ISOVECTOR PROPERTIES OF COLLECTIVE STATES AND THE IBA-2 MODEL

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The IBA-2 model of nuclear structure makes explicit predictions of isovector properties of collective states which have never before been tested. We have made the first measurements of separate neutron (Mn) and proton (Mp) matrix elements for the first 2+ and 3- states in $^{104,106,108,110}$Pd by means of inelastic scattering of 180 MeV $\pi^+$ and $\pi^-$. It is found that the IBA-2 model with core polarization can explain the trend of the data for Mp, and $M_n/M_p$, but only if its parameters are empirically adjusted.

Isovector (or equivalently proton–neutron) properties of nuclear states provide extremely sensitive tests of nuclear structure models [1,2]. For heavy nuclei one model, which has gained increasing attention in recent years is the Interacting Boson Approximation (IBA) model [3]. In the latest version of this model, called the IBA-2 model [4] the active part of the nucleus (outside a supposedly inert core) is described in terms of a small number of mutually interacting proton and neutron bosons which can be either s-state or d-state. This model is able to make explicit predictions regarding the isovector properties of nuclear transitions. Here we report on the first successful experiment specifically designed to test these predictions.

In the pioneering work of Iversen et al. [2] on $^{18}$O it was clearly shown that comparison of inelastic scattering of positive and negative pions of energies near the (3, 3) resonance is a unique and powerful tool in the study of isovector characteristics. The unique advantage of pion scattering is that both the experimental method and the method of data analysis are completely symmetric with respect to $\pi^+$ and $\pi^-$, and minimum uncertainty can be expected in the extraction of relative quantities such as $M_n/M_p$, the ratio of neutron and proton matrix elements for inelastic excitations. Pion inelastic scattering is therefore ideally suited for the study of isovector properties of IBA-2 [5]. However, pion beam intensity and energy resolution introduce severe practical limitations and no such studies have been reported so far. In the present experiment we report on the first such measurement of $M_n$ and $M_p$ for a series of palladium isotopes for which detailed IBA-2 calculations are available [6].

We have measured differential cross sections for elastic and inelastic scattering of 180 MeV $\pi^+$ and $\pi^-$ from enriched targets ($\geq 95\%$) of $^{104,106,108,110}$Pd at the EPICS spectrometer facility at LAMPF. As illustrated in fig. 1, the energy resolution obtained was $\approx 165$ MeV. The absolute cross section normalization was determined both by normalizing elastic scattering to optical model predictions which are described later, and by measuring $\pi^+p$ scattering at several angles from a CH$_2$ target and comparing it with phase shift predictions. The two methods were found to lead to consistent absolute normalization to within $\pm 5\%$, for
all the four isotopes. The relative errors in the final cross-sections between the isotopes are estimated to be $\leq 3\%$. In fig. 2, where we show the elastic and inelastic $2^+_1$ and $3^+_1$ angular distributions only the statistical errors are indicated. In order to extract $M_\pi$ and $M_\rho$ from our data we have made distorted wave impulse approximation (DWIA) calculations with the coordinate space computer code DWPI [7] using Kisslinger type optical potential and collective form factors. In order to obtain simultaneous good fits for $\pi^+$ and $\pi^-$ elastic and inelastic angular distributions it was found necessary to evaluate the $\pi$-nucleon $t$-matrix at an energy $30$ keV less than the incident energy [8]. We find that if the measured cross sections are renormalized by optical model fits to the elastic scattering, and the inelastic scattering is analyzed in terms of DWIA using the same optical model, the resulting deformation parameters $\beta$ are quite stable with respect to minor changes in the optical model parameters. This procedure also minimizes the effect of systematic errors and

Fig. 1. Excitation energy spectra for the reaction Pd($\pi^+$, $\pi^-$) at $\theta_{lab} = 42.5^\circ$.

Fig. 2. Angular distributions for elastic and inelastic scattering for the reaction Pd($\pi, \pi'$). The solid lines are DWPI predictions.
we have adopted it for the analysis of the present data. The neutron and proton deformation parameters $\beta_n$ and $\beta_p$ for a given transition were varied till simultaneous, best $\chi^2$ fit to both $\pi^+$ and $\pi^-$ inelastic angular distributions was obtained. As is well known the proton transition matrix element $M_p$ is related to $\langle E1\rangle$ as:

$$B(E1) = M_p^2 \approx (3ZR^4/4\pi)\beta_p^2.$$ $M_n$ is defined in an exactly similar manner in terms of $N$ and $\beta_n$. The radius parameter $R$ is sometimes related to the half density radius of the potential well used in the DWIA analysis. However, for strongly surface absorbed pions, the effective value of $R$ is known to occur farther out in the surface, and we have set $R = 1.20 A^{1/3}$ fm. In the present case ($c = 1.06 A^{1/3}$ fm and $a = 0.55$ fm) this $R$ corresponds to the point where the density has fallen to 23% of its central value. With this choice of $R$ we obtain the results shown in table 1. We note that our values of $M_p$ for the $2^+$ states agree well within errors ($<5\%$) with those from lifetime measurements [9]. For the $3^-\pi$ states no other direct results for $M_p$ are available for comparison.

In the IBA-2 model the neutron and proton quadrupole transition matrix elements are defined as [6]:

$$Q_p = \alpha_p A_p + \beta_p B_p,$$

$$Q_n = \alpha_n A_n + \beta_n B_n.$$

The nuclear structure information is contained in the reduced boson proton–neutron-transition matrix elements $A_{p,n}$ and $B_{p,n}$ for the d-boson non-conserving and d-boson conserving parts, respectively. $\alpha_{p,n}$ and $\beta_{p,n}$ are the corresponding values of the quadrupole operators.

We relate our experimental results to the IBA-2 model at two different levels. We can consider the IBA-2 model with or without core polarization. Further, we can treat $\alpha_{n,p}$ and $\beta_{n,p}$ as adjustable parameters, as was done in ref. [6], or we can use the values calculated in a generalized seniority scheme using a surface delta interaction [10]. These calculations give (all in fm$^2$) $\alpha_n = 18.5, 16.6, 15.6, 14.8; \beta_n = -8.2, -8.2, -3.7, 2.4$ and $\alpha_p = 14.7, 15.1, 15.3, 15.6, \beta_p = -4.3, -3.9, -2.6, -3.3$ for palladium 104, 106, 108 and 110 respectively. In all these comparisons we take $A_i$ and $B_i$ from ref. [6].

We first consider the IBA-2 model with core polarization. If we follow the phenomenological procedure, use $x_p = \alpha_p/\beta_p$, and $x_n = \alpha_n/\beta_n$ as optimized in ref. [6], and set $M_n = eQ_n, M_p = eQ_p$ for the $2^+$ states, we obtain the average values ($\alpha_n = 21.2$ fm$^2$, and $\langle e_n \rangle = 0.49$ e, calculated $\alpha$'s and $\beta$'s.

### Table 1
Summary of results for excitation of $2^+$ states in Pd isotopes $a$).

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$A$</th>
<th>Experimental results</th>
<th>Model predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ref. [8] ($n, n'$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_p$</td>
<td>$M_p$</td>
</tr>
<tr>
<td>$2^+$</td>
<td>104</td>
<td>74(2)</td>
<td>77(2)</td>
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<tr>
<td></td>
<td>106</td>
<td>83(3)</td>
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</tr>
<tr>
<td></td>
<td>108</td>
<td>88(3)</td>
<td>89(2)</td>
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<tr>
<td></td>
<td>110</td>
<td>94(3)</td>
<td>98(2)</td>
</tr>
<tr>
<td>$2^+$</td>
<td>104</td>
<td>16(1)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>106</td>
<td>13(1)</td>
<td>10</td>
</tr>
<tr>
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<td>108</td>
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<td>9</td>
</tr>
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<td></td>
<td>110</td>
<td>11(1)</td>
<td>14</td>
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<tr>
<td>$3^-$</td>
<td>104</td>
<td>365(9)</td>
<td>542(14)</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>110</td>
<td>315(8)</td>
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</table>

$a$) $M_{p,n}$ are in units of $e$ fm$^2$ for $2^+$, and $e$ fm$^3$ for $3^-$.  

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b) Average $\langle e_p \rangle = 1.13 e$, $\langle e_n \rangle = 0.49 e$, and empirical $\alpha$'s and $\beta$'s.

c) Average $\langle e_p \rangle = 1.17 e$, $\langle e_n \rangle = 0.48 e$, calculated $\alpha$'s and $\beta$'s.
Fig. 3. $(M_n/M_p)/(N/Z)$ for the $2^+$ states in Pd isotopes. The curves refer to IBA-2 calculations described in the text. Without core-polarization: thin solid line for empirically adjusted $(\alpha, \beta)$, thin dashed line for microscopically calculated $\alpha$, $\beta$. With core-polarization: thick solid for empirically adjusted $(\alpha)$ and $(\beta)$ and $(e_p) = 1.12 \, e$, $(e_n) = 0.49 \, e$, thick dashed line for microscopically calculated $\alpha$, $\beta$ and $(e_p) = 1.17 \, e$, $(e_n) = 0.48 \, e$. Solid curve in fig. 3. Neither of these results are in agreement with the data. Further, $M_p$ for the $2^+$ states are predicted to be $\approx 2.5$ larger than the experimental ones. If we use the calculated values of $\alpha$ and $\beta$, we obtain even poorer agreement with the data. We obtain $M_n$ which are $\approx 75(\pm8)$ of the experimental ones, $M_p$ which are $\approx 41(\pm4)$ of the experimental ones, and $M_n/M_p$ which are uniform factor $\approx 2$ larger than the experimental results (thin dashed curve in fig. 3). $M_p$ for the $2^+_2$ states fare better, being only $\approx 21(\pm5)$% smaller than the experimental values.

From the above results we conclude that irrespective of what approach one adopts towards $\alpha$'s and $\beta$'s, we must compensate for the truncation of the valence boson space by including core polarization. As in the shell model one can do this by introducing “effective” charges $e_p$ and $e_n$ for proton and neutron bosons and note that for a self-conjugate core (here $N = Z = 50$):

$$M_p = e_p Q_p + e_n Q_n, \quad M_n = e_n Q_p + e_p Q_n.$$  

For a given nucleus once $Q_p$ and $Q_n$ are known, $e_p$ and $e_n$ are determined uniquely by fitting the measured values of $M_p$ and $M_n$. We first use $\chi$'s as empirically optimized in ref. [6] and $\langle \alpha_n \rangle = 16.4 \, \text{fm}^2$, $\langle \alpha_p \rangle = 15.2 \, \text{fm}^2$ (which are averages of the calculated results), to obtain $Q_p$ and $Q_n$ for each isotope. Comparison with $M_p$ and $M_n$ then leads to a set of $e_p$ and $e_n$ which are found to be essentially the same (within $\approx \pm 5\%$) for all isotopes: $\langle e_p \rangle = 1.13 \, e$, $\langle e_n \rangle = 0.49 \, e$. The results for $M_p$ and $M_n/M_p$ obtained by using these effective charges are shown in table 1 and by the thick solid curve in fig. 3. The fit to the data is quite good. As shown in table 1 the same effective charges predict $M_p$ for the $2^+_2$ states which are in fair agreement with the data. In addition they lead to very distinctive predictions for $M_n/M_p$ which should be tested in future experiments.

Unfortunately, if we use $\alpha$'s and $\beta$'s as obtained from the microscopic calculations referred to earlier, the results are much poorer. Individual $e_n$ are found to be fairly constant $(e_n = 0.46 \, e \to 0.50 \, e)$ but $e_p$ shows an unacceptable amount of variation $(e_p = 1.02 \, e \to 1.35 \, e)$ in going from $^{104}\text{Pd}$ to $^{110}\text{Pd}$. Equivalently, we note that if we use the average values: $(e_p) = 0.48 \, e$, $(e_n) = 1.17 \, e$, we fail to reproduce the trend of $M_p$ and $M_n$ individually, although $M_n/M_p$ agrees with the data (thick dashed curve in fig. 3). For the $2^+_2$ states the predicted $M_p$ differ from the experimental ones by $43(\pm23)$% and the predicted $M_n/M_p$ differ by a factor two from those with empirically adjusted $\alpha$'s and $\beta$'s.

Because of the limited success obtained above with the IBA-2 model without empirically adjusted parameters, it is instructive to examine what other simple phenomenological models predict. In the hydrodynamical model, nuclei in the Pd region are well described as vibrational nuclei, with

$$E(\lambda) = \hbar \langle C_{\lambda} / B_{\lambda} \rangle^{1/2},$$

$$M_p^2 = \hbar (\lambda + 1/2) / [(3/4) \pi Z e R_{\lambda}^2 (B_{\lambda} C_{\lambda})^{1/2}.$$  

By analogy, $M_n$ is defined in terms of $N$, $R_n$ and $(B_{\lambda}, C_{\lambda})_n$. The parameters $B_{\lambda}$ (related to the mass transport associated with the vibration) $C_{\lambda}$ (related to the effective surface tension or restoring force) and $R$ can, in principle, vary from nucleus to nucleus and between neutron and proton fluids, but are generally considered constant in a small region of $N$ and $Z$. Thus this model predicts $M_p = \text{constant}$ for a series of isotopes and $M_n/M_p = N/Z$. As seen in table 1, the experimental $M_p$ increases by $\approx 25\%$ from $^{104}\text{Pd}$ to $^{110}\text{Pd}$. Also the experimentally observed ratios of $M_n/M_p$ for the $2^+_1$ and $3^+_1$ states are respectively $\approx 12\%$ and $\approx 18\%$ higher than $N/Z$ (see table 1). Better agreement with the experimental results can be obtained by treating $B_{\lambda}, C_{\lambda}$, and $R$ as adjustable parameters. For example, we can
obtain excellent fits to the measured $M_p$ by simply varying $C_2$ with neutron number. Similarly, the experimental $M_n/M_p$ for both $2^+_1$ and $3^-_1$ can be fitted perfectly by assuming either that the effective radius parameters $R_n$ and $R_p$ are different, or that $(B_x, C_x)_p$ and $(B_x, C_x)_n$ are different.

We can summarize our results as follows. The IBA-2 model without core-polarization is clearly unsuccessful in explaining the experimental results. With the inclusion of core polarization effects, via effective charges, the IBA-2 model can provide a good description of the data as long one allows empirical adjustment of its parameters. However, at this level, it is worthwhile to remember that other, even simpler models, can also explain the experimental results provided their parameters are also adjusted from isotope to isotope. Our first attempts to calculate some of the IBA-2 parameters on a microscopic basis have so far been unsuccessful. Clearly more theoretical work is required. At the experimental front measurement of $M_n/M_p$ for other quadrupole transitions, particularly $0^+_1$ to $2^+_2$ would help narrow the range of alternative explanations.

References