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THE ASYMPTOTIC PHOTON PROPAGATOR IN A NONPERTURBATIVE ITERATION SCHEME OF QUANTUM ELECTRODYNAMICS

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In connection with the possibility of a finite theory of quantum electrodynamics, the asymptotic photon-propagator is investigated in lowest order of a nonperturbative iteration technique. The result is that at this level of approximation the theory cannot be finite.

A nonperturbative approximation technique initiated by Salam [1] has received new interest as an iterative scheme for the solution of the Dyson–Schwinger equations in quantum electrodynamics [2,3]. One approximates the exact vertex functions in the Dyson–Schwinger equations by an ansatz which respects the Ward–Takahashi identities and hopes that, at least in some respects, this method can supersede truncated perturbation theory.

The spectral function ρ of the renormalized electron propagator $S(p)$, defined as

$$S(p) = \int_{-\infty}^{\infty} dW \rho(W) [\gamma \cdot p - W + i\epsilon(W)]^{-1}, \quad (1)$$

has been calculated [2] in lowest order in the Landau gauge, with the renormalized photon propagator $D^{\mu\nu}(k)$ approximated by the free one, i.e.

$$e^2 D^{\mu\nu}(k) = (-g^{\mu\nu} + k^\mu k^\nu / k^2) e^2 / k^2, \quad (2)$$

where e is the electron charge.

The result is

$$\begin{aligned} \rho(W) = & 6\lambda R(\lambda) (W^2/m^2 - 1)^{-1-6\lambda} \\ & \times [W^{-1} F(-3\lambda, -3\lambda; -6\lambda; 1 - W^2/m^2) \\ & + m^{-1} F(-3\lambda, 1 - 3\lambda; -6\lambda; 1 - W^2/m^2)], \end{aligned} \quad (3)$$

where $\lambda = e^2/16\pi^2$, m is the renormalized mass of the electron and $R(\lambda)$ is an arbitrary function with $R(0) = 1$.

The bare mass of the electron, m_0 and the inverse of the electron's wavefunction renormalization constant Z_2 , have also been calculated [4]:

$$m_0 = 0, \quad (4)$$

$$Z_2^{-1} = R(\lambda) [\Gamma(1 + 3\lambda)]^2 \Gamma(1 - 6\lambda). \quad (5)$$

With these results in mind, a connection with the work of Johnson–Baker–Willey [5] on finite quantum electrodynamics has been suggested in ref. [4].

The JBW program states that quantum electrodynamics can be a finite theory if $m_0 = 0$ and if the renormalized photon propagator is asymptotically finite, which means for example in the Landau gauge

$$e^2 D^{\mu\nu}(k) = (-g^{\mu\nu} + k^\mu k^\nu / k^2) e_0^2 / k^2, \quad (6)$$

for $-k^2/m^2 \rightarrow \infty$, where e_0^2 is the asymptotic coupling strength. However, the consistency of the Salam technique with the hypothesis (6) has not been further explored in ref. [4], so we propose to examine it here.

If we stick to the Landau gauge then $D^{\mu\nu}(k)$ can be calculated from

$$D^{\mu\nu}(k) = (-g^{\mu\nu} + k^\mu k^\nu / k^2) k^{-2} (1 + e^2 \Pi_R(k^2))^{-1}, \quad (7)$$

where $\Pi_R(k^2)$ is the renormalized vacuum polarization, defined as

$$\Pi_R(k^2) = \Pi(k^2) - \Pi(0), \quad (8)$$

where

$$\Pi(k^2) = -(1/3) \Pi^\mu{}_\mu(k) / k^2, \quad (9)$$

and

$$\Pi^{\mu\nu}(k) = ie^2 Z_2 \times \int (2\pi)^{-4} d^4 q \operatorname{Tr} [\gamma^\mu S(q) \Gamma^\nu(q, q-k) S(q-k)]. \quad (10)$$

The exact equation (7) can only be consistent with (6) for $-k^2/m^2 \rightarrow \infty$ if $\Pi_R(k^2)$ is asymptotically finite. To check this, it is sufficient to calculate $\Pi(k^2)$ from the JBW model of vacuum polarization which is identical to the exact $\Pi(k^2)$ in perturbation theory but without internal photon self-energy insertions and all internal photon propagators replaced by the asymptotic form of eq. (6). Then one can show [5] that for large $-k^2/m^2$, $\Pi_R(k^2)$ behaves as

$$\Pi_R(k^2) \sim f(e_0^2) \log(-k^2/m^2) + \text{bounded terms for } -k^2/m^2 \rightarrow \infty, \quad (11)$$

where the function $f(x)$ can be calculated from perturbation theory. To obtain consistency with (6), the logarithmic dependence on k^2 must be suppressed and this leads to the eigenvalue condition

$$f(x) = 0. \quad (12)$$

If (12) is realised for some positive x then e_0^2 is fixed as a root of (12). Now let us calculate the vacuum polarization with the Salam technique using the ansatz

$$S(p') \Gamma_\mu(p', p) S(p) = \int_{-\infty}^{\infty} dW \rho(W) [\gamma \cdot p' - W]^{-1} \gamma_\mu [\gamma \cdot p - W]^{-1}, \quad (13)$$

in eq. (10). Then the vacuum polarization becomes a weighted integral over the second order vacuum polarization of perturbation theory, with electron mass W and weight function ρ .

Via (8) and (9), Π_R is obtained

$$\Pi_R(k^2) = \frac{Z_2 k^2}{12\pi^2} \int_{-\infty}^{\infty} dW \rho(W) \int_{4W^2}^{\infty} \frac{dt}{t(t-k^2-i\epsilon)} \times \left(1 + \frac{2W^2}{t}\right) \left(1 - \frac{4W^2}{t}\right)^{1/2}, \quad (14)$$

and only Z_2 and $\rho(W)$ have still to be determined.

For the construction of $f(x)$ in the JBW program

it is sufficient to know the leading behavior of $S(p)$ for large $-p^2/m^2$. For this purpose the exact photon propagator can be replaced by (6) but comparence with (2) shows that the calculation will be the same as in [2]. So the leading behaviour of ρ will be given by eq. (3) but with e_0^2 instead of e^2 . Then

$$\Pi_R(k^2) = \frac{Z_2}{12\pi^2} \int_{-\infty}^{\infty} dW \rho(W) \left\{ -\log(-k^2/m^2) + \left[\frac{5}{3} + \frac{4W^2}{k^2} + \log \frac{W^2}{m^2} + \log(-k^2/W^2) - \left(1 + \frac{2W^2}{k^2}\right) \sqrt{1 - \frac{4W^2}{k^2}} \right] \times \log \frac{(1 - 4W^2/k^2)^{1/2} + 1}{(1 - 4W^2/k^2)^{1/2} - 1} \right\}, \quad (15)$$

results. The remaining unknown, Z_2 , can be calculated with the Lehmann formula

$$Z_2^{-1} = \int_{-\infty}^{\infty} dW \rho(W), \quad (16)$$

and it is consistent to use the same ρ as in eq. (15). The function $f(x)$ appears to be equal to $-1/12\pi^2$, so there cannot be a solution of (12).

We conclude that the Salam technique in lowest order gives no prospect of a finite quantum electrodynamic theory.

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