University of Groningen

Configuration mixing in the interacting boson model

Duval, Philip D.; Barrett, Bruce R.

Published in:
Physics Letters B

DOI:
[10.1016/0370-2693\(81\)90321-X](https://doi.org/10.1016/0370-2693(81)90321-X)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
1981

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Duval, P. D., & Barrett, B. R. (1981). Configuration mixing in the interacting boson model. *Physics Letters B*, 100(3), 223-227. [https://doi.org/10.1016/0370-2693\(81\)90321-X](https://doi.org/10.1016/0370-2693(81)90321-X)

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

CONFIGURATION MIXING IN THE INTERACTING BOSON MODEL

Philip D. DUVAL and Bruce R. BARRETT

*Kernfysisch Versneller Instituut, Groningen, The Netherlands**and Department of Physics and the Theoretical Physics Division, University of Arizona, Tucson, AZ 85721, USA*¹

Received 30 December 1980

We present a new, general technique for providing a quantitative description of configuration mixing in nuclei. The technique uses the Interacting Boson Model to make separate descriptions of the different configurations, which are then mixed using an appropriate mixing hamiltonian. An example is given using the mercury isotopes.

In many nuclei the coexistence of two quite different structures in the same energy region is manifest in the experimental data. Generally there is configuration mixing between these structures. A particularly striking example of this effect is in the light mercury isotopes [1–3]. In this paper we present an extension of the Interacting Boson Model (IBM) of Arima and Iachello [4], which allows us to describe in a completely general and quantitative manner the mixing of two different configurations of any arbitrary structure.

The method consists of first describing the general features of the two different configurations in terms of two different IBM calculations and then mixing the results of these two calculations using an appropriate IBM mixing hamiltonian. Our technique is most easily understood in terms of a specific application, which we choose to do for the mercury isotopes in the region $^{184}_{80}\text{Hg}_{104}$ to $^{188}_{80}\text{Hg}_{108}$. These are particularly interesting isotopes to study since they simultaneously show one set of states which are distinctly rotational in character and a second set which are vibrational in structure. Such simultaneous rotational and vibrational structures are a common feature of nuclei which have either their proton number or their neutron number near a closed-shell value, another example being the tin isotopes [5]. Although previous descriptions of this phenomenon in the Hg isotopes have been

given [1–3], we will see that in the present technique the rotational and vibrational characters of the different configurations arise quite *naturally* through the IBM formalism.

To illustrate this effect we show in fig. 1 the energy spectra of the Hg isotopes [6]. One clearly sees the

Fig. 1. Experimental energy levels for $^{182}_{80}\text{Hg}_{102}$ to $^{192}_{80}\text{Hg}_{112}$. The solid lines connect states of vibrational character. The dashed lines connect states of rotational character. The data are from ref. [6].

¹ Permanent address.

two different configurations, namely the vibrational states, which change very little from isotope to isotope, and the rotational levels which come very low in energy at $^{184}_{80}\text{Hg}_{104}$.

We now present a description of our technique and specifically apply it to the Hg isotopes. In the IBM, correlated valence neutron pairs and proton pairs are assumed to be bosons of $J = 0$ (s bosons) and $J = 2$ (d bosons). Particle bosons are counted from the beginning to the middle of a shell; hole bosons, from the middle to the end. For example, $^{182}_{80}\text{Hg}_{102}$ has ten neutron-particle bosons and one proton-hole boson. In the IBM the properties of nuclei are assumed to arise from interactions among the bosons. The IBM hamiltonian which incorporates the essential features of the underlying fermionic interactions is of the form [4,7,8]

$$H = \epsilon \hat{n}_d + \kappa \hat{Q}_\pi \cdot \hat{Q}_\nu + \hat{V}_{\nu\nu} + \hat{V}_{\pi\pi}, \quad (1)$$

where ϵ is related to the difference in the s- and d-boson energies and $\hat{n}_d = \hat{n}_{d\pi} + \hat{n}_{d\nu}$ is the d-boson number. The parameter κ is the strength of the quadrupole-quadrupole interaction, and $\hat{Q}_\pi$ and $\hat{Q}_\nu$ are the proton and neutron quadrupole operators, respectively, which are given by:

$$\hat{Q}_\rho = (s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger s_\rho)^{(2)} + \chi_\rho (d_\rho^\dagger \tilde{d}_\rho)^{(2)}, \quad \rho = \pi, \nu, \quad (2)$$

where χ_ρ is the ratio of the seniority-conserving part of the quadrupole operator to the seniority-nonconserving part. The term $\hat{V}_{\nu\nu}$ ($\hat{V}_{\pi\pi}$) is the residual interaction among the neutron (proton) bosons, which we consider to conserve d-boson number [4].

In the present work we consider the simultaneous vibrational and rotational structures in the Hg isotopes to arise from two different boson configurations. The vibrational spectrum is assumed to come from the standard IBM picture of the interaction of the one proton-hole boson ($N_\pi = 1$) with the valence neutron bosons (N_ν), as shown in fig. 2a. In this case, the effect of the $\kappa \hat{Q}_\pi \cdot \hat{Q}_\nu$ interaction is small, since $N_\pi = 1$, and the hamiltonian (1) possesses more or less a U(5) or vibrational symmetry [7,8]. The rotational spectrum is assumed to arise from the excitation of a proton boson (i.e. a proton pair) above the 82 shell gap, so as to produce a configuration of two proton-hole bosons and one proton-particle boson ($N_\pi = 3$), as shown in fig. 2b. In this case we have *three* active proton bosons to interact with the active

Fig. 2. (a) The single-particle proton and neutron configurations for the even-even Hg isotopes. (b) The single-particle proton and neutron configurations for the even-even Hg isotopes when a proton pair has been excited across the 82 shell gap.

neutron bosons through the $\kappa \hat{Q}_\pi \cdot \hat{Q}_\nu$ term. This interaction strongly mixes states of different numbers of s and d bosons and has been shown to lead to a rotational spectrum when several active proton and several active neutron bosons are present. So for $N_\pi = 3$ the hamiltonian (1) possesses more or less a SU(3) or rotational symmetry [7,8].

We would also expect there to be some mixing between these different configurations. There is evidence for such mixing in the experimental data [9]. It can also be seen in fig. 1, e.g. the 4^+ states for $N = 106$ and 108 . To account for this mixing we introduce a boson mixing hamiltonian between the $N_\pi = 1$ configuration and the $N_\pi = 3$ configuration. In the specific case considered here the most general mixing hamiltonian that is two body in the fermion space and connects the two configurations is:

$$H_{\text{mix}} = \alpha (s_\pi^\dagger s_\pi^\dagger + s_\pi s_\pi)^{(0)} + \beta (d_\pi^\dagger d_\pi^\dagger + \tilde{d}_\pi \tilde{d}_\pi)^{(0)}. \quad (3)$$

In the case of the mixing of configurations in other nuclei, H_{mix} may have a different form.

The complete IBM calculation of the Hg isotopes now involves two separate calculations (one for $N_\pi = 1$, another for $N_\pi = 3$) plus a mixing calculation. In the former two independent calculations, the hamiltonian [eq. (1)] is diagonalized in the appropriate space. Each of these calculations would appear to in-

volve several different parameters. However, we have insisted that the neutron-number dependent parameters be the same for each isotope in *both* calculations. The $\hat{V}_{\pi\pi}$ term is not used since N_π is small, i.e. one or three. Also, the value for χ_π in each calculation is kept fixed for *all* isotopes. In the mixing calculation we take the first four states for each angular momentum in the $N_\pi = 1$ configuration and mix them with the first four states with the corresponding angular momentum in the $N_\pi = 3$ configuration according to eq. (3). We keep the parameters α and β *constant* for *all* isotopes. We also need another parameter which gives the amount of energy needed to excite a proton boson into the next major shell. This parameter, which we call Δ , is also kept fixed at a value of 4 MeV for all isotopes.

In figs. 3a and b we give the results of a preliminary calculation. Fig. 3a shows an unmixed calculation, i.e. $\alpha = \beta = 0$. In this case we do two separate calculations and add an amount Δ to the $N_\pi = 3$ eigenvalues. Fig. 3b shows the result of including mixing among

Fig. 3. Comparison of theoretical versus experimental energy levels (a) for no configuration mixing and (b) for configuration mixing, with parameters $\alpha = 0.1$ MeV and $\beta = 0.2$ MeV [see eq. (3)]. Solid lines are the $N_\pi = 1$ configurations, labelled by (1); dashed lines are the $N_\pi = 3$ configurations, labelled by (3). Only the first two 0^+ bands are shown. Experimental states are ordered in increasing angular momentum. Open circles are vibrational levels. Solid circles are rotational levels.

the states in fig. 3a. The mixing is seen to cause the states to “move” in exactly the manner described by the experimental states.

Using the wave functions from the previous calculations, we can determine the $B(E2)$ values by considering an E2 transition operator of the form:

$$T^{(E2)} = e_1(\hat{Q}_{1\pi} + \hat{Q}_{1\nu}) + e_3(\hat{Q}_{3\pi} + \hat{Q}_{3\nu}), \quad (4)$$

where e_1 has units of e b and gives the effective charge of the proton and neutron bosons in the $N_\pi = 1$ configuration. The proton and neutron bosons are treated as having the same effective charge for simplicity. Similarly, e_3 is the effective boson charge in the $N_\pi = 3$ configuration. The $\hat{Q}$ operators are the same as those given in eq. (2). The subscripts 1 and 3 refer to the proton-boson space in which the operator acts.

In table 1 we compare our theoretical calculations for $B(E2)$ transition rates and branching ratios with experiment. For absolute $B(E2)$ values we have only to determine the values of e_1 and e_3 . For the sake of simplicity we assume the ratio e_1/e_3 is given by κ_1/κ_3 where κ_1 and κ_3 are the strengths of the quadrupole interaction in the $N_\pi = 1$ and $N_\pi = 3$ configurations, respectively. We do this since both the effective charge and the quadrupole interaction strength are proportional to the mean square proton radius. Thus by fitting to one of the experimental $B(E2)$ values we determine both e_1 and e_3 . For the branching ratios, however, only the ratio e_1/e_3 is important, so there are no new free parameters in this case.

For the most part, the theoretical calculations of energy levels and $B(E2)$ values compare very well with experiment, with the exception of the ratio $B(E2; 2_2^+ \rightarrow 2_1^+)/B(E2; 2_2^+ \rightarrow 0_1^+)$ for $^{186}_{80}\text{Hg}_{106}$. Since the amount of mixing between states depends upon the location of the energy levels before they are mixed as well as the mixing parameters, α and β , improving the fit to the energy levels should improve the agreement of the calculated $B(E2)$ values with experiment.

We have performed our calculations using an excited configuration built on three identical proton bosons. A more realistic calculation should make a distinction between the two proton-boson holes and the excited proton-boson particle. However, if we consider only symmetric combinations of these different proton bosons, then the calculation can still be performed treating them as identical. The argument is the same as that given for performing an IBM-1 calcu-

Table 1

Comparison of experimental and theoretical $B(E2)$ values.

$B(E2; J_i \rightarrow J_f)$						$B(E2; J_i \rightarrow J_f)/B(E2; J_i' \rightarrow J_f')$								
$2_1^+ \rightarrow 0_1^+$		$4_1^+ \rightarrow 2_1^+$		$6_1^+ \rightarrow 4_1^+$		$\frac{2_2^+ \rightarrow 2_1^+}{2_2^+ \rightarrow 0_1^+}$		$\frac{4_1^+ \rightarrow 2_2^+}{4_1^+ \rightarrow 2_1^+}$		$\frac{4_2^+ \rightarrow 2_1^+}{4_2^+ \rightarrow 2_1^+}$		$\frac{6_2^+ \rightarrow 4_2^+}{6_2^+ \rightarrow 4_1^+}$		
exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.	exp.	th.	
$^{188}_{80}\text{Hg}_{108}$	—	0.41	—	0.62	—	0.70	15 c)	11.1	—	0.16	81 d)	19.5	1.35 d)	4.8
$^{186}_{80}\text{Hg}_{106}$	0.28 a)	0.39	0.6 a)	0.63	1.9 a)	1.06	>1200 c)	6.8	<0.9 c)	0.54	1.17 c)	4.55	30 c)	28.8
$^{184}_{80}\text{Hg}_{104}$	0.39 b)	0.39	1.13 b)	0.88	2.1 b)	1.19	—	1.48	—	0.28	—	6.27	—	84.2

a) Ref. [10]. b) Ref. [11]. c) Ref. [12]. d) Ref. [13].

lation, where neutron bosons and proton bosons are treated as identical [8].

As we have already seen in the calculations for the Hg isotopes, even though the spectrum for $N_\pi = 3$ is shifted up in energy by 4 MeV with respect to the $N_\pi = 1$ spectrum, sufficient energy is gained through the attractive $\kappa \hat{Q}_\pi \cdot \hat{Q}_\nu$ interaction due to the increase in the number of active protons to overcome most of this shift. This effect is not present in the heavy Hg isotopes, since the number of active neutron bosons is small. The $N_\pi = 3$ spectrum lies very high in energy for these isotopes [14].

One might expect that states formed by exciting two proton bosons into the next major shell are also important. However, this will depend upon the strength of the repulsive particle-hole (p-h) interaction. If we take the p-h interaction strength to be B , where $B > 0$, then for one particle and two holes, an amount of energy equal to $2B$ is lost to the p-h interaction. But for two particles and three holes, an amount of energy $6B$ is lost. For three particles and four holes, $12B$ is lost, etc., so that as far as *low-lying energy levels are concerned*, this process must eventually cut itself off. In the Hg isotopes it appears that only one boson p-h excitation of this kind is important.

In conclusion, we have presented a new technique which is of general use for describing configuration mixing in nuclei, although a specific application has been given only for the Hg isotopes. This technique involves describing the different configurations in terms of separate IBM calculations and then mixing the results of these calculations with the appropriate IBM mixing hamiltonian. In the case of the Hg isotopes we saw that the vibrational and rotational characters of

the energy levels arose *naturally* in our approach, due to the U(5) symmetry of the $N_\pi = 1$ configuration and the SU(3) symmetry of the $N_\pi = 3$ configuration [7,8]. The present calculations are only preliminary; more extensive calculations are being carried out for the entire chain of Hg isotopes and will be reported in a forthcoming paper. In closing, we emphasize that our technique is quite general and is also being applied to other nuclei, such as the platinum and molybdenum isotopes, which also exhibit the features of the mixing of two distinctly different configurations [15].

We wish to thank Professor F. Iachello for calling this problem to our attention and for numerous helpful and encouraging discussions, and Dr. J.L. Wood for providing us with information regarding recent experimental data for the mercury isotopes. We also thank Professor R. Siemssen for the hospitality provided to us at the Kernfysisch Versneller Instituut, Groningen, where this work was carried out. This work was funded in part by the Stichting voor Fundamenteel Onderzoek der Materie (FOM), by the National Science Foundation Grant No. PHY-7902654-A01 and by the Theoretical Physics Division, Arizona Research Laboratories, University of Arizona, Tucson.

References

- [1] F. Dickmann and K. Dietrich, Z. Phys. 271 (1974) 417.
- [2] D. Kolb and C.Y. Wong, Nucl. Phys. A245 (1975) 205.
- [3] S. Frauendorf, F.R. May and V.V. Pashkevich, in: Future directions in studies of nuclei far from stability, eds. J.H. Hamilton et al. (North-Holland, Amsterdam, 1980) p. 133, and references therein.

- [4] A. Arima and F. Iachello, *Ann. Phys. (NY)* 99 (1976) 253; 111 (1978) 201;
O. Scholten, F. Iachello and A. Arima, *Ann. Phys. (NY)* 115 (1978) 325;
A. Arima, T. Otsuka, F. Iachello and I. Talmi, *Phys. Lett.* 66B (1977) 205;
T. Otsuka, A. Arima, F. Iachello and I. Talmi, *Phys. Lett.* 76B (1978) 139.
- [5] J. Bron et al., *Nucl. Phys. A* 318 (1979) 335.
- [6] C.M. Lederer and V.S. Shirley, eds., *Table of isotopes*, 7th Ed. (Wiley, New York, 1978).
- [7] F. Iachello, ed., *Interacting bosons in nuclear physics* (Plenum, New York, 1979).
- [8] O. Scholten, *The interacting boson approximation model and applications*, Ph.D. thesis, Univ. of Groningen (1980).
- [9] P. Dabkiewicz et al., *Proc. Intern. Conf. Nuclear structure (Tokyo, 1977)*, *J. Phys. Soc. Japan Suppl.* 44 (1978) 503.
- [10] D. Proetel et al., *Phys. Rev. Lett.* 31 (1973) 896.
- [11] N. Rud et al., *Phys. Rev. Lett.* 31 (1973) 1421.
- [12] R. Béraud et al., *Nucl. Phys. A* 284 (1977) 221.
- [13] J.H. Hamilton et al., *Phys. Rev. Lett.* 35 (1975) 562.
- [14] R.E. Anderson et al., *Phys. Rev. C* 19 (1979) 2138.
- [15] J.L. Wood, in: *Frontier research in nuclear physics*, eds. D.H. Feng et al. (Plenum, New York, 1981).