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A TEST OF THE NUCLEUS–NUCLEUS POTENTIAL USING FUSION EXCITATION FUNCTIONS

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A numerous set of experimentally measured fusion excitation functions was analysed in the framework of the Thomas–Huizenga–Igo model with transmission coefficients calculated in the parabolic approximation. It is shown that the fusion data provide very valuable information about the nucleus–nucleus interaction which can be used, particularly, for verification of different theoretical potentials.

It is well known that analysing angular distributions of heavy-ion elastic scattering one cannot determine the real part of the optical model potential unambiguously. The ambiguity is a consequence of a very strong absorption (with respect to the elastic scattering channel) even at relatively large distances where only a weak tail of the nucleus–nucleus potential influences the elastic scattering. Studying nuclear fusion one can probe the nucleus–nucleus potential deeper because the fusion process is sensitive to the height and shape of the potential energy barrier which is situated some 1–1.5 fm inwards with respect to the distance where different ambiguous potentials (describing elastic scattering) cross each other [1,2].

To test the peripheral region of an effective energy surface in fusion reactions we have chosen a numerous set of experimentally measured fusion excitation functions in the energy range close to the interaction barrier. Due to this limitation of the energy range we could neglect effects associated with the energy dissipation outside the potential energy barrier. Assuming that fusion always takes place when the barrier B_l has been passed, the fusion cross section can be expressed as

$$\sigma_{fu} = \pi \tilde{\lambda}^2 \sum_{l=0}^{\infty} (2l+1) T_l, \quad (1)$$

where $\tilde{\lambda}$ is the reduced wave length, and T_l is the transmission coefficient which is calculated in the parabolic approximation [3–5]:

$$T_l = \{1 + \exp[2\pi(B_l - E)/\hbar\omega_l]\}^{-1}, \quad (2)$$

$$\hbar\omega_l = \left| \frac{\hbar^2}{\mu} \frac{d^2 V_l}{dr^2} \right|_{r=r_B(l)}^{1/2}. \quad (3)$$

Here $B_l = V_l\{r_B(l)\}$ is the height of the barrier for angular momentum l , $r_B(l)$ its position, E the c.m. energy, μ the reduced mass, and V_l the total nucleus–nucleus potential consisting of nuclear, Coulomb and centrifugal components:

$$V_l(r) = V_N(r) + Z_1 Z_2 e^2/r + l(l+1)\hbar^2/2\mu r^2. \quad (4)$$

The only free parameters entering this simple model are those describing the nuclear potential $V_N(r)$. Using expressions (1)–(4) the predicted fusion cross sections were fitted to the experimental fusion excitation functions. With an assumed shape of $V_N(r)$ (e.g., the

Fig. 1. Contour plot of the constant χ^2 value of the 2-parameter (V_0 and a) fit to the $^{32}\text{S} + ^{27}\text{Al}$ fusion excitation function [6]. The nuclear potential is assumed to be of the Saxon-Woods shape with $R_0 = C_1 + C_2 = 6.44$ fm.

Saxon-Woods potential) the free parameters were determined using the least-squares procedure.

Results of the calculations show that the fusion excitation functions put stronger constraints on the parameters of the nucleus-nucleus potential as compared with the scattering data. Fig. 1 shows the contour plot of constant χ^2 value for the $^{32}\text{S} + ^{27}\text{Al}$ fusion excitation function [6], calculated assuming the Saxon-Woods shape of $V_N(r)$ with the depth and diffuseness parameters (V_0 and a) variable, and the radius parameter R_0 fixed. It is seen from this plot that the continuous ambiguity of the Igo type (correlation between V_0 and a) is reduced to a rather limited area in V_0, a space.

Using the model described above we have tested the validity of the proximity potential [7] and the Saxon-Woods potential with the parameter derived from boundary conditions following the liquid drop model [8]. Another commonly used potential, namely, the energy-density potential [9] is nearly identical with the proximity potential. Therefore, we did not test the energy-density potential separately.

The proximity potential [7] has the following form:

$$V_{\text{prox}} = 4\pi\gamma [C_1 C_2 / (C_1 + C_2)] b \Phi(\xi), \quad (5)$$

Table 1

Normalization factors, N_{prox} and N_{SW} , needed to fit the near-to-barrier fusion excitation functions with the proximity potential [7], and the Saxon-Woods potential with the parameters derived from boundary conditions following the liquid drop model [8].

Reaction	Ref.	Energy range a)		N_{prox}	N_{SW}
		E_{min}	E_{max}		
$^{12}\text{C} + ^{152}\text{Sm}$	[13]	1.00	1.32	1.54	1.07
$^{16}\text{O} + ^{63}\text{Cu}$	[14]	1.12	1.61	1.12	0.97
$^{16}\text{O} + ^{148}\text{Nd}$	[13]	1.05	1.30	1.78	1.05
$^{16}\text{O} + ^{150}\text{Nd}$	[13]	1.01	1.21	1.79	1.05
$^{16}\text{O} + ^{208}\text{Pb}$	[15]	1.06	1.28	2.08	1.08
$^{18}\text{O} + ^{148}\text{Nd}$	[13]	1.01	1.21	1.33	0.94
$^{32}\text{S} + ^{24}\text{Mg}$	[6]	1.05	1.35	1.36	1.00
$^{32}\text{S} + ^{27}\text{Al}$	[6]	1.01	1.36	1.56	1.01
$^{32}\text{S} + ^{40}\text{Ca}$	[6]	1.02	1.24	2.05	1.03
$^{35}\text{Cl} + ^{27}\text{Al}$	[16]	1.02	1.78	1.63	1.02
$^{35}\text{Cl} + ^{58}\text{Ni}$	[16]	1.01	1.14	2.17	1.01
$^{35}\text{Cl} + ^{60}\text{Ni}$	[16]	0.99	1.11	2.39	1.02
$^{35}\text{Cl} + ^{62}\text{Ni}$	[16]	1.00	1.18	2.29	1.00
$^{35}\text{Cl} + ^{64}\text{Ni}$	[16]	0.99	1.15	2.44	1.01
$^{35}\text{Cl} + ^{90}\text{Zr}$	[16]	1.09	1.31	1.72	0.93
$^{35}\text{Cl} + ^{116}\text{Sn}$	[16]	1.14	1.39	1.35	0.86
$^{35}\text{Cl} + ^{124}\text{Sn}$	[16]	1.18	1.43	0.99	0.78
$^{40}\text{Ca} + ^{40}\text{Ca}$	[17]	1.02	1.28	3.32	1.13

a) In $Z_1 Z_2 e^2 / 1.6 \text{ fm} (A_1^{1/3} + A_2^{1/3})$ units.

where $\Phi(\xi)$ is approximated by

$$\Phi(\xi < 1.2511) = -0.5(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3, \quad (6)$$

$$\Phi(\xi > 1.2511) = -3.437 \exp(-\xi/0.75). \quad (7)$$

C_1 and C_2 denote the central radii [10] of the colliding nuclei,

$$C = R - b^2/R, \quad (8)$$

where R is the effective sharp radius [10],

$$R = (1.13 + 0.0002A)A^{1/3} \text{ fm}, \quad (9)$$

and $b \approx 1 \text{ fm}$ is the surface width [10]. The variable ξ is defined as $\xi = (r - C_1 - C_2)/b$, and γ is the surface tension coefficient [11],

$$\gamma = 0.9517 \{1 - 1.7826(N - Z)^2/A^2\} \text{ MeV/fm}^2. \quad (10)$$

Another potential tested in this work is also based on the liquid drop model. Its shape is postulated to be

Fig. 2. Normalization factors, N_{prox} and N_{SW} , plotted as a function of the reduced mass number of the fusing system (see table 1).

of the Saxon-Woods form,

$$V_{\text{SW}} = -V_0 [1 + \exp((r - C_1 - C_2)/a)]^{-1}, \quad (11)$$

and the parameters V_0 and a are determined [8] from boundary conditions following the liquid drop model:

$$V_0 = b_{\text{surf}} \{A_1^{2/3} + A_2^{2/3} - (A_1 + A_2)^{2/3}\}, \quad (12)$$

$$a = \frac{(C_1 + C_2)b_{\text{surf}} \{A_1^{2/3} + A_2^{2/3} - (A_1 + A_2)^{2/3}\}}{16\pi\gamma C_1 C_2}. \quad (13)$$

Here, $b_{\text{surf}} \approx 17$ MeV is the surface energy coefficient, and A_1 and A_2 are the mass numbers of the colliding nuclei. (C_1 , C_2 and γ are the same as defined formerly.)

Both potentials, (5)–(7) and (11)–(13), have the same value of the first derivative at $r = C_1 + C_2$ (the contact force [12]), but while the proximity potential (5) is proportional to this force in the whole range of relative distances, the Saxon-Woods potential (11)–(13) is normalized accordingly with the assumption that the surface energy changes adiabatically all the way down to its minimum at $r = 0$.

Results of our tests are presented in table 1. For each reaction we have determined the normalization factor, N , which must stand in front of the potentials V_{prox} and/or V_{SW} (eqs. (5) and (11), respectively), so as to obtain the best fit to the experimental fusion excitation function:

$$V_N(\text{least } \chi^2) = N_{\text{prox}} V_{\text{prox}}, \quad (14)$$

$$V_N(\text{least } \chi^2) = N_{\text{SW}} V_{\text{SW}}. \quad (15)$$

It is seen from table 1 (and also from fig. 2) that the Saxon-Woods potential with the parameters determined from boundary conditions following the liquid drop model gives perfect agreement with the experimental data. The experimentally determined depths of the potential do not differ from the values predicted by eq. (12) more than by 20%. On the other hand, results obtained from the proximity potential show a considerable dispersion of the N_{prox} values (mostly above $N_{\text{prox}} = 1$). This indicates that to obtain a good fit using the proximity-type potential it is not sufficient to increase the normalization factor N_{prox} or to extend (independently) the tail of the potential by an increase of the constant under the exponent in eq. (7).

The success of the potential (11)–(13) in explaining the experimental fusion cross sections at low energies seems to prove the reality of the coupling between the depth and slope of the potential that follows the assumption of adiabaticity of the collision. At this energy range the proximity potential fails, but it is very likely that at higher energies the proximity potential and also the energy-density potential which are based on the sudden approximation will turn out to be more adequate for description of nuclear fusion.

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