Supplementary Material

We estimate the maximum age that will be reached by Japanese women before 2070 based on data on the current age structure of older Japanese women, an estimation of the death probabilities at older ages and a projection of future changes in the death probabilities. For our analysis we use population data and unsmoothed mortality data from the Human Mortality Database (HMD) obtained on October 11, 2016\cite{1}. To estimate death probabilities for ages beyond 109 and for projecting future age-specific death probabilities, we used the CoDe 2.0 model.

The CoDe 2.0 model

For almost all countries, the age pattern of human mortality has common characteristics. We observe high mortality levels in infancy followed by a strong decrease in early childhood and a sharp increase in the teenage years. Mortality increases exponentially during adulthood, but levels off at older ages\cite{2}. The CoDe mortality model which was recently introduced and validated\cite{2}, describes the full age pattern of mortality by including a power function to describe death probabilities at young ages, a logistic model to describe mortality in adolescence and a logistic-type function to model mortality in adulthood and old age. For our current aim of estimating the death probabilities for ages beyond 109 and for projecting future age-specific death probabilities, we use the CoDe 2.0 model, which is a continuous version of the CoDe model\cite{2}. The CoDe 2.0 model is formulated by:

\[
q_{x,t}^c = \frac{A_t}{x+B_t} + \frac{a_t e^{x-16}}{1+e^{x-16}} + \frac{b_{x,t} e^{b_{x,t}(x-M_t)}}{1+b_{x,t} e^{b_{x,t}(x-M_t)}} \quad \text{with} \quad b_{x,t} := \beta_1 + \frac{\beta_2 + \beta_3 e^{\beta_3(x-c_t)}}{1+\beta_3 e^{\beta_3(x-c_t)}}
\]

where \(q_{x,t}^c\) denotes the death probability at age \(x\) and calendar year \(t\). The model describes mortality at young ages by a power function (first term), whereby parameter \(A_t\) captures the level of infant mortality and parameter \(B_t\) affects the rate of decrease of mortality in childhood. A logistic term (second term) describes both the teenage ‘hump’ and the level of background mortality, parameter \(a_t\). The model uses a logistic-type function with an age-dependent slope.
to describe mortality at adult and advanced ages (third term). The slope parameter \( b_{x,t} \) accounts for the compression of mortality. We include the modal age at death \( M_t \) as a parameter to account for the delay in mortality. Note that unlike in the original CoDe model\(^1\) the slope parameter \( b_{x,t} \) is continuous over age. The slope \( b_{x,t} \) increases with age (see Extended Data Figure 3d). At old ages, the slope is asymptotically equal to \( \beta_{1,t} + \beta_{3,t} \). Parameter \( \beta_{2,t} \) represents the slope of the increase in \( b_{x,t} \) and parameter \( c_t \) determines around which age \( b_{x,t} \) increases.

A pioneering model that has been presented in the literature to describe the increase of the death probabilities in adulthood is the Gompertz model.\(^3\) While the Gompertz model provides an accurate fit for most adult ages, it tends to overestimate the increase in mortality at the oldest ages\(^4,5\). The overestimation of mortality at the oldest ages can be addressed by using a logistic-type model.\(^6\) Extended Data Figure 1 shows that when the Gompertz model is fitted to the death probabilities of Japanese women for ages 70-100 and the fitted parameter values are used to project the death probabilities for ages 101 and over, the model overestimates the increase of the death probabilities for the oldest ages. By contrast, the logistic-type model used in the CoDe 2.0 model produces accurate projections for old ages. Extended Data Figure 2 shows that the increase in the death probabilities levels off at ages 90 and over. The logistic-type model used in the CoDe 2.0 model provides an accurate description of this levelling off whereas the Gompertz model assumes an acceleration of the increase in old age. Note that our statistical procedure implies a deceleration of the death probabilities at the oldest ages because of the theoretical upper limit equal to one and does not lead to estimated death probabilities that suddenly become one.

For each calendar year we estimate the nine parameters of the model: namely \( A_t, B_t, a_t, M_t, g_t, c_t \) and \( \beta_{j,t} \) for \( j = 1,2,3 \) by non-linear least squares. More specifically we minimize the Mean Squared Errors (MSE) of the death probabilities \( q_{x,t} \), the logarithm of the death probabilities \( \log q_{x,t} \) and the density of the age-at-death distribution \( d_{x,t} \) whereby we use the respective variances as weights\(^2\). That is, we minimize

\[
\frac{1}{3} \left[ \frac{\text{MSE}(q_{x,t})}{\text{Var}(q_{x,t})} + \frac{\text{MSE}(\log q_{x,t})}{\text{Var}(\log q_{x,t})} + \frac{\text{MSE}(d_{x,t})}{\text{Var}(d_{x,t})} \right]
\]
This implies that the mean value of $R^2$ for $q_{x,t}$, $\log q_{x,t}$ and $d_{x,t}$ is maximized. We decided to use this criterion because we want the age pattern of mortality to have a good fit across all ages. The errors in $\log q_{x,t}$ give a relatively high weight to the errors at younger ages, while the errors in $d_{x,t}$ give a high weight to the errors around the modal age at death and the errors in $q_{x,t}$ give a high weight to mortality at older ages².

We fit the model to the unsmoothed death probabilities of Japanese women for ages zero to 109 for calendar years 1960 to 2014. The corresponding numbers of deaths and exposures are available from the Human Mortality Database (HMD)¹. For ages above 100, the numbers of deaths and exposures are either not available or fluctuate heavily, especially in the earlier years. Thus, for ages above 100 we use data only when the number of women at that age in that calendar year is higher than 50.

The CoDe 2.0 model provides an excellent fit for both the death probabilities and the age-at-death distribution among Japanese women for each of the years in the period 1960-2014. The average value of $R^2$ is 99.93% for the logarithm of the death probabilities and is 99.94% for the age-at-death distribution. The value of $R^2$ is somewhat lower for the death probabilities due to the relatively large fluctuations in the death probabilities at older ages: average $R^2$ equals 99.57%. Extended Data Figure 3 shows that the model perfectly fits the whole age pattern of mortality, in both 1960 and 2014. Extended Data Figures 3a, 3b and 3c show that the model provides a good fit at older ages, at young ages, and at ages around the modal age at death respectively. Extended Data Figure 3d shows the age pattern of the compression parameter $b_{x,t}$. The values of $b_{x,t}$ are higher at older ages in 2014 than in 1960. This implies that there is more compression of mortality at older ages in 2014 than in 1960.

The main reason why we assume that $b_{x,t}$ varies by age is that this assumption allows us to assess the compression of mortality at older ages. In a conventional logistic model including a constant value of $b_{x,t}$ across ages i.e. $b_{x,t} = \beta_{1,t}$, a change in the value of $b_{x,t}$ affects the death probabilities at all adult ages. For example an increase in the value of $b_{x,t}$ leads to lower death probabilities at young adult ages and higher probabilities at older ages. The age dependency of $b_{x,t}$ in the CoDe 2.0 model allows us to model the compression of mortality at older ages without
affecting the death probabilities at young or middle adult ages. Since in 1960 the value of $b_{x,t}$ hardly changes across ages, a conventional logistic model that assumes that $b_{x,t}$ is constant across ages provides an accurate fit. But such a model provides a poor fit in 2014. Extended Data Figure 4 shows that assuming a constant value of $b_{x,t} = \beta_{1,t}$ does not provide an accurate fit for the death probabilities at young adult and middle ages, but it does provide an accurate fit at ages 70+. This result explains why the conventional logistic model with a constant slope parameter provides an accurate description of the death probabilities at older ages\textsuperscript{2,6,5,7,8}. For example, the HMD uses the Kannisto\textsuperscript{6} logistic model to smooth the death rates at older ages. One main difference between our model and the Kannisto model is that in the latter model the asymptotic value for the death rates equals one, which is equivalent to an asymptotic value of 0.67 for the death probabilities, whereas in the CoDe 2.0 model the asymptotic value $g_t$ is one of the parameters to be estimated and it is allowed to vary across years.

The death probabilities cannot be observed for ages beyond 109 due to the lack of data\textsuperscript{1}. Nonetheless the death probabilities for these ages are important for the estimation of the maximum age at death. An advantage of the CoDe 2.0 model is that it allows for the estimation of the death probabilities for ages beyond 109 without any additional assumptions. Indeed, by using equation (1) and the estimated values for the parameters we can estimate the death probabilities for ages beyond 109. We estimate that the death probabilities increase from 0.5 at age 110 to 0.6 at ages above 120. This is slightly higher than the estimate by Gampe, who concluded, based on the analysis of 637 supercentenarians, that the death probabilities at ages 110 and above are flat at a constant level of around 0.5\textsuperscript{9}. In contrast, based on American data, Gavrilova and Gavrilov conclude that death rates continue to increase in very old age\textsuperscript{10}. However, they note that the death probabilities will decelerate at advanced ages because the probability has an upper limit equal to one. Furthermore, their estimate of the death rate at age 106 corresponds with a death probability close to 0.5, their results are not inconsistent with our findings.

**Mortality projection using the CoDe 2.0 model**
We project the age-specific death probabilities by projecting the CoDe 2.0 model parameters using a multivariate time series model. We capture the interdependencies among the parameters by modeling them as a vector; namely

$$u_t := (M_t, A_t, B_t, a_t, g_t, c_t, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}).$$

A well-known framework for jointly modeling time series is the Vector Auto Regression (VAR) model, which is a generalization of the univariate autoregressive model (AR). However, because our time series are not stationary — that is, the mean value and the variance are not time invariant — we could not use the original VAR model. Instead, we used a Vector Error Correction Model (VECM) that includes an additional term that corrects for the co-integration of our time series that we observed based on the Johansen\textsuperscript{11} test. The VECM is formulated by

$$\Delta u_t = \Phi D_t + \sum_{i=1}^{k-1} I_i \Delta u_{t-1} + \zeta \lambda_{t-1} + \epsilon_t \text{ for } t=k+1, \ldots, T,$$

where $D_t$ is the deterministic term, in our case a linear trend, weighted with parameter matrix $\Phi \in R^{9x2}$ and $\zeta, \lambda$ are $9 \times r$ matrices with $\text{rank}(\zeta) = \text{rank}(\lambda) = r$, which is the order of co-integration. The error terms $\epsilon_t$ are independently and identically distributed (i.i.d) multivariate normal distributed with zero mean and covariance matrix $\Sigma \in R^{9x9}$.

We estimate the model on the basis of data for the years 1960-2014 to make projections for the years 2015-2070. In Extended Data Figure 5 we present the median values of the parameter projections and the corresponding 95% projections intervals.

Using the projected median values of the parameters we project the death probabilities for the years 2015-2070. In Extended Data Figures 3a and 3b we present the projections of $q_{x,t}$ and $\log q_{x,t}$ for the calendar year 2070. Since 1960 the age pattern of death probabilities has shifted to the right, which indicates that mortality has been delayed to older ages\textsuperscript{2} (Extended Data Figure 3b). In addition, mortality compression has occurred, because the death probabilities at the oldest ages have declined less strongly than at younger ages\textsuperscript{2}. The projected increase in the value of $b_{x,t}$ at older ages (Extended Data Figure 3d) implies that we anticipate further compression of mortality at older ages. In Extended Data Figure 3c, we plot the projection of the age-at-death
distribution for the year 2070. We observe a continued delay (increase in the modal age at death) and a continued compression of mortality (a larger share of deaths around the modal age).

**Projection of the maximum age at death**

The maximum reported age at death depends on both the size of the population at risk and the level of death probabilities at the oldest ages. If the annual death probability at age 115 equals 0.5, the probability that a woman aged 115 survives to age 116 equals 50 percent. If five women are aged 115 the probability that at least one of them survives to age 116 equals 96 per cent (see Extended Data Figure 6). The probability that a woman aged 115 survives to age 120 is as small as 2 per cent. But if the number of women aged 115 increases to 20, the probability that at least one of them will reach age 120 increases to one third. Thus even though the probability for an individual woman to survive to age 120 is small, the increase in the number of centenarians will make it more likely that at least one woman will survive to such a high age.

Using the projected death probabilities for the years 2015-2070 and the observed age structure of the Japanese female population on January 1, 2015\(^1\), we project the number of older Japanese women by age. The number of women by age is projected by

\[
N_{x+1,t+1} = (1 - q_{x+1,t})N_{x,t}
\]

where \(N_{x,t}\) denotes the number of women aged \(x\) on January 1 of year \(t\) and \(q_{x+1,t}\) denotes the probability that a woman aged \(x\) on January 1 of year \(t\) will die before January 1 of year \(t + 1\). We assume that there is no migration. Note that we project the population on January 1. This implies that if the oldest woman on January 1 of year \(t\) is aged \(x_{\text{max}}\) and reaches age \(x_{\text{max}} + 1\) during year \(t\) but dies before January 1 of year \(t+1\), we regard \(x_{\text{max}}\) as the maximum age. This implies that our estimate of the maximum age is conservative. The actual maximum age at death will be several months higher than our estimate. Note, however, that we report the maximum age in years and not in months.
We define the maximum age $x_{\text{max}}$ on January 1 of year $t$ as the age at which we expect that at least one woman will still be alive, whereas we expect that no woman will be alive at age $x_{\text{max}} + 1$. This does not imply that it is impossible that a woman who is aged $x_{\text{max}}$ on January 1 of year $t$ will survive to reach age $x_{\text{max}} + 1$ on January 1 of year $t + 1$, but we expect the probability that she will survive to be smaller than the probability that she will die before January 1 of year $t + 1$. Thus, we do not consider $x_{\text{max}}$ to represent a natural limit to human lifespan, because we do not assume that the probability of surviving to year $t + 1$ is zero. We calculate the maximum age at death in year $t$ as the age at which the probability that at least one woman will survive to age $x_{\text{max}} + 1$ on January 1 of year $t + 1$ is smaller than 0.5. For example when two women are alive at age 120 and the death probability equals 0.6, the probability that at least one woman will survive to age 121 equals 0.64 (because the probability that both women will die before reaching age 121 equals 0.36). The probability that at least one woman will survive to age 122 equals 0.29 since the probability that both women will die before age 122 equals 0.71. We consider 121 to be the maximum age because the probability that at least one woman will survive to age 121 exceeds 0.5, while the probability that at least one woman will survive to age 122 is smaller than 0.5.

References


**Extended Data Figures**

**Extended Data Figure 1. Death probabilities, Japanese women, 2012-2014.** The solid black line shows the average observed values for the years 2012-2014 for ages 70-109. The dashed blue line shows the fit of the Gompertz model for ages 70-100 and the projection for ages 101-109. The dashed red line shows the fit and projection of the logistic-type function used in the CoDe 2.0 model. Even though the fit of both models for ages 70-100 is equally accurate, the projections of the Gompertz model overestimate the increase of the death probabilities at ages 100 and over, while the CoDe 2.0 model provides an accurate projection for the oldest ages.
Extended Data Figure 2. Change in death probabilities Japanese women, 2012-2014. The solid black line shows the changes in the average observed values of the death probabilities between successive ages for the years 2012-2014 for ages 71-109. The dashed blue line shows that the Gompertz model projects an acceleration of the increase in the death probabilities. The Gompertz model underestimates the increase in the death probabilities between ages 85 and 100 and overestimates the increase for ages 100 and over. The dashed red line shows that the logistic-type model used in the CoDe 2.0 model describes the levelling off of the increase in the death probabilities at ages 90 and over.
Extended Data Figure 3. Death probabilities, age-at-death distribution and slope parameter, Japanese women, 1960, 2014 and 2070. The red lines show the values for 1960, the blue lines those for 2014 and the green lines the projection for 2070. The dashed lines show the observed values, the solid lines show the fit of the CoDe 2.0 model. The grey area represents the ages for which we extrapolate our estimations. The projections for 2070 are calculated using the CoDe 2.0 model in which the parameter values are assumed to equal the median values of the projections of the parameters for 2070.  

a: The increase in the death probabilities $q_{x,t}$ levels off at older ages. In 2014 the maximum death probability is lower than in 1960. The decrease in death probabilities at the oldest ages between 2014 and 2070 is smaller than between 1960 and 2014. As a result, the rate of increase in the death probabilities $q_{x,t}$ above age 100 in 2070 exceeds that in 2014. 

b: The decrease in the logarithm of the death probabilities between 1960 and 2014 is smaller at older ages than at young and adult ages. Due to delay of mortality death probabilities around age 100 are projected to decrease strongly up to 2070. Due to compression of mortality in old age the decrease of death probabilities at the oldest ages is smaller than around age 100. 

c: 

d:
The age-at-death distribution in 2014 is more compressed around the modal at death than in 1960. In 2070 both the density of the age-at-death distribution at the modal age at death and the modal age are higher than in 2014, due to further compression and delay of mortality respectively. The slope parameter $b_{x,t}$ of the logistic term increases more strongly at older ages in 2014 than in 1960. This results in compression of mortality in old age. This development is projected to continue up to 2070.
Extended Data Figure 4. Logarithm of death probabilities, Japanese women, 2014. The dashed line shows the observed values. The solid line shows the fit of the CoDe 2.0 model assuming that $b_{x,t}$ does not change with age. The solid line shows that if $b_{x,t}$ does not change with age, the model does not provide an accurate fit of the whole age pattern. While the fit of the model at older ages is accurate, the death probabilities at middle ages are underestimated. Thus the logistic model with a constant slope parameter underestimates the compression of mortality around the modal age at death. The fitted model overestimates the levelling off of the increase of the death probabilities across adult ages.
Extended Data Figure 5. Estimates and projections of the parameter values of the CoDe 2.0 model, Japanese women. The solid lines show the estimates for the period 1960-2014 and the median values of the projections for the period 2015-2070. The dashed lines show the 95% projections intervals. The increase in the modal age at death $M_t$ implies that the CoDe 2.0 model projects a continuation of the delay of mortality. Note that the projection interval is relatively narrow. Thus based on the observed increase in the modal age at death in the 1960-2014 period, our time series model projects that it is highly likely that the delay of mortality to older ages will continue. The projected decrease in the values of $A_t$ and $a_t$ imply further reduction in infant and adolescent mortality contributing to compression of mortality. The increase in the value of $\beta_{3,t}$ implies that a continuation of the compression of mortality at older ages is projected. The projected constant value of $g_t$ implies that the asymptotic value of the death probabilities in very old age will be constant.
Extended Data Figure 6. Probability that at least one woman aged 115 will reach age 116 or age 120. The probability that at least one woman survives to a certain age depends on both the size of the population at risk and the level of death probabilities. Using the CoDe 2.0 model we estimate that the probability for a Japanese woman aged 115 to survive to age 116 equals 0.5 (see Extended Data Figure 3a). The blue line shows that the probability that at least one woman will reach age 116 increases sharply if the number of women aged 115 increases. Since we expect that the number of centenarians among Japanese women will increase in the foreseeable future, the probability that at least one of them will reach age 116 will increase considerably. The death probability increases slightly between ages 115 and 120 (see Extended Data Figure 3a). As a result the probability that a Japanese woman aged 115 will survive to age 120 equals 2 per cent. Thus it is not very likely that in the near future a Japanese woman will reach age 120. However, the red line shows that when the number of Japanese women aged 115 increases to 20, the probability that at least one woman will reach age 120 will increase to one third. Even if death probabilities among centenarians will not decline, the number of women aged 115 can be expected to increase to 20 due to the strong increase in the number of centenarians. Thus in the long run it will no longer be exceptional that a woman survives to age 120.