front limb.

If \(W_v\) is decreasing:

\[
W_{\text{back}} = W_{y,\text{back}} - |W_v| ; \tag{16A}
\]

if \(W_v\) is increasing:

\[
W_{\text{back}} = W_{y,\text{back}} . \tag{16B}
\]

The front limb also slows down the forward movement of the COM following foot contact. If energy transfer is allowed, the absorbed energy can be used by the back limb to lift the COM again (\(W_v\) is increasing during the second half of double support in Fig. 3B). Thus the work performed by the front limb equals the horizontal work of the front limb plus the amount of positive vertical work transferred to the back limb.

If \(W_v\) is decreasing:

\[
W_{\text{front}} = W_{y,\text{front}} ; \tag{17A}
\]

if \(W_v\) is increasing:

\[
W_{\text{front}} = W_{y,\text{front}} + |W_v| . \tag{17B}
\]

Adding \(W_{\text{front}}\) and \(W_{\text{back}}\) instant-by-instant results in the amount of work performed on the COM during double contact (\(W_{\text{com}}\)). To obtain the work resulting from the opposite action of both legs on the COM (\(W_{\text{int,dc}}\)), the positive increments in \(W_{\text{com}}\) have to be subtracted from the sum of the positive increments in \(W_{\text{back}}\) and \(W_{\text{front}}\).

\[
W_{\text{int,dc}} = W_{y,\text{front}}^+ + W_{y,\text{back}}^+ - W_{\text{com}}^+ . \tag{18}
\]

To be able to compare the ILM and the method proposed by Bastien et al. (2003), \(W_{\text{ILM}}\) and the sum of \(W_{\text{ext}}\) and \(W_{\text{int,dc}}\) were plotted as a function of gait cycle duration. The oscillations in \(W_p\) are largest and almost completely determine the total mechanical energy fluctuations. (B) The average kinetic energy fluctuations in the forward (\(y\)), lateral (\(z\)) and vertical (\(z\)) directions are plotted as a function of gait cycle duration. \(E_k\) is entirely determined by the forward kinetic energy fluctuations.
Mechanical energy in toddler gait

**Total mechanical work**

To determine total mechanical work, $W_{\text{ext}}$, $W_{\text{int},k}$ and $W_{\text{int,dc}}$ were added and plotted as a function of walking speed and froude number. The obtained result was compared to the sum of $W_{\text{ILM}}$ and $W_{\text{int,k}}$.

**The effect of walking experience**

Since the participating children show a fairly large range of walking experiences (from 2 weeks to 6 months), the effect of increasing experience in walking on average walking speed, the inverted pendulum (IP) mechanism, $W_{\text{ext}}$, $W_{\text{int,k}}$ and $W_{\text{int,dc}}$ were also investigated.

**Results**

**The inverted pendulum mechanism**

The average mechanical energy fluctuations in toddlers show a sinusoidal oscillation, with two maxima and two minima occurring during one gait cycle (Fig. 4A). The oscillations in $E_p$ are largest in amplitude and almost completely determine the total mechanical energy fluctuations. Oscillations in $E_k$ are dependent upon walking speed. With increasing walking speed, the $E_k$ oscillations increase (cf. the decrease in $RA$ with increasing walking speed, which will be discussed below). The forward kinetic energy is the most important component, almost completely determining the fluctuations in $E_k$ (Fig. 4B).

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**Fig. 5.** (A) Recoveries $R$ are plotted as a function of walking speed for both adults (triangles) and toddlers (circles). (Adult data were reproduced from Willems et al., 1995.) In both age groups $R$ shows an inverted U-shape relationship with walking speed. $R$ reaches a maximum at lower speeds in toddlers. (B) Adult (triangles) and toddler (circles) $R$-values are plotted as a function of the dimensionless froude number. In both age groups $R$ is maximal around froude number 0.4. In toddlers, however, $R$ is smaller than in adults. (C) % Congruity expresses the % of the gait cycle during which $E_k$ and $E_p$ are oscillating in-phase. % Congruity shows a U-shaped relationship with speed, reaching a minimum at the speed when $R$ was maximal. (D) % Congruity shows a U-shaped relationship with froude number and is minimal at froude number 0.4. (E,F) Relative amplitude $RA$. The amplitude of $E_k$ is much larger than the amplitude of $E_p$. With increasing walking speed (or froude number) the $E_k$ oscillations increase. However, they never exceed the fluctuations in $E_p$ (cf. $RA$ always $>1$). $r^2$ values apply to toddlers only.
Fig. 6. (A) External work $W_{\text{ext}}$ is plotted as a function of walking speed for both adults (triangles) and toddlers (circles). (Adult data were reproduced from Willems et al., 1995.) $W_{\text{ext}}$ shows a U-shaped relationship with walking speed, reaching a minimum at speeds around 0.6 m s$^{-1}$. Toddlers seem to produce a higher amount of external mechanical work to walk at the same speed than adults. (B) Differences between adults and toddlers do not disappear when $W_{\text{ext}}$ is plotted as a function of the dimensionless froude number. This suggests that other factors, apart from their small stature, make toddlers consume more energy than adults. (C) Positive mechanical work $W_{\text{int,k}}$ required to move the body segments relative to the centre of mass is plotted as a function of walking speed for both adults (triangles) and toddlers (circles). (Adult data were reproduced from Willems et al., 1995.) $W_{\text{int,k}}$ seems to be larger in toddlers. (D) If $W_{\text{int,k}}$ is plotted as a function of dimensionless froude number, $W_{\text{int,k}}$ is smaller in toddlers. (E,F) Work during double contact. $W_{\text{ext}}$ (black line), $W_{\text{ILM}}$ (circles) and the sum of $W_{\text{ext}}$ and $W_{\text{int,dc}}$ (+) are compared as a function of walking speed and froude number. As expected, $W_{\text{ILM}}$ is larger than the sum of $W_{\text{ext}}$ and $W_{\text{int,dc}}$. However, the shape of the curves and the position of the optimum walking speed do not change, regardless of the method used. (G) Total mechanical work $W_{\text{tot}}$ is plotted as a function of walking speed for both adults (triangles) and toddlers (circles). (Adult data were reproduced from Willems et al., 1995.) $W_{\text{ext}}$ shows a U-shaped relationship with walking speed, reaching a minimum at speeds around 0.6 m s$^{-1}$. (H) If $W_{\text{tot}}$ is plotted as a function of froude number (and differences in size between adults and toddlers are taken into account), $W_{\text{tot}}$ is comparable between adults and toddlers around froude number 0.4. At lower and higher froude numbers sharp increases in $W_{\text{tot}}$ are observed. $r^2$ values apply to toddlers only.
Recovery values are plotted as a function of walking speed in Fig. 5A. $R$ shows an inverted U-shape relationship with speed ($r^2=0.63$). At optimal speed almost 40% of the external mechanical energy required to lift and accelerate the COM can be recovered. Compared to adults, optimal speed in toddlers is much smaller (0.6 m s$^{-1}$ in toddlers compared to 1.65 m s$^{-1}$ in adults, Fig. 5A). To correct for the differences in size between adults and toddlers, $R$ was also plotted as a function of froude number (Fig. 5A). At equal froude number, $R$ is much smaller in toddlers compared to adults. In both age groups, energy exchange seems to be optimal around froude number 0.4.

Relative amplitudes are plotted as a function of walking speed (Fig. 5E) and froude number (Fig. 5F). With increasing walking speed (and also increasing froude number), $RA$ decreases but the oscillations in $E_p$ remain larger than the oscillations in $E_k$ (RA never below 1).

The phase relationship between $E_p$ and $E_k$ is expressed as % congruity (Fig. 5C,D), which equals the percentage of the gait cycle during which $E_p$ and $E_k$ are moving in the same direction. During as much as 25–50% of the gait cycle $E_p$ and $E_k$ are oscillating in phase. % Congruity shows a U-shaped relationship with speed (Fig. 5C; $r^2=0.46$), reaching a minimum at approximately 0.6 m s$^{-1}$ (around froude number 0.4).

**External mechanical work**

$W_{ex}$ plotted as a function of walking speed shows a U-shaped curve (Fig. 6A; $r^2=0.71$) reaching a minimum at a speed of 0.6 m s$^{-1}$, which coincides with the speed when $R$ was maximal. When compared to adults, toddlers show a tendency to produce a higher amount of mass-specific work per unit distance over the entire speed range observed (0.1–1.0 m s$^{-1}$). Contrary to what is seen in children above 3 years of age, the

![Fig. 7. The kinetic energies of the body segments are plotted as a function of gait cycle duration. The kinetic energies of the head, trunk and arms are small since the upper body is kept relatively stiff during walking and there is no arm swing. The kinetic energies of the thigh and shank are most important.](image-url)
difference in external work production between adults and toddlers does not disappear when $W_{\text{ext}}$ is plotted as a function of the dimensionless froude number (Fig. 6B). $W_{\text{ext}}$ remains larger in toddlers than in adults.

**Classical internal mechanical work**

$W_{\text{int,k}}$ shows a positive linear relationship with walking speed (Fig. 6C, $r^2=0.60$). Compared with adults, toddlers seem to perform more mass-specific internal work when walking at the same speed. To eliminate differences due to different body proportions, $W_{\text{int,k}}$ was plotted as a function of the dimensionless froude number in Fig. 6D. If body size is taken into account, $W_{\text{int,k}}$ is smaller in toddlers compared to adults.

The kinetic energies of the body segments are plotted in Fig. 7. Kinetic energies of the head, the trunk and the arms are relatively small compared to the kinetic energies of the thigh and shank.

**Internal work during double contact**

In Fig. 6E,F $W_{\text{ILM}}$ and the sum of $W_{\text{ext}}$ and $W_{\text{int,dc}}$ are compared to $W_{\text{ext}}$ calculated by the classical combined limbs method. Also in toddlers, $W_{\text{ILM}}$ as well as $W_{\text{int,dc}}+W_{\text{ext}}$ are larger. For example, at optimal speed $W_{\text{ext}}=0.84 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$, $W_{\text{ext}}+W_{\text{int,dc}}=0.88 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ and $W_{\text{ILM}}=0.94 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$.

On average, the work performed by the back limb to overcome the braking action of the front limbs accounts for 7% (if the method of Bastien et al., 2003 is used) to 16% (if the ILM is used) of the work performed on the COM.

**Total mechanical work**

$W_{\text{tot}}$ shows a U-shaped relationship with walking speed, reaching a minimum around 0.6 m s$^{-1}$, in contrast to adults, who show a positive linear relationship with speed (Fig. 6G). If the ILM is used, $W_{\text{tot}}$ values are slightly higher than when summing $W_{\text{ext}}$, $W_{\text{int}}$ and $W_{\text{int,k}}$ (Fig. 6G: compare at optimal speed: $W_{\text{ext}}+W_{\text{int,dc}}+W_{\text{int,k}}=1.15 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ while $W_{\text{ILM}}+W_{\text{int,k}}=1.24 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$). However, the choice of method has no influence on the position of the optimal walking speed.

At optimal speed, toddlers seem to use more than twice the energy per unit body mass that adults need to walk at the same speed. At speeds slower and faster than the optimal speed the difference is even larger.

$W_{\text{tot}}$ was also plotted as a function of froude number (Fig. 6H) to eliminate differences between adults and toddlers due to body size. Energy consumption is minimal around froude number 0.4. At this froude number, $W_{\text{tot}}$ is comparable in adults and toddlers. But when walking faster or slower, toddlers again consume a lot more energy than adults.

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![Fig. 8](image-url). To investigate the effect (mechanical energy fluctuations) of walking experience (WE) on the inverted pendulum mechanism, individual mean $E_k$ and $E_p$ profiles (+ s.d.) were inspected. The grey bars indicate double support. In the youngest children, the $E_k$ and $E_p$ curves are irregular. The sinusoidal pattern becomes a lot smoother after 3 months of walking experience (28 weeks WE), but variation remains large.
The effect of walking experience

The individual mean $E_p$ and $E_k$ curves are plotted in Fig. 8. In the earliest walkers the sinusoidal oscillations of the energy curves appear to be irregular. After 3 months of independent walking the stereotype pattern with two maxima and two minima begins to show. However, variation remains large, especially in $E_p$.

The effects of walking experience on preferred walking speed, mechanical work production and recovery values are shown in Fig. 9. With increasing walking experience, there is a slight increase in preferred walking speed. Total mechanical work as well as $W_{ext}$ show a U-shaped relationship with walking experience. No relationship is found between the $W_{int,k}$, $R$ values or work during double contact and walking experience ($r^2<0.01$).

Discussion

The inverted pendulum mechanism

Our results suggest that in order to minimize energy expenditure, toddlers are (at least partially) able to use the IP mechanism of energy exchange. At optimal speed, 40% of external mechanical energy can be recovered. However, the IP mechanism seems to be imperfect in toddlers. The relative amplitudes show that the potential energy fluctuations are larger than kinetic energy fluctuations, a feature that is disadvantageous for energy exchange to occur. The large decreases in $E_p$ during the second half of the swing phase can be used to increase $E_k$ when the centre of mass accelerates downwards. But the oscillations in $E_k$ are too small to perform the required amount of work to lift the COM against gravity. The large difference in relative magnitude of the $E_p$ and $E_k$...
Oscillations can be explained by toddlers’ tossing gait (large vertical oscillations of the centre of mass) in combination with their low walking speed.

Also the phase relationship between $E_p$ and $E_k$ is sub-optimal, as the % congruity measurements show that during as much as 25–50% of the gait cycle, $E_p$ and $E_k$ are moving in the same direction. Fig. 8 shows that in some of the children (e.g. a child with 13 weeks of walking experience) after foot contact the COM is still moving down (decrease in $E_p$) during the first half of single support. At this time $E_k$ is also decreasing as this downward movement of the COM is slowed down by flexing the hip and knee (A.H., personal observations).

Positive external work

Positive external work in toddlers is minimal at the speed when energy exchange is optimal (at approximately 0.6 m s$^{-1}$). Despite the fact that the IP mechanism of energy exchange is observed in toddlers, they perform a greater amount of work
Mechanical energy in toddler gait

per unit distance to walk at a given speed than adults, even when their small stature is taken into account (by expressing speed as the dimensionless Froude number). As argued above, an explanation can be found in the differences in gait pattern between adults and toddlers. Both their slow walking speed and tossing gait are disadvantageous for the pendular exchange of energy. Consequently, less energy can be recovered and more work has to be performed to lift and accelerate the COM.

Classical internal work

Positive internal work performed to accelerate the body segments relative to the COM is a second important component of total positive mechanical work performed. It is almost completely dependent on the work performed to swing the limb forward. Work performed on the arms and upper body is negligible since the upper body is kept relatively stiff and arm swing has not yet developed in these children. Exactly these features cause $W_{\text{int,k}}$ to be smaller in toddlers compared to adults.

Mechanical internal work during double contact

Work performed due to the fact that one leg is pushing against the other in toddlers is also a contributing factor to mechanical work production. Contrary to what might be expected due to the prolonged phase of double support in toddlers, this amount of work is small compared to adults. Using the ILM, it accounts for 16% of $W_{\text{ext}}$, whereas Donelan et al. (2002) reported values of $W_{\text{int,dc}}$ reaching up to 33% of $W_{\text{ext}}$ in adults. Bastien et al. (2003) reported that $W_{\text{int,dc}}$ reaches 40% of $W_{\text{ext}}$ in older children and adults, while in toddlers this value is only 7%. Again, an explanation can be found in the combination of a slow walking speed with a tossing gait. As a consequence of these features of immature gait, work resulting from the vertical ground reaction force components is much larger than opposite work resulting from the fore–aft force components (compare the instantaneous work traces in Fig. 10). In other words, because of toddlers' tossing gait the most important component of work during double contact is the work that has to be performed to lift the COM against gravity. Compared to this, the amount of work that has to be performed to overcome the opposite action of the front and back limb in order to maintain a (slow) constant walking speed is relatively small.
Due to different assumptions of the model, the ILM leads to higher work values than the method proposed by Bastien et al. (2003). Fig. 11 shows the difference between both methods. Allowing for energy exchange to occur between the front and back limb flattens the instantaneous work traces (\(W_{\text{front}}\) and \(W_{\text{back}}\)) and reduces the amount of opposite work during double contact. While quantitative results differ upon using one or the other method, the conclusions of both methods remain the same.

**Total mechanical work**

Work performed on the COM is the major contributing factor of total mechanical work performed during walking in toddlers. The second most important contributor to \(W_{\text{tot}}\) is internal work performed to accelerate the body segments relative to the COM. If opposite work performed during double contact is ignored, \(W_{\text{tot}}\) will be underestimated. However, this amount of work is not a determining factor of \(W_{\text{tot}}\) since it does not influence the position of the optimal walking speed.

At froude number 0.4, \(W_{\text{tot}}\) is comparable between adults and toddlers. Recoveries are optimal at this dimensionless speed (at froude number 0.4, \(R\) reaches a maximum of 40%) and thus \(W_{\text{ext}}\) is minimal. Nevertheless it is still larger than the external work production in adults at comparable froude number. Despite this, \(W_{\text{tot}}\) is still comparable between adults and toddlers since \(W_{\text{int,k}}\) is much smaller in toddlers due to the stiff upper body and the absence of arm swing.

**The effect of walking experience**

The individual \(E_p\) and \(E_k\) traces suggest the IP is not fully mastered at the onset of independent walking and starts to mature after 3 months of walking experience. However, these observations were not confirmed by the \(R\) values, which did not change over the range of walking experiences. Possibly by averaging different trials per individual, speed effects are masked. Also, a cross-sectional study is not the best set-up for exploring maturational effects, since in each child the speed at which gait matures will differ. While this study is very valuable for giving insight into the overall mechanisms of energy transfer and mechanical energy costs of bipedal gait in very young walkers, a longitudinal set-up would be more appropriate for investigating subtle changes in mechanical energy expenditure due to motor development and growth. A longitudinal follow-up study of young walkers, is therefore our future goal.

The \(U\)-shaped relationship of \(W_{\text{ext}}\) and consequently also \(W_{\text{tot}}\) with walking experience can be explained by the increase in preferred walking speed. After approximately 3 months of independent walking, toddlers walk at a speed at which energy exchange is optimal for their length. However, when growing older they choose to walk at speeds above the optimal speed and thus mechanical energy production rises again.

**Conclusion**

\(W_{\text{ext}}\) is the major contributing factor of total mechanical work in toddlers. They perform more mass specific work per unit distance than adults to lift and accelerate the COM. An explanation can be found in the fact that, despite the fact that the IP mechanism is observed in toddlers to a certain degree, it has not yet been completely mastered. Because of their tossing gait and slow walking speed, energy exchange is imperfect. Apart from external work performed on the COM, internal work is performed to swing each limb forward during walking. \(W_{\text{int,k}}\) increases linearly with speed. If the small stature of toddlers is taken into account, toddlers perform less internal work compared to adults since arm swing is not yet present. Another component of internal work is the work performed during double contact, due to the fact that one leg is pushing against the other. This component of work is rather small in toddlers and is not a major contributing factor to total mechanical work production.

**List of Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_x), (A_y), (A_z)</td>
<td>3-D linear accelerations of the COM</td>
</tr>
<tr>
<td>(a)</td>
<td>origin</td>
</tr>
<tr>
<td>(b)</td>
<td>endpoint</td>
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<tr>
<td>COM</td>
<td>centre of mass of the body</td>
</tr>
<tr>
<td>(\text{com}_i)</td>
<td>centre of mass of the body segments</td>
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<tr>
<td>(E_k)</td>
<td>kinetic energy of the centre of mass of the body</td>
</tr>
<tr>
<td>(E_{k,\text{int}})</td>
<td>kinetic energy of the body segments, which is the sum of translational and rotational kinetic energy</td>
</tr>
<tr>
<td>(E_p)</td>
<td>potential energy of the centre of mass of the body</td>
</tr>
<tr>
<td>(E_{\text{tot}})</td>
<td>total mechanical energy of the centre of mass of the body (which is the sum of kinetic and potential energy)</td>
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<td>(F_x)</td>
<td>lateral force component</td>
</tr>
<tr>
<td>(F_y)</td>
<td>fore–aft force component</td>
</tr>
<tr>
<td>(F_z)</td>
<td>vertical force component</td>
</tr>
<tr>
<td>(F_{y,\text{back}})</td>
<td>fore–aft force component of the back leg</td>
</tr>
<tr>
<td>(F_{y,\text{front}})</td>
<td>fore–aft force component of the front leg</td>
</tr>
<tr>
<td>(F_{z,\text{back}})</td>
<td>vertical force component of the back leg</td>
</tr>
<tr>
<td>(F_{z,\text{front}})</td>
<td>vertical force component of the front leg</td>
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<tr>
<td>(I_i)</td>
<td>moment of inertia of the body segments in the sagittal plane</td>
</tr>
<tr>
<td>ILM</td>
<td>individual limbs method</td>
</tr>
<tr>
<td>IP</td>
<td>inverted pendulum mechanism</td>
</tr>
<tr>
<td>(m_i)</td>
<td>mass of the body segments</td>
</tr>
<tr>
<td>(M_{\text{tot}})</td>
<td>total body mass</td>
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<tr>
<td>(R)</td>
<td>% of recovered energy</td>
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<td>relative amplitude</td>
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<td>(V_y)</td>
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<tr>
<td>(V_z)</td>
<td>vertical velocity of the centre of mass of the body</td>
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<td>(v)</td>
<td>walking speed</td>
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<td>(v_{x,i})</td>
<td>lateral velocity of the segmental centre of mass</td>
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<tr>
<td>(v_{y,i})</td>
<td>forward velocity of the segmental centre of mass</td>
</tr>
<tr>
<td>(v_{z,i})</td>
<td>vertical velocity of the segmental centre of mass</td>
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</tbody>
</table>
Mechanical energy in toddler gait

We wish to thank all participating children and their parents for their time and effort. Also we would like to thank the HIKE for their willing participation in our research project. This project was funded by a personal grant to A. Hallemans and a BOF project to P. Aerts.

References

\[ W_{\text{back}} \] positive work performed by the back leg
\[ W_{\text{front}} \] positive work performed by the front leg
\[ W_{\text{com}} \] work performed by the back leg
\[ W_{\text{ext}} \] the amount of positive work performed during a gait cycle to lift and accelerate the centre of mass
\[ W_{\text{front}} \] work performed by the front leg
\[ W_{\text{ILM}} \] the amount of work calculated by the ILM, which is the sum of external mechanical work performed on the centre of mass and some internal work due to the opposite action of the front and back limb during double support
\[ W_{\text{int,dc}} \] the amount of work performed during double contact due to the fact that both legs are working against each other, calculated by the Bastien method
\[ W_{\text{int,k}} \] the amount of positive work performed during a gait cycle to accelerate the body segments relative to the centre of mass
\[ W_y,\text{back} \] work performed by the horizontal force component of the back leg
\[ W_y,\text{front} \] work performed by the horizontal force component of the front leg
\[ W_z,\text{back} \] work performed by the vertical force component of the back leg
\[ W_z,\text{front} \] work performed by the vertical force component of the front leg
\[ w_v \] work performed by both legs in the vertical direction
\[ w_y \] work performed by both legs in the horizontal direction
\[ z \] vertical displacement of COM
\[ \alpha \] angular rotation of the body segments in the sagittal plane
\[ \omega \] angular velocity of the body segments in the sagittal plane