A calculation of low-lying collective states in even-even nuclei
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We present results of a calculation of the low-lying collective quadrupole states in even–even nuclei within the framework of the proton–neutron interacting boson model.

The purpose of this note is to point out that the interacting proton–neutron boson model \[1,2\] may provide a simple and yet detailed phenomenological description of low-lying collective states in even–even nuclei. This description appears to be equally good irrespectively of the intrinsic nature of the collective states (vibrational, axially symmetric rotational, γ-unstable, etc.). In addition, once the boson–boson interaction has been determined from one set of experiments this phenomenological description allows to predict, in a straightforward way, states in other nuclei, thus providing a powerful tool for nuclear studies. Finally, since the parameters appearing in the interacting proton–neutron boson hamiltonian are directly related to the underlying effective nucleon–nucleon interaction, this phenomenological study will provide the input for testing microscopic theories of collective states.

In this letter, we illustrate these points by presenting results of calculations of the isotopic chains 54Xe, 56Ba, 58Ce, 39 nuclei in all, of which only 10 are experimentally well known. Calculations of other isotopic chains, which have already been performed or which are being performed at present, will be presented in a longer paper. The starting hamiltonian for these calculations is the proton–neutron interacting boson hamiltonian \[2\]

\[
H = H_\pi + H_\nu + V_{\pi\nu},
\]

where the index \(\pi\) (\(\nu\)) denotes proton (neutron) bosons. The boson–boson interactions and boson energies appearing in \(H_\pi\), \(H_\nu\) and \(V_{\pi\nu}\) can in principle be derived from a microscopic theory, using either the approach described in refs. \[2\] and \[3\] or the nuclear field theory (NFT) of Bes, Broglia and collaborators \[4\]. From a phenomenological point of view, one may attempt to extract an effective boson–boson interaction from one set of experiments and then use this effective interaction to correlate a larger set of experimental results, much in the same way as it is done in the shell-model description of light nuclei \[5\]. In order to extract the effective interaction without much complication, we make use of a simpler form of \(H\) given by \[2\]

\[
H = e(n_{d_\pi} + n_{d_\nu}) + \kappa Q_\pi \cdot Q_\nu + aM,
\]

where

\[
Q_\pi(\nu) = (d^\dagger \times s + s^\dagger \times d)^{(2)}_{\pi(\nu)} + \chi_{\pi(\nu)} (d^\dagger \times d)^{(2)}_{\pi(\nu)}. \tag{3}
\]

In terms of the underlying microscopic picture, this simpler form will arise from a nucleon–nucleon interaction with a strong pairing component in the identical particle channel (whose properties determine \(e\), taken equal, for simplicity, for protons and neutrons, \(e_\pi = \)

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$e' = e$), and with a strong quadrupole component in the nonidentical particle channel (whose properties determine $\kappa$, $x_{\pi}$ and $x_{\nu}$). The last term in (2), proportional to the Majorana operator $M$, fixes the location of the states with mixed U(6) symmetry $[N_{\pi} + N_{\nu} - 1, 1]$, $[N_{\pi} + N_{\nu} - 2, 2]$, ... with respect to the totally symmetric states $[N_{\pi} + N_{\nu}, 0]$, $N_{\pi(\nu)}$ being the total number of proton (neutron) pairs, as in ref. [2]. Since there is no experimental evidence for states of this sort, all calculations have been done with a large value of $a$, thus placing the mixed states at high energy. The effect of the last term on the low-lying states is then small.

The quantities $e$, $\kappa$, $x_{\pi}$, $x_{\nu}$, which, in the parameterization (2), determine the structures of the calculated spectrum, may depend, in principle, on both proton and neutron number $N_{\pi}, N_{\nu}$. However, guided by our microscopic calculations, we have assumed that only $e$ and $\kappa$ depend both on $N_{\pi}$ and $N_{\nu}$, while $x_{\pi}$ depends only on $N_{\pi}$ and $x_{\nu}$ on $N_{\nu}$. If this is so, by determining the values of the parameters along one isotopic (fixed $N_{\pi}$) and one isotonic (fixed $N_{\nu}$) chain, we should not only be able to describe nuclei along these chains but also predict states in other neighbouring nuclei.

We have applied the procedure to the 50–82 major shell, and the results for the isotopic chains $^{54}\text{Xe}$, $^{56}\text{Ba}$, $^{58}\text{Ce}$ are shown in figs. 1, 2, 3. The neutron dependence of the parameters has been determined by combining the results for the isotopic chain $^{46}\text{Pd}$ (not shown here) in the region $54 \leq \text{neutron number} \leq 64$ with those

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**Fig. 1.** Calculated energy spectra in $^{54}\text{Xe}$. The circles, squares, triangles are the experimental values [9].

**Fig. 2.** Calculated energy spectra in $^{56}\text{Ba}$. The circles, squares, triangles are the experimental values [9].

**Fig. 3.** Calculated energy spectra in $^{58}\text{Ce}$. The circles, squares, triangles are the experimental values [9].
for the isotopic chain $^{54}\text{Xe}$ in the region $64 \leq \text{neutron number} \leq 78$. The proton dependence has been determined by using the results for the isotonic chain with neutron number $= 76$ and $54 \leq \text{proton number} \leq 58$. The values of the parameters are shown in figs. 4a and 4b. One may note that the values of the parameters are smooth enough that simple extrapolations can be made from known to unknown regions. Moreover, it appears that in the neutron shell $50-82$ and at the beginning of the proton shell $54 \leq Z \leq 58$, the parameters change in accordance with a simple microscopic theory in which all single-particle levels in a major shell are considered to be degenerate [2,3]. However, in addition to being able to predict states in other nuclei, the most striking feature of figs. 1, 2 and 3 is their ability to describe in an accurate fashion widely different intrinsic collective structures ranging from those typical of an anharmonic vibrator (SU(5) symmetry in the IBA [6]) to those typical of an axially symmetric rotor (SU(3) symmetry in the IBA [7]), to those typical of a $\gamma$-unstable rotor (SO(6) symmetry in the IBA [8]).

Finally, we have investigated the effect of adding a neutron–neutron and proton–proton boson interaction in (2). For the isotopic chains presented here which have a small number of proton bosons ($N_{\pi} = 2, 3, 4$) the effects of the proton–proton interaction are expected to be small. We have therefore set this interaction to zero and only considered a neutron–neutron boson interaction of the form

$$V_{\nu\nu} = \sum_{L=0,2,4} c_{L}^{(v)} \frac{1}{2}(2L + 1)^{1/2}$$

$$\times [(d_\nu^+ \times d_\nu^+)^{(L)} \times (\bar{d}_\nu \times \bar{d}_\nu)^{(L)}]^{(0)}. \quad (4)$$

Fig. 4. (a) Behaviour of $\epsilon$, $\kappa$, $x_\nu$ as a function of neutron number. (b) Behavior of $\epsilon$, $\kappa$, $x_\pi$ as a function of proton number. (c) The neutron–neutron interaction parameters $c_{L}^{(\nu)}$ as a function of neutron number. $c_{4}^{(\nu)}$ was kept equal to zero. (d) The proton–proton interaction parameters $c_{L}^{(\pi)}$ were kept equal to zero.
An interaction of this form is expected from microscopic calculations [3,4]. The neutron–neutron interaction produces sizable effects only in nuclei with few proton bosons and many neutron bosons, for example in $^{120}_{54}$Xe$_{66}$ ($N_p = 2, N_n = 8$). The structure of nuclei with both $N_p$ and $N_n$ large is on the contrary mostly determined by the strong $Q_{n} - Q_{p}$ interaction in (2). The values of the parameters $c_{J}^{(0)}$ used in the calculation are shown in fig. 4c. The neutron–neutron interaction was taken to be the same for all isotopic chains. Figs. 2, 3 and 4 include the effects of this interaction.

In conclusion, we present a set of calculations based on the proton–neutron boson model which appear to describe nuclei with widely different intrinsic structure. These calculations include energies (shown here), E0, E2, E4 transition rates, isotope and isomer shifts, $(p,t)$ and $(t,p)$ transfer strengths. The most appealing aspect of these calculations is that they provide a unified framework in which all even–even nuclei can be described. Moreover, they make specific predictions for nuclear levels not yet known. With the rapid development of nuclear spectroscopy which is taking place at present, it will be interesting to see whether or not these predictions turn out to be correct. For example, the lighter Ba isotopes with $120 \leq A \leq 128$ should be easily accessible at UNISOR, ISOLDE and similar facilities.

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