The binding energy of the deeply bound $0g_{9/2}$ hole coupled to active protons in the same shell

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The binding energy of the $0g_{9/2}$ hole state observed in neutron pick-up reactions is calculated in the framework of the shell model. Isospin symmetry and coupling between the hole and active protons in the same shell are taken into account.

The broad bump observed recently in neutron pick-up reactions on medium weight nuclei [1–3] can be understood as the formation of a deeply bound hole in the $0g_{9/2}$ shell. The hole binding energy estimated from the observed position of the peak of the broad bump has been compared [2] with the calculation of Veje [4] who considered one nucleon motion in a potential well. Agreements are reasonable, but the calculated binding energies in Zr and Mo isotopes are a little too small.

The aim of the present work is to evaluate the hole binding energy in the framework of the shell model, treating isospin symmetry in a proper way and taking into account strong coupling between the hole and active nucleons in the same shell. We attempt to describe the hole states in the nucleus with $40 \leq Z \leq 48$ and $50 < N$, in which many experimental data have been accumulated [2], by the coupling scheme shown in fig. 1a, where $T_1$ represents the isospin of nucleons above the $Z = N = 50$ core and $T_0$ is the isospin of nucleons in the $0g_{9/2}$ shell. The isospin of the target nucleus is expressed as the stretch coupling between $T_1$ and $T'$, where $T'$ is the isospin in the $0g_{9/2}$ shell of the target. Note that the hole must in general be partially a neutron and partially a proton in order that $T_0$ should be kept a constant. For comparison, we show the core-hole coupling scheme in fig. 1b. The present coupling scheme is valid if the hole is strongly coupled with active nucleons in the same shell through the two-body interaction and the Pauli-principle. The hole state (c) in fig. 1a will represent the peak of the broad bump, since the spectroscopic factor in the neutron pick-up reaction is concentrated in it:

$$\langle f|a_j|i\rangle^2 = \frac{2j'+1}{2j+1} S,$$

$$S = \frac{n}{2j+2-n+m} \text{ for the state (a), } \frac{mn}{(2j+2-n)(2j+2-n+m)} \text{ for (b), } \frac{(2j+2)(2j+1-n)}{2j+2-n} \text{ for (c),}$$

where $a_j$ is neutron annihilation operator in the $f = 0g_{9/2}$ shell. The states $|i\rangle$ and $|f\rangle$ are defined as

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Fig. 1a shows the $0g_{9/2}$ hole states (a), (b) and (c) proposed in the present work. For comparison, the core-hole coupling scheme is shown in fig. 1b.
\[ |i\rangle = |j^{n+2j+1} (v_0 = 0 \quad T' = (2j + 1 - n)/2 \quad J' = 0) \rangle \quad m(T_1 = m/2 \quad J) \quad T = T' + T_1 \]

\[ |f\rangle = |j^{n+2j} (v_0 = 1 \quad T_0 = j) \rangle \quad m(T_1 = m/2) \quad T \quad J', \]

where \( j = 0g_{9/2} \) and \( j = 1d_{5/2}, 0g_{7/2} \) etc. The \( ^{80}Zr \) core and the seniority scheme in the \( 0g_{9/2} \) shell are assumed. For simplicity, we discuss the case of even-proton nuclei only.

In the \( Z > 50 \) nuclei, the state \( (c) \) does not exist, since the \( 0g_{9/2} \) proton shell is fully occupied, and the state \( (b) \) represents the peak of the broad bump. The treatment is essentially the same as discussed by Bansal and French [5]. In the \( N \leq 50 \) nuclei, there is no state \( (b) \).

The binding energies of the states \( (a) \), \( (b) \) and \( (c) \) are evaluated using the expression

\[ B.E. = \langle f|H|i\rangle - \langle i|H|i\rangle; \quad H = H_1 + H_2 + \text{Coulomb int.}, \]

where \( H_1 \) represents the interaction between particles in the \( 0g_{9/2} \) shell, and \( H_2 \) is the interaction between the nucleon in the \( 0g_{9/2} \) shell and the nucleon in the \( 1d_{5/2}, 0g_{7/2} \) etc. The Coulomb energy will be approximated as a function of \( Z \) and \( A \) only, independent of the coupling scheme, and it is neglected here. The matrix element of \( H_1 \) is expressed as [6]

\[ \langle j^{nu}TJJ_h|H_1|j^{nu}TJJ \rangle = nC + an(n - 1)/2 + \{T(T + 1) - 3n/4\}b + [n/2]C_p, \]

where seniority \( v = 0, 1 \) and parameters \( a, b \) and \( C_p \) are related to the two-body matrix elements [6]. We assume the following form [5] for \( H_2 \):

\[ H_2 = A + 2B \sum_{i,k} (t_i^* t_k), \]

where symbol \( l \) represents the nucleon in the \( 0g_{9/2} \) shell and \( k \) the nucleon in the \( 1d_{5/2}, 0g_{7/2} \) etc. As to the coupling constants \( A \) and \( B \), we use, for simplicity, the same set of values \( A_1 \) and \( B_1 \) for the \( 1d_{5/2} \) and \( 0g_{7/2} \) shells and another set of values \( A_2 \) and \( B_2 \) for \( 2s_{1/2}, 1d_{3/2} \) and \( 0h_{11/2} \) shells. The binding energies of the states \( (a), (b) \) and \( (c) \) in the nucleus with \( 40 \leq Z \leq 48 \) and \( 50 < N < 64 \) are expressed as

\[ \text{B.E. of (a)} = \text{B.E. of (c)} + (2n' + 2 - n)b + mB_1, \]

\[ \text{B.E. of (b)} = \text{B.E. of (c)} + (2j + 2 - n)(b - B_1), \]

\[ \text{B.E. of (c)} = n\left(\frac{b}{2} - a\right) - m\left(\frac{B_1}{2} + A_1\right) - C', \]

where \( n \) is the number of nucleons in the \( 0g_{9/2} \) shell and \( m \) is the number of neutrons above the \( N = 50 \) core. The constant \( C' \) is defined by

\[ C' = 2Ja + jb + C_p + C. \]

The excitation energy of the state \( (c) \) is the smallest among the states \( (a), (b) \) and \( (c) \), since in general \( b > B_1 > 0 \).

In table 1, we show the calculated binding energies of the hole states in various nuclei together with binding energies estimated from the observed peak of the broad bump. We have used the expression (8) for the nucleus with \( Z \leq 48 \) and \( N \leq 64 \), and the expression (7) for the nucleus with \( 50 \leq Z \) and \( N \leq 64 \). The following expression for the state \( (b) \) is used for the nucleus with \( Z = 50 + n' \) \( (n' \leq 14) \) and \( N = 64 + m' \):

\[ \text{B.E.} = 10(b/2 - a) + b - C' - (n' + 14)A_1 - m' A_2 - (16 - n' + m') \{(14 - n')B_1 + m'B_2\}/ \{2(14 - n' + m')\}. \]

We have used the following values of parameters:

\[ b = 0.816 \text{(MeV)}, \quad B_1 = 0.496, \quad B_2 = 0.16, \quad C' = -11.98, \quad B/a = B_1/A_1 = B_2/A_2 = -8. \]

In table 2, calculated energy differences between the isobaric analog state, whose energy is expressed by eq. (6), and the peak of the broad bump are compared with those deduced from experimental data [7]. The difference of
Table 1
Binding energies of the peak of the broad bump observed in neutron pick-up reactions (in MeV)

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$^{90}$Zr</td>
<td>11.98</td>
<td>11.98</td>
</tr>
<tr>
<td>$^{94}$Mo</td>
<td>12.32</td>
<td>12.63</td>
</tr>
<tr>
<td>$^{95}$Mo</td>
<td>12.33</td>
<td>12.44</td>
</tr>
<tr>
<td>$^{98}$Mo</td>
<td>12.09</td>
<td>11.88</td>
</tr>
<tr>
<td>$^{100}$Mo</td>
<td>11.50</td>
<td>11.51</td>
</tr>
<tr>
<td>$^{106}$Pd</td>
<td>12.56</td>
<td>13.18</td>
</tr>
<tr>
<td>$^{111}$Cd</td>
<td>13.73</td>
<td>13.64</td>
</tr>
<tr>
<td>$^{112}$Cd</td>
<td>13.25</td>
<td>13.46</td>
</tr>
<tr>
<td>$^{116}$Sn</td>
<td>15.10</td>
<td>15.17</td>
</tr>
<tr>
<td>$^{118}$Sn</td>
<td>14.02</td>
<td>14.72</td>
</tr>
<tr>
<td>$^{128}$Sn</td>
<td>14.83</td>
<td>14.64</td>
</tr>
<tr>
<td>$^{126}$Te</td>
<td>14.14</td>
<td>14.34</td>
</tr>
<tr>
<td>$^{130}$Te</td>
<td>14.71</td>
<td>14.97</td>
</tr>
<tr>
<td>$^{142}$Nd</td>
<td>16.66</td>
<td>17.10</td>
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</tbody>
</table>

The expressions (7) and (8) is essential to get a good fit to experimental data in various nuclei from Zr isotopes of Te isotopes. The parameter $b$ deduced independently from energy spectra of the $(0g_{9/2})_p(0g_{9/2})_n^{-1}$ configuration in $^{90}$Nb is 0.67, a value in fairly good agreement with the value in (11). We conclude from these results that the present coupling scheme of the hole states holds very well.

The large value of $-b/a$, which has been obtained also in the previous works [5, 6], suggests us that the isospin dependent interaction is very strong. This fact does not contradict the weak $(N - Z)$ dependence of the potential used by Veje [4], which can be understood if we rewrite, for example, the expression (7) in the following form:

\[
\begin{align*}
-\text{eq. (7)} & = C' + 40 \left( b/2 - a \right) - 50 \left( b_1/2 + A_1 \right) - 51 \left( b - B_1 \right) + A \left( 2a + b + 2A_1 - B_1 \right) / 4 \\
+ (N - Z) & \left( 2a - b + 2A_1 + 3B_1 \right) / 4 \\
& = -17.2 - 0.002A + 0.188 \left( N - Z \right).
\end{align*}
\]

As an extension of the present work, we are studying the width of the broad bump, considering the fragmentation of the state(s) (b) and/or (c) among many states with different intermediate isospins $T_0$ and $T_1$. Using isospin dependent interaction, the sharp isotope-dependence of the width observed in Sn isotopes [3] will be reproduced in this model.

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References