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ON TRIAXIAL FEATURES IN THE NEUTRON-PROTON IBA

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We show that the neutron-proton IBA model contains a dynamical symmetry that can be interpreted as a triaxial rotor in a geometrical approach. It is suggested that the energy spectrum and electric quadrupole matrix elements of the nucleus ^{104}Ru can be described by a situation intermediate between the triaxial and the γ -unstable limit.

It has been known for a long time that in certain regions of the periodic system nuclei with static deformation show deviations from a rigid axially symmetric rotor picture. In the geometrical approach these nuclear properties are usually interpreted in terms of either a γ -unstable rotor model [1], or a rigid triaxial rotor (with $0^\circ < \gamma < 60^\circ$) model [2]. Since most experimentally observable quantities are rather insensitive to the difference between these two shape parametrizations it has been difficult to really judge the validity of either approach.

More recently data from Coulomb excitation measurements [3,4] that seem to suggest a more complex and possibly intermediate situation between γ -rigid and γ -unstable properties, have stimulated renewed interest in the nature of the γ -potential. In this note we address the question of the description of non-axially symmetric features in terms of the neutron-proton IBA model [5]. We show that two of the dynamical symmetries occurring in this model can be interpreted as γ -unstable and triaxial rotors, respectively.

The interacting boson model in its original form (IBA-1) [6-8] provides a unified description of rotation-vibration degrees of freedom of nuclei in terms of two kinds of bosons, an $L = 0$ s-boson, and an $L = 2$ d-boson. The states of the N -boson system can be classified according to the totally symmetric irreducible representation $[N]$ of $\text{SU}(6)$. The most general two-body hamiltonian within the boson space

can then be written in terms of the generators g_i of this group. In three limiting cases H can be expressed in terms of the generators of a subgroup $G \subset \text{SU}(6)$. These so-called dynamical symmetries are characterized by the group chains

$$\begin{aligned} \text{SU}(6) \supset \text{U}(5) \supset \text{O}(5) \supset \text{O}(3) & \quad (\text{a}), \\ & \supset \text{SU}(3) \supset \text{O}(3) & \quad (\text{b}), \\ & \supset \text{O}(6) \supset \text{O}(5) \supset \text{O}(3) & \quad (\text{c}). \end{aligned} \quad (1)$$

These cases can be identified [9,10] with the anharmonic vibrator (a), the γ -unstable rotor (b), and the axially symmetric deformed rotor (c), of the geometrical model, respectively. Intermediate situations can be described by numerical diagonalization of the full hamiltonian. Although many experimentally observed collective properties can be interpreted surprisingly well in terms of this simple model some small but systematic differences occur between the predicted and experimental energy spectra especially in the $\text{O}(6)$, or γ -unstable region, which seems to indicate the presence of at least some degree of triaxiality [3,4,11,12].

It has been noted [9,10] that in the IBA-1 no stable shapes occur with $0^\circ < \gamma < 60^\circ$, unless one introduces cubic interaction terms in the hamiltonian of the type $\sum_{\lambda} c_{\lambda} (d^{\dagger} d^{\dagger} d^{\dagger})^{(\lambda)} \cdot (\tilde{d} \tilde{d} \tilde{d})^{(\lambda)}$. At present a quantitative microscopic derivation of the form and the parameters of the IBA hamiltonian is still lacking and therefore inclusion of higher order terms seems rather ad hoc. However, in this note we wish to point out that in the framework of the neutron-proton IBA

(IBA-2) [5] triaxial shapes can occur in a more natural way. In the IBA-2 approach the basis states span the irreducible representations $[N_p] \otimes [N_n]$ of the group $SU^{(p)}(6) \otimes SU^{(n)}(6)$. Dynamical symmetries for this case can be found by an obvious generalization of the chains (1)

$$\begin{aligned} &SU^{(n)}(6) \otimes SU^{(p)}(6) \\ &\supset U^{(n)}(5) \otimes U^{(p)}(5) \supset U(5) \supset O(5) \supset O(3), \\ &\supset SU^{(n)}(3) \otimes SU^{(p)}(3) \supset SU(3) \supset O(3), \\ &\supset O^{(n)}(6) \otimes O^{(p)}(6) \supset O(6) \supset O(5) \supset O(3). \end{aligned} \quad (2)$$

To be able to describe these extreme cases as well as intermediate situations we consider a schematic hamiltonian

$$H = \epsilon \hat{n}_d + \kappa Q^{(2)} \cdot Q^{(2)}. \quad (3)$$

Here $\hat{n}_d$ denotes the number operator of the d-bosons:

$$\hat{n}_d = \hat{n}_{d_p} + \hat{n}_{d_n} = a_p^\dagger \cdot \tilde{a}_p + a_n^\dagger \cdot \tilde{a}_n, \quad (4)$$

and $Q_\mu^{(2)}$ is the quadrupole operator $Q_\mu^{(2)} = Q_{p,\mu}^{(2)} + Q_{n,\mu}^{(2)}$,

$$Q_{k,\mu}^{(2)} = (s_k^\dagger \times \tilde{a}_k + a_k^\dagger \times \tilde{s}_k)_\mu^{(2)} + \chi_k (a_k^\dagger \times \tilde{a}_k)_\mu^{(2)}, \quad (5)$$

with $k = p$ (proton) or n (neutron).

The dynamical symmetries (2) occur for a special choice of the parameters ϵ, κ, χ_p and χ_n . If $\kappa = 0$ we have the $SU(5)$ case. However, in this note we restrict ourselves to a discussion of the other possibilities which occur if $\epsilon = 0$ and $\kappa < 0$.

For the $SU(3)$ limit we can distinguish two possibilities (i) $\chi_p = \chi_n = \pm \frac{1}{2} \sqrt{7}$, and (ii) $\chi_p = -\chi_n = \pm \frac{1}{2} \sqrt{7}$.

To examine this limit in more detail it is convenient to rewrite (3) in terms of the Casimir invariants of the $SU(3)$ and $O(3)$ groups:

$$H = \frac{1}{2} \kappa C_{2SU(3)} - \frac{3}{8} \kappa L^{(1)} \cdot L^{(1)}, \quad (6)$$

where $L^{(1)} = L_p^{(1)} + L_n^{(1)}$ is the angular momentum operator.

The corresponding energy spectrum can be expressed as

$$E(\lambda, \mu, L) = \frac{1}{2} \kappa C(\lambda, \mu) - \frac{3}{8} \kappa L(L+1), \quad (7)$$

where

$$C(\lambda, \mu) = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu), \quad (8)$$

and (λ, μ) denote the irreps of $SU(3)$.

Case (i): $\chi_p = \chi_n = -\frac{1}{2} \sqrt{7}$.

In this case the irreps of $SU^{(n)}(3)$ are given by

$$(\lambda_n, \mu_n) = (2N_n, 0) \oplus (2N_n - 4, 2) \oplus \dots,$$

and those of $SU^{(p)}(3)$ by

$$(2N_p, 0) \oplus (2N_p - 4, 2) \oplus \dots$$

The allowed values of (λ, μ) can be obtained by decomposition of the direct product $(\lambda_n, \mu_n) \otimes (\lambda_p, \mu_p)$ into the irreps (λ, μ) of $SU(3)$. Obviously the lowest irrep is given by $(2N_n + 2N_p, 0)$, which can be identified with the $(2N, 0)$ representation in the IBA-1 approach [7]. We note that in addition new representations occur which are not totally symmetric in the neutron-proton degree of freedom (and therefore do not occur in IBA-1). An example is the representation $(\lambda = 2N_p + 2N_n - 2, \mu = 1)$; its geometrical interpretation will be discussed elsewhere. (If $\chi_p = \chi_n = \frac{1}{2} \sqrt{7}$ the allowed representations (λ, μ) of $SU(3)$ can be obtained by interchanging λ and μ of the previous case leaving the energy spectrum invariant.)

Case (ii): without loss of generality we can take

$$\chi_p = -\chi_n = \frac{1}{2} \sqrt{7}.$$

In this case the irreps of $SU^{(n)}(3)$ are the same as above, but those of $SU^{(p)}(3)$ are given by

$$(\lambda_p, \mu_p) = (0, 2N_p) \oplus (2, 2N_p - 4) \oplus \dots$$

The allowed values of (λ, μ) are found to be[†]

$$(\lambda, \mu) = (2N_n, 2N_p) \oplus (2N_n - 4, 2N_p + 2)$$

$$\oplus (2N_n + 2, 2N_p - 4) \oplus (2N_n - 1, 2N_p - 1) \oplus \dots$$

The K -quantum numbers (that can be introduced to distinguish states with the same L within one irrep (λ, μ)) take on the values: $K = \min(\lambda, \mu), \min(\lambda, \mu) - 2, \dots, 1$ or 0 , and the allowed L values are given by $K \leq L \leq K + \max(\lambda, \mu)$ with the restriction that if $K = 0$: $L = \max(\lambda, \mu), \max(\lambda, \mu) - 2, \dots, 1$ or 0 .

We note that the energy spectrum of the lowest representation $(\lambda = 2N_n, \mu = 2N_p)$ as shown in fig. 1 is similar to that of the triaxial rotor with $\gamma = 30^\circ$ of the geometrical model [2]. To split the degeneracy of the states with the same L within a given $SU(3)$ multiplet it is sufficient to add a small $SU(3)$ symmetry breaking term to the hamiltonian (7). It is also seen that in the

[†] In the following we will refer to this case as $SU^*(3)$ to distinguish it from the $SU(3)$ of case (i).

Fig. 1. Energy spectrum of the hamiltonian (6) with $\chi_p = -\chi_n = \frac{1}{2}\sqrt{7}$, $N_p = 3$, $N_n = 4$.

present formulation higher representations, including excited 0^+ states, occur in a natural way, whereas it is difficult to describe these states in the geometrical approach [2,13].

Finally, the $O(6)$ limit of IBA-2 is realized if $\chi_p = \chi_n = 0$. The corresponding energy spectrum can in general be expressed in terms of the eigenvalues of the $O(6)$, $O(5)$ and $O(3)$ Casimir invariants:

$$E(\sigma_1, \sigma_2, \tau_1, \tau_2, L) = A[\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2)] + B[\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)] + CL(L + 1). \quad (9)$$

In this special case the constants A , B , C can be related to κ : $A = \kappa$, $B = -\kappa$, $C = 0$. The lowest representation has $\sigma_1 = N_p + N_n$, $\sigma_2 = \tau_2 = 0$, $\tau_1 = \sigma_1$, $\sigma_1 - 1, \dots, 0$, and can be identified with the $\sigma = N$ representation of the $O(6)$ limit in IBA-1 [8]. We note that compared to IBA-1, where $\sigma_2 = \tau_2 = 0$, new possibilities occur which correspond to vibrations of the neutron against the proton bosons.

It is instructive to examine the geometrical interpretation of the $SU^*(3)$ limit (for large boson number N). This can be done by constructing the classical limit of the boson hamiltonian [9,10,14], given by the expectation value of H in the coherent state representation. In the present case the generalized $SU^{(p)}(6) \otimes SU^{(n)}(6)$ coherent states can be expressed as

$$|N_p \alpha_p, N_n \alpha_n\rangle = \mathcal{N} (s_p^\dagger + \alpha_p \cdot d_p^\dagger)^{N_p} (s_n^\dagger + \alpha_n \cdot d_n^\dagger)^{N_n} |0\rangle, \quad (10)$$

where α_p and α_n represent five in general complex

quadrupole variables. Since we consider here only ground state (static) properties we take without loss of generality the α 's real. It is then convenient to transform to the intrinsic frame of the neutron and proton bosons separately ($k = p, n$):

$$a_{k,\mu} = \sum_\nu D_{\mu\nu}^{(2)}(\Omega_k) \alpha_{k,\nu}, \quad (11)$$

where $a_{k,0} = \beta_k \cos \gamma_k$, and $a_{k,2} = a_{k,-2} = 2^{-1/2} \beta_k \times \sin \gamma_k$.

Subsequently one can introduce a transformation from the (Ω_p, Ω_n) system to the Euler angles Ω for the orientation of the mass distribution and three angles (χ_1, χ_2, χ_3) describing the relative orientation of the neutron intrinsic frame with respect to that of the proton. In this way using the rotational invariance of H results in an energy surface

$$E(\alpha_p, \alpha_n) = \langle N_p \alpha_p, N_n \alpha_n | H | N_p \alpha_p, N_n \alpha_n \rangle, \quad (12)$$

which depends in general on seven intrinsic variables: $\beta_n, \beta_p, \gamma_n, \gamma_p$ and χ_1, χ_2, χ_3 . Minimization with respect to these variables defines the equilibrium shape. Application to the $SU^*(3)$ case leads to the following result: because of the axial symmetry of the lowest neutron and proton intrinsic states the relative orientation of the neutron and proton symmetry axis is completely determined by one angle χ . Its equilibrium value is $\bar{\chi} = 90^\circ$ and furthermore $\bar{\beta}_n = \bar{\beta}_p = \sqrt{2}$, $\bar{\gamma}_p = 0^\circ$ and $\bar{\gamma}_n = 60^\circ$. The geometrical interpretation is that of a prolate and an oblate axial rotor coupled so as to maximize their overlap. The resulting mass distribution can be parametrized by an asymmetry parameter $\bar{\gamma}$ in the following way [15] $\tan \bar{\gamma} = \sqrt{2} Q_{m,2} / Q_{m,0}$, where $Q_{m,0} = \langle 2z^2 - x^2 - y^2 \rangle$ and $Q_{m,2} = (3/2)^{1/2} \langle x^2 - y^2 \rangle$, characterize the intrinsic mass quadrupole distribution (sum of neutron and proton contributions). By taking the prolate proton distribution with respect to the z -axis and the oblate neutron distribution with respect to the y -axis one finds $Q_{m,0} = Q_{p,0} - \frac{1}{2} Q_{n,0}$ and $Q_{m,2} = -\frac{1}{2} (3/2)^{1/2} Q_{n,0}$. For $N_p = N_n$ one has $Q_{n,0} = -Q_{p,0}$ and therefore one finds $\bar{\gamma} = 30^\circ$, corroborating the interpretation of a triaxial rotor.

Whereas probably no examples of nuclei with pure $SU^*(3)$ symmetry can be found in nature several regions of the periodic system show evidence for a situation intermediate between the $SU^*(3)$ and $O(6)$ extreme cases. In IBA-2 such a transitional region can

conveniently be described in terms of a variation of the parameters $\chi_n = -\chi_p$ between $\frac{1}{2}\sqrt{7}$ and 0. As an illustration we discuss ^{104}Ru . Recent Coulomb excitation measurements of electromagnetic properties [4] suggest an interplay between γ -unstable $[\text{O}(6)]$ and γ -rigid $\text{SU}^*(3)$ features. For example, besides the fact that the experimental ratio $R \equiv E_{31}/(E_{21} + E_{22}) = 1.00$ precisely agrees with the triaxial rotor value, the measured values of the quadrupole moments $\langle Q \rangle_I$ in the ground state band seem to follow the γ -rigid prediction. On the other hand the energies of higher spin states in the γ -band do not follow the strong clustering predicted by the triaxial rotor model and the observed $B(E2)$ values and ratios for γ - to ground state band transitions are in general closer to the predictions of the γ -unstable approach.

The nucleus ^{104}Ru has been described before in terms of IBA-1 [16] and IBA-2 [17]. We have repeated the calculation of ref. [17] using the schematic hamiltonian (3). The only essential difference with ref. [17] is the inclusion of the term $\kappa(Q_p^{(2)} \cdot Q_p^{(2)} + Q_n^{(2)} \cdot Q_n^{(2)})$, in the absence of which no triaxial shapes occur. Since the protons are assumed to be hole-like ($\bar{N}_p = 3$) and the neutrons particle-like ($N_n = 5$) (with respect to the $N = Z = 50$ shell) we have assumed opposite and almost equal values of χ_p and χ_n : $\chi_p = +0.80$, $\chi_n = -0.90$. The values of $\epsilon = 0.70$ MeV and $\kappa = -0.10$ MeV have been fitted to the overall properties of the ground state and γ -bands of the spectrum. From fig. 2 where the experimental and calculated spectrum are compared two features emerge: in contrast to the re-

Fig. 2. Comparison of the experimental energy spectrum of ^{104}Ru with the results of the IBA-2 calculation of ref. [17] (A), the present result (B), and the asymmetric rotor prediction (C) (taken from ref. [4]).

Fig. 3. The quadrupole matrix elements in the ground state band as a function of the spin I [4], compared with the calculation of ref. [17] (IP), the asymmetric rotor model (AR), and the present result.

sults of ref. [17] the odd-even level spacings in the γ -band are quite well described, and the known O_2^+ state at 1 MeV is not reproduced. However, there are several indications [4,17] that this state does not belong to the IBA model space consisting of correlated pairs of valence nucleons, but instead arises by promoting a pair of protons across the $Z = 50$ shell.

The electric quadrupole matrix elements have been calculated by taking the simplest possible E2-operator

$$Q_\mu^{(2)} = e(Q_{p,\mu}^{(2)} + Q_{n,\mu}^{(2)}),$$

and the boson effective charge, e , has been fixed by fitting the observed $B(E2, 2_1^+ \rightarrow 0_1^+)$. From fig. 3 it is seen that the calculated values of $\langle Q \rangle_I$ in the ground state band fall in between the triaxial rotor and γ -unstable rotor predictions, in fair agreement with experiment. The rapid fall off with the $B(E2, I_\gamma \rightarrow I_g - 2)$ with increasing I_γ , which is predicted in the γ -unstable model, is very well reproduced in the present calculation (fig. 4).

We note, however, that it is difficult to draw definite conclusions from the present analysis. In partic-

Fig. 4. Same as fig. 3 for the $B(E2; I_\gamma \rightarrow I_g - 2)$.

ular the observed strong enhancement of the $B(E2; 6_1^+ \rightarrow 4_1^+)$ over the rotational value [4] suggests that yet another degree of freedom plays a role.

Finally we note that although we have not yet investigated them in detail several other nuclei have properties which are very similar to the case considered above, e.g. $^{110}_{46}\text{Pd}$ [3] and $^{134}_{56}\text{Ba}$. These nuclei have in common that the neutrons are hole-like and the protons are particle-like, or vice versa, and therefore the neutrons and protons are likely to have opposite intrinsic quadrupole deformation. On the basis of a microscopic picture one expects in these cases an $O(6)$, $SU^*(3)$ or an intermediate situation. A necessary condition for $SU^*(3)$ characteristics is the presence of a quadrupole-quadrupole interaction among the like bosons in addition to the neutron-proton force. It would be interesting to investigate whether such an interaction can be derived from a microscopic model.

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