Whether this is good enough depends on why the equation was specified and estimated. But, in any case, it should be emphasized that least squares does estimate the parameters (except the constant) of the frontier function. I think the concept of the "average" function is more or less a red herring; if we define the "average" function as that which is consistently estimated by least squares, then (under the above assumptions) it differs from the frontier function only in its constant term.

Finally, my article considered the properties of the Aigner-Chu estimators under very favorable assumptions about the disturbances—assumptions under which these estimators are in fact maximum likelihood estimators. Even under these favorable assumptions I could not determine their properties (though perhaps someone else could). Chu objects to these assumptions as being too simple, and I agree. I have elsewhere proposed a slightly more complicated set of assumptions that I think are superior—see Aigner, Lovell and Schmidt (1977). I encourage Chu to be explicit about what he would assume about the disturbance; only then is it possible to compare properties of various estimators. However, my intuition suggests that complicated assumptions will do more damage to the properties of the programming estimators, which are fairly sophisticated, and hence sensitive, than they will to the properties of least squares, which are reasonably robust.

REFERENCES

ENVIRONMENTAL REPERCUSSIONS AND THE ECONOMIC STRUCTURE:
FURTHER COMMENTS

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I. Introduction

The input-output approach to the modeling of environmental repercussions on the economic system has received much attention in recent years. A very well known model was presented by Leontief (1970, 1973) that extended regular input-output analysis in two ways. Firstly, the one-product vector of output coefficients of each of the conventional sectors was replaced by a multi-product output vector that accounted for the involuntary production of a number of pollutants next to the "proper" product. Secondly, so called anti-pollution sectors were introduced in which pollutants from the conventional sectors were eliminated. By this procedure the amount of emitted pollutants could be endogenized.

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It seemed that Leontief's extension was made at the cost of a very attractive feature of the (open) input-output model. The existence of a nonnegative vector of gross output levels for any final demand (now to include tolerated-pollution levels as well) is not guaranteed any more. A counter-example, requiring a negative level for some anti-pollution sector, was provided by Flick (1974) in this REVIEW. We shall show that this counter-example is not appropriate because a certain dependency between the model's equations has not been accounted for; the dependency being that tolerated pollution cannot exceed total generated pollution in Leontief's formulation. A result on square nonnegative matrices then gives a condition that assures that a nonnegative gross output vector for the entire extended system results.

Furthermore, it will be shown that a substitution is possible in Leontief's environmental repercussions model that generates an adjusted system; the adjustment corresponding to the extent to which industries have to participate in anti-pollution
measures. Next to adjusted technical coefficients, adjusted value-added coefficients can be found by the same substitution. Then price systems can be determined as a function of the degree to which the "polluter pays principle" is applied in the economy. In this way, price effects on conventional goods as a result of environmental policy can be analysed more clearly.

II. Leontief's Extended System

It will be convenient to write down the structural coefficient matrix of Leontief's extended model in the following partitional form (Leontief and Ford, 1972).

\[ A_{11} \begin{bmatrix} \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots \end{bmatrix} A_{12} \]

\[ A_{21} \begin{bmatrix} \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots \\ \vdots & \vdots & \ldots & \vdots \end{bmatrix} A_{22} \]

The contents of the matrices \( A_{11}, A_{12}, A_{21} \) and \( A_{22} \) are described on the right-hand side. The various types of coefficients are defined as follows:

- \( a_{ij} \) = input of good \( i \) per unit of output good \( j \) (produced by sector \( j \)); \( i,j = 1,2,\ldots,m \).
- \( a_{ig} \) = input of good \( i \) per unit of eliminated pollutant \( g \) (eliminated by sector \( g \)); \( i = 1,2,\ldots,m; g = m+1,m+2,\ldots,n \).
- \( a_{pi} \) = output of pollutant \( g \) per unit of good \( i \) (produced by sector \( i \)); \( i = 1,2,\ldots,m; g = m+1,m+2,\ldots,n \).
- \( a_{gk} \) = output of pollutant \( g \) per unit of eliminated pollutant \( k \) (eliminated by sector \( k \)); \( g,k = m+1,m+2,\ldots,n \).

If the coefficients of matrices \( A_{21} \) and \( A_{22} \) are entered with a negative sign (Leontief and Ford, 1972, pp. 10–11), the physical input-output balance reads

\[
\begin{bmatrix}
I - A_{11} & -A_{12} \\
A_{21} & -I + A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
c_1 \\
c_2
\end{bmatrix}.
\]

III. Existence of Nonnegative Solutions

Serious criticism was offered by Flick (1974), which can be summarized as follows: Can it be shown for the environmental repercussions model that for any given final demand vector \( c > 0 \), there exists an \( x > 0 \) such that \((I - A)x = c\)? By way of a counter-example Flick showed that this is not the case. Flick's suggestion to introduce an "environmental services" sector does not seem to be a satisfactory way out as pointed out by Leontief (1974) in his reply. Apart from its conceptual vagueness, it bypasses the point that the levels of tolerated and eliminated pollution are not independent in the model.

The identities of the extended input-output system are the following:

\[(I - A_{11})x_1 - A_{12}x_2 = c_1\]

or

\[(I - A_{11})x_1 = c_1 + A_{12}x_2. \quad (1)\]

The Hawkins-Simon conditions on \((I - A_{11})\) guarantee that this identity can be met for any nonnegative \( x_2 \).

\[A_{21}x_1 + A_{22}x_2 - IX_2 = c_2\]

or

\[(I - A_{22})x_2 = A_{21}x_1 - c_2, \quad (2)\]

which gives

\[x_2 = (I - A_{22})^{-1}(A_{21}x_1 - c_2).\]

\((I - A_{22})\) will generally satisfy the Hawkins-Simon conditions; these conditions make economic sense: anti-pollution activities should not produce more pollution than they eliminate. Stated differently, \((I - A_{22})\) will be nonsingular and \((I - A_{22})^{-1}\) will be (semi)positive. Therefore, a nonnegative solution \( x_2 \) will depend on the relation between \( x_1 \) and \( c_2 \). Thus, if \((A_{21}x_1 - c_2)\) is negative for some pollutant \( g \), the corresponding element of \( x_2 \) can be negative, depending on the elements of \( A_{22} \). However, if produced pollution \( A_{21}x_1 \) exceeds tolerated pollution levels \( c_2 \), a nonnegative \( x_2 \) is always found. This explains why in practice there seems to be no problem. In polluted areas produced pollution will normally exceed desired or tolerated levels. If this is not the case, the extended model simply does not apply: the original input-output model should be applied.

IV. A Measure of the Degree of Pollution Abatement

Leontief (1970) gives an example of an economy in which one pollutant is distinguished. A corre-
sponding disposal activity has been introduced. The system's structural coefficient matrix is

\[
\begin{bmatrix}
0.25 & 0.40 & 0 \\
0.14 & 0.12 & 0.20 \\
0.50 & 0.20 & 0 \\
0.80 & 3.60 & 2.00
\end{bmatrix}
\]

In the quoted example, given \( c_1 = (55, 30)' \) and \( c_2 = 30 \), \( x_1 = (104.50, 58.43)' \) and \( x_2 = 33.93 \) are determined.

Up to now \( c_2 \) has been fixed a priori. However, we can also look in a different way at the model. An interesting question is "How much of total emissions has been removed?" In the one-pollutant case of the example we can indicate this by a parameter \( a \) in a definition relation: \( c_2 = ax_2 \) \((a > 0)\), relating (tolerated) emissions \( c_2 \) to eliminated pollution \( x_2 \). In Leontief's example \( a = 30/33.93 \). This means that 53.07 \((= 100 \cdot (1 + a)^{-1})\)% of pollution has been abated. We see that if \( a = 0 \) no pollution is allowed: abatement is 100%. In the case \( a = 1 \), the amount of eliminated pollution \( x_2 \) equals \( c_2 \), the delivery to the environment. Abatement now is 50%.

If environmental policy prescribes that 100 \cdot (1 + a)^{-1}\)% of generated pollution must be eliminated, we see that business in that case, in order to produce the same final consumption basket \( c_1 \), has to increase total output of both (conventional) commodities. (In the example above, output has risen from 100 to 104.50 for the first good, and from 50 to 58.43 for the second good.) This means that because of the environmental policy more inputs than before are needed now to produce the same consumption vector. This result can be interpreted as a change in input structure. The change obviously is a function of the abatement policy prevailing, i.e., (in this model) of \( a \).

Can something be said about this structural change, invoked by the choice of a particular abatement policy? We have the following system:

\[
(I - A_{11})x_1 - A_{12}x_2 = c_1
\]
\[
A_{21}x_1 + (A_{22} - I)x_2 = c_2
\]
\[
\alpha x_2 = c_2 \quad (\alpha \text{ fixed})
\]

We start by eliminating \( c_2 \). This gives

\[
A_{21}x_1 - (I - A_{22})x_2 = \alpha x_2
\]
or

\[
-(I - A_{22})x_2 = \alpha x_2 - A_{21}x_1
\]
or

\[
((1 + \alpha)I - A_{22})x_2 = A_{21}x_1.
\]

Thus,

\[
x_2 = ((1 + \alpha)I - A_{22})^{-1}A_{21}x_1,
\]
which gives

\[
(I - A_{11})x_1 - A_{12}((1 + \alpha)I - A_{22})^{-1}A_{21}x_1 = c_1
\]
or

\[
x_1 = (I - A^*)^{-1}c_1
\]
with

\[
A^* = A_{11} + A_{12}((1 + \alpha)I - A_{22})^{-1}A_{21},
\]
which gives \( x_1 \) as a function of \( a \).

\( x_2 \) is then found from (6). A next step gives \( c_2 \) from (5). Adjusted value-added coefficients can be found in a similar way:

\[
L = v_1x_1 + v_2x_2.
\]

We know:

\[
x_2 = ((1 + \alpha)I - A_{22})^{-1}A_{21}x_1;
\]
this gives

\[
L = \left[ v_1 + v_2((1 + \alpha)I - A_{22})^{-1}A_{21} \right] x_1 = v^*_1x_1
\]
with \( v^*_1 \) the adjusted value-added coefficients, and \( L \) total labor demand.

The increase in total output of original sectors can be shown in a diagram. Figure 1 shows the change in gross output of a two by two economy, in which at least one pollutant is abated up to a certain extent, indicated by \( a = a_0 \). The original system's equations are (no abatement)

\[
L_1: (1-a_{11})x_1 - a_{12}x_2 = c_1
\]
\[
L_2: -a_{21}x_1 + (1-a_{22})x_2 = c_2.
\]

The point of intersection \( P \) determines \( x_1 \) and \( x_2 \), after which total labor demand can be calculated from

\[
L = v_1x_1 + v_2x_2.
\]

If pollutants are eliminated up to the desired extent, the equations of the adjusted system become

\[
L_{1}^*: (1-a_{11}^*)x_1 - a_{12}^*x_2 = c_1
\]
\[
L_{2}^*: -a_{21}^*x_1 + (1-a_{22}^*)x_2 = c_2
\]
and

\[
L^*: v_{1}^*x_1 + v_2^*x_2
\]
with \( a_{ij}^*, v_{ij}^* \) \((i,j = 1, 2)\) determined by \( a_0 \) according to formulas (7) and (8).

The substitution thus allows us to confine ourselves to the original \( m \times m \) system, instead of working with Leontief's extended system. Correcting the model's original processes for anti-pollution efforts can be useful in some cases. For instance, a comparison of input-output tables over a rather long

\[\text{2In the case of more pollutants a vector of similar coefficients must be introduced.}\]

\[\text{3For a digression on a diagram like this, see, e.g., Dorfman, Samuelson, and Solow (1958).}\]
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FIGURE 1.—GROSS OUTPUTS OF CONVENTIONAL COMMODITIES BEFORE AND AFTER ADJUSTMENT

Note: Gross outputs of conventional commodities and the required labor supply before adjustment are indicated by \((x_1, x_2, \text{and } L)\), and after adjustment by \((x_1^*, x_2^*, \text{and } L^*)\).

Period, i.e., before and after the introduction of environmental policy measures of a prescribed form, is now possible because the sector classification has remained the same over time. Another argument might be that anti-pollution facilities become gradually integrated in normal production processes because abatement at the source becomes more and more imperative. The necessity to distinguish separate anti-pollution activities might decrease therefore.

V. The "Polluter Pays Principle"

The polluter pays principle roughly states that a polluter should pay for any damage caused by his action and/or should pay for required measures to abate pollution up to a prescribed extent. Many rules have been introduced to operationalize the principle. In practice many exceptions to the strict definition are observed: government policy may be that costs are borne not only by business but also (to some extent at least) by the public. Especially, initial costs are often financed by the government.

The polluter may be charged various prices for his polluting activity, depending on the degree to which the polluter pays principle is applied. Price consequences probably are made more transparent by rewriting the system, defined by equations (3–5), as follows:

\[
(I - A_{11})x_1 - A_{12}x_2 = c_1
\]

\[-(1 + \alpha)^{-1} A_{21}x_1 + (I - (1 + \alpha)^{-1} A_{22})x_2 = 0.
\]

If prices are determined from

\[
p_1(I - A) + p_2(-(1 + \alpha)^{-1} A_{21}) = v_1
\]

\[
p_1(- A_{12}) + p_2(I - (1 + \alpha)^{-1} A_{22}) = v_2,
\]

all expenses of abatement up to a physical level of 100\% of real costs \((0 \leq p < 1)\), prices are determined from

\[
p_1(I - A_{11}) + p_2\left(- \frac{\beta}{1 + \alpha} A_{21}\right) = v_1
\]

\[
p_1(- A_{12}) + p_2\left(I - \frac{\beta}{1 + \alpha} A_{22}\right) = v_2.
\]

We see, if \(\beta = 1\), the polluter pays principle is applied in a "pure" form; polluters are charged all costs made to abate pollution up to the desired 100\% of real costs. If \(\beta = 0\), polluters are not charged any elimination costs. For intermediate values, polluters are charged only partially.

VI. Some Numerical Examples

We shall illustrate the correction factor using Leontief's data (section IX of his 1970 article). The essentials of the Leontief-Flick (1974) discussion on prices may become more transparent this way. If 50\% of pollution is eliminated, and if enterprises are fully charged, we get \((\alpha = \beta = 1)\):

\[
0.25 x_1 + 0.40 x_2 + 50 = x_1
\]

\[
0.14 x_1 + 0.12 x_2 + 0.20 x_3 + 30 = x_2
\]

\[
0.50 x_1 + 0.20 x_2 - c_3 = x_3
\]

Elimination of \(c_3\) and \(x_3\) from the system gives

\[
0.25 x_1 + 0.40 x_2 + 55 = x_1
\]

\[
0.14 x_1 + 0.12 x_2 + 30 = x_2.
\]

Hence the adjusted technology matrix \((I - A^*)\) is

\[
(I - A^*) = \begin{bmatrix}
0.75 & 0.40 \\
-0.71 & 0.86
\end{bmatrix}.
\]

Prices can be calculated from the adjusted technology matrix \((I - A^*)\) after \(v_1^*\) and \(v_2^*\) (value-added of original sectors 1 and 2) have been adjusted. This gives in Leontief's example

\[
v_1^* = 0.80 + 2.00(0.50/2) = 1.30
\]
and

\[ e_2^* = 3.60 + 2.00(0.20/2) = 3.80. \]

Prices are now calculated from

\[
\begin{align*}
0.75p_1 - 0.19p_2 &= 1.30 \\
-0.40p_1 + 0.86p_2 &= 3.80.
\end{align*}
\]

This gives values \( p_1 = 3.23 \) and \( p_2 = 5.92 \), which equal Leontief's values. If polluting industries are not charged any abatement costs, which amounts to putting \( \beta \) equal to zero in equations (11) and (12), the price structure is determined by the original system:

\[ p_1(I - A_{11}) = v. \]

This gives Leontief's values, \( p_1 = 2.00 \) and \( p_2 = 5.00 \).

If total abatement is desired and the polluter is fully charged (\( \alpha = 0, \beta = 1 \)), prices are determined from

\[
(p_1 \ p_2) \begin{pmatrix} 0.75 & -0.40 \\ -0.24 & 0.84 \end{pmatrix} = (1.80 \ 4.00).
\]

This gives \( p_1 = 4.63 \) and \( p_2 = 6.96 \).

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