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ON THE EFFECTIVE NUMBER OF BOSONS IN THE INTERACTING BOSON MODEL

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A procedure for calculating the effective number of bosons in the Interacting Boson Model is proposed. As an example, a calculation is presented for the 50–82 major shell for protons in which there is a subshell closure at $Z = 64$.

In the Interacting Boson Model [1] (IBM) the structure of medium heavy and heavy nuclei are calculated as a system of interacting bosons. In a microscopic interpretation of the model these bosons are considered as collective pairs of fermions [2]. The number of bosons for a given nucleus is therefore equal to half the number of particles (or holes, whichever is less) in the valence shell. In some cases this definition can be ambiguous. In the 50–82 major shell for protons for example there exists strong evidence that $Z = 64$ acts as a subshell closure [3]. A nucleus with $Z = 60$ (Nd) for example can therefore be regarded as having 5 particle-like proton bosons in the 50–82 major shell or as having 2 hole-like proton bosons in the 50–64 subshell. To resolve these kinds of ambiguities, in this letter a method will be outlined for obtaining the effective number of bosons from a microscopic calculation.

In the IBM one considers *s* and *d* bosons. The microscopic equivalent of the *s* boson in a shell model picture is the *S*-pair state [2], a collective pair of fermions coupled to angular momentum $J = 0$,

$$S^+ = \sum_j a_j \frac{1}{2} \hat{j} (a_j^\dagger a_j^\dagger)^{(0)}, \quad (1)$$

where $a_j^\dagger$ is the creation operator for a fermion. Similarly the *d* boson is related to a *D*-pair state, a collective pair of fermions coupled to $J = 2$. The coefficients α_j in eq. (1) can be calculated in a microscopic model for the bosons [4]. In order to see the significance of the values of α_j we will consider two extreme cases:

(A) the coefficients q_j are all equal, and (B) one is much larger than the other. We will first discuss these two extreme cases and then the realistic case which is, as usual, somewhere in between.

Case A: When all coefficients α_j in eq. (1) are equal, $\alpha_j = 1$ it can be shown that the S^+ operator, together with its adjoint, S and $S_0 = \frac{1}{2} [S^+, S]$ generate a quasi-spin algebra and as a consequence the problem is completely equivalent to that of a large- j shell with a pair degeneracy Ω , equal to the total of the orbits involved in the sum in eq. (1); $\Omega = \sum_j \Omega_j$ where $\Omega_j = (2j + 1)/2$. The particles are distributed equally over the orbits,

$$\langle S^N | \hat{n}_j | S^N \rangle = 2\Omega_j N / \Omega,$$

where n_j is the number operator for the shell- j . This case corresponds to that of several degenerate orbits and the number of bosons is unambiguously defined as the number of particle or hole pairs, whichever is less, $N_B = \min(N, \Omega - N)$.

Case B: Let us consider the case of three orbits with coefficients α_j such that $\alpha_1^2 \gg \alpha_2^2 \gg \alpha_3^2$ and a number of fermion pairs N such that $\Omega_1 < N < \Omega_1 + \Omega_2$. It is instructive to calculate the number of particles in each shell [5],

$$\langle S^N | \hat{n}_1 | S^N \rangle = 2\Omega_1 + O(\alpha_2^2/\alpha_1^2),$$

$$\langle S^N | \hat{n}_2 | S^N \rangle = 2(N - \Omega_1) + O(\alpha_2^2/\alpha_1^2),$$

$$\langle S^N | \hat{n}_3 | S^N \rangle = 0 + O(\alpha_3^2/\alpha_1^2)$$

In leading order therefore, orbit 1 is completely filled, orbit 3 completely empty while orbit 2 is partially occupied, with $M = (N - \Omega_1)$ pairs. In this extreme case one would consider orbit 2 (which can be the equivalent of several degenerate orbits, case A) as the valence shell. The number of bosons in this case can thus also be defined unambiguously as the number of particle or hole pairs in shell 2; $N_B = \min(M, \Omega_2 - M)$.

As was remarked before, a realistic case will be in between these two extreme cases, but it is possible to make a formal separation into three orbits of which one behaves effectively as completely filled, one as completely empty and one as a valence orbit. Each of these three orbits must be seen as an effective orbit as was previously discussed in case A. To make this separation possible the task is to construct operators that work in the full space that measure the number of nucleons in the "valence shell". In the example of case B, it is possible to calculate in leading order the quantities

$$\langle S^{N-1} | SS^+ | S^{N-1} \rangle = Z(\Omega_2 - M + 1)N^2/M, \quad (2a)$$

$$\begin{aligned} \langle S^{N-1} | SSS^+ S^+ | S^{N-1} \rangle \\ = Z^2(\Omega_2 - M)(\Omega_2 - M + 1)N^2(N+1)^2/M(M+1). \end{aligned} \quad (2b)$$

$$\langle S^{N-1} | D | SS^+ | S^{N-1} \rangle = Z(\Omega_2 - M - 1)N^2/M, \quad (2c)$$

where $|S^N D\rangle$ is a normalized $J = 2, v = 2$ state corresponding in the IBA model to the one d boson state. The constant Z introduced in eqs. (2) can be regarded as a normalization factor and is equal to α_2^2 . When the coefficients α_j , that enter in eq. (1), have been determined from a microscopic model for the boson structure (such as is outlined in ref. [4]) the left-hand sides of eqs. (2) can be calculated. Next the three equations (2a)–(2c) can be solved for the three unknowns, Ω_2 , the size of the effective valence shell; M , the effective number of particle pairs in the valence shell; and Z . The effective number of bosons can now thus be defined as

$$N_{\text{eff}} = \min(M, \Omega_2 - M). \quad (3)$$

The number of inactive nucleon pairs is $\Omega_1 = N - M$ and these act as a kind of inert core. Eq. (2) thus allow for an unambiguous determination of the effective

Fig. 1. The calculated values for Ω_1 , Ω_2 and N_{eff} , as discussed in the text, as a function of the number of protons in the 50–82 major shell.

number of bosons once the microscopic structure of the bosons themselves has been determined. In the IBA model the number of bosons is an integer, while N_{eff} calculated from the proposed procedure is in general not an integer, as can be seen in fig. 1. A possible solution to this problem would be to choose the nearest integer to N_{eff} as the actual number of IBA bosons.

The three quantities in eqs. (2) have been selected since they lead to the simplest linear independent equations that yield a unique solution. The operator, $2S_0 = [S^+, S]$ for example, leads to the introduction of another unknown parameter, α_1^2 .

As an example, the effective number of proton bosons in the 50–83 major shell is calculated. The S-pair structure coefficients α_j were determined by a diagonalization in a basis including states with generalized seniority [6] $w \leq 2$. In this basis, the ground state is the $w = 0$ state and can thus be written as $S^{+N} |0\rangle$ where S^+ is defined in eq. (1). In the calculation the

Fig. 2. The coefficients α_j as a function of the number of protons in the 50–82 major shell. The coefficient $\alpha_{g_{7/2}}$ has been taken equal to 1 since the normalization of the $|S^N\rangle$ state has been treated separately.

α_j were allowed to vary as a function of the number of protons and are plotted in fig. 2. The hamiltonian was taken from a shell model calculation by Kruse and Wildenthal [7] where the single-particle energies and the interaction have been adjusted as to give a best reproduction of the experimental energies in the even and odd mass $N = 82$ isotones. It has been verified that the calculation in the generalized seniority basis reproduces the shell model results for the lowest levels of each spin [8]. The variation of α_j with Z indicates that the interaction contains terms that break generalized seniority. The Z -dependence of α_j remains qualitatively the same when a surface delta interaction is used. The d boson state is calculated as the first excited 2^+ state. Using the thus determined values of α_j the left-hand side of eqs. (2) can be calculated [5] and eqs. (2) can subsequently be solved for Ω_1 , Ω_2 and N_{eff} . In fig. 1 the results are given in addition to the values obtained if one considers $Z = 64$ as a full subshell closure. Clearly the $Z = 64$ shell closure has an important influ-

ence, without it Ω_1 would be zero, $\Omega_2 = 16$ and $N_{\text{eff}} = 8$ for $Z = 66$. However there is also a considerable washing out of the shell closure. For $Z = 62$ for example, $N_{\text{eff}} = 3.2$ rather than 1, what one would expect from the subshell closure.

The calculation of the effective number of bosons for the 50–82 proton shell shows that the $Z = 64$ subshell closure has a strong influence and can be clearly recognized in the dependence of Ω and N_{eff} on Z . The effective number of bosons shows a clear minimum at the $Z = 64$ subshell closure, it is however still several units higher than what would have been obtained by taking the $Z = 64$ subshell closure as was suggested by Wolf et al. [9] on the basis of a calculation of g -factors. The fact that it is important to include the proper number of bosons in IBA model calculations is clearly demonstrated in refs. [9] and [10]. It should be noticed that the introduction of the neutron–proton interaction, which will become increasingly important when there are several neutrons outside the $N = 82$ closed shell, will tend to wash out the $Z = 64$ shell closure [11]. More detailed calculations are in progress.

It should be noted that although the procedure proposed in this paper to calculate an effective pair degeneracy Ω_2 is distinctly different from the one given in ref. [12], the obtained values from the two methods are approximately the same.

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