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DYNAMIC SYMMETRIES IN THE PROTON-NEUTRON INTERACTING BOSON MODEL (IBA-2)

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Dynamic symmetries that occur in the proton-neutron IBM are examined. It is pointed out that these dynamic symmetries do not require the equality of neutron-proton and the neutron-neutron or proton-proton boson interactions.

The original version (IBA-1) of the interacting boson model [1]^{†1} deals with s- and d-bosons. The hamiltonian and other operators are constructed from generators of the SU(6) Lie group. Motivated by microscopic considerations a more detailed version – IBA-2 – was introduced in which proton and neutron bosons are distinguished [3]. The basis states of IBA-2 span the irreducible representations $[N_\pi] \otimes [N_\nu]$ of the direct product $SU^{(\pi)}(6) \otimes SU^{(\nu)}(6)$.

Using the notation $b_{\alpha,\rho}^+$ for the creation operators $d_{\mu,\rho}^+$, s_ρ^+ , where ρ stands for π or ν , and similarly $b_{\alpha,\rho}$, the 36 + 36 generators are given by $G_{\alpha\beta}^{(\rho)} = b_{\alpha,\rho}^+ b_{\beta,\rho}$. The rotationally invariant IBA-2 hamiltonian can then be expressed by the *scalar part* of

$$\sum_{\alpha\beta\rho} \epsilon_{\alpha\beta}^{(\rho)} G_{\alpha\beta}^{(\rho)} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta\rho\rho'} u_{\alpha\beta\gamma\delta}^{(\rho\rho')} G_{\alpha\beta}^{(\rho)} G_{\gamma\delta}^{(\rho')}. \quad (1)$$

The eigenvalues can be obtained by writing the hamiltonian matrix in a given basis and diagonalizing it. Still, there is considerable interest in special cases where analytic solutions can be found. Such cases, so-called dynamic symmetries, provide direct insight into the physics of the system.

In a recent paper [4] some aspects of such dynamic symmetries of IBA-2 were discussed. Three of these symmetries are straightforward generalizations of those of IBA-1 [1], and they can be obtained by considering the linear combinations

$$G_{\alpha\beta}^{(\pi+\nu)} = G_{\alpha\beta}^{(\pi)} + G_{\alpha\beta}^{(\nu)}, \quad (2)$$

which are generators of the group $SU^{(\pi+\nu)}(6)$. The latter group is a subgroup of $SU^{(\pi)}(6) \otimes SU^{(\nu)}(6)$. A hamiltonian constructed from the generators (2) is fully symmetric in proton and neutron bosons, namely $\epsilon_{\alpha\beta}^{(\pi)} = \epsilon_{\alpha\beta}^{(\nu)}$ and $u_{\alpha\beta\gamma\delta}^{(\pi,\pi)} = u_{\alpha\beta\gamma\delta}^{(\pi,\nu)} = u_{\alpha\beta\gamma\delta}^{(\nu,\nu)}$. Its eigenstates have definite symmetry types which can be characterized by definite values of F -spin [3]. The lowest states are fully symmetric in proton and neutron bosons with maximum value of F -spin, $F_{\max} = \frac{1}{2}(N_\pi + N_\nu)$ and are in one-to-one correspondence with IBA-1 states. The three dynamic symmetries correspond to the following chains of groups:

$$SU^{(\pi)}(6) \otimes SU^{(\nu)}(6) \supset SU^{(\pi+\nu)}(6)$$

$$\supset U^{(\pi+\nu)}(5), \quad (3a)$$

$$\supset SU^{(\pi+\nu)}(3), \quad (3b)$$

$$\supset O^{(\pi+\nu)}(6). \quad (3c)$$

The various subgroups of $U(5)$, $SU(3)$ and $O(6)$ are omitted in (3) for brevity.

^{†1} For recent references see ref. [2].

A much more interesting dynamic symmetry discovered in ref. [4] was obtained by first considering the symmetry within $SU^{(\pi)}(6)$ and $SU^{(\nu)}(6)$. The various chains of groups in that case are given by

$$SU^{(\pi)}(6) \otimes SU^{(\nu)}(6)$$

$$\supset U^{(\pi)}(5) \supset U^{(\nu)}(5) \supset U^{(\pi+\nu)}(5), \quad (4a)$$

$$\supset SU^{(\pi)}(3) \supset SU^{(\nu)}(3) \supset SU^{(\pi+\nu)}(3), \quad (4b)$$

$$\supset SU^{(\pi)}(3) \supset \overline{SU^{(\nu)}(3)} \supset SU^{(\pi+\nu)*}(3), \quad (4b^*)$$

$$\supset O^{(\pi)}(6) \supset O^{(\nu)}(6) \supset O^{(\pi+\nu)}(6). \quad (4c)$$

In addition to the symmetries (3), the new symmetry (4b*) is obtained by taking for the neutron bosons the group $\overline{SU^{(\nu)}(3)}$ which corresponds to a $SU(3)$ group realization *different* from that of the proton bosons $SU^{(\pi)}(3)$. Hence, no symmetry between proton and neutron bosons is required and F -spin is no longer a good quantum number.

The aim of the present note is to discuss the possible IBA-2 hamiltonians which have the dynamic symmetries (4b), (4b*), (4c) corresponding to the deformed symmetries [$\epsilon = 0$ in (1)]. In ref. [4] only a brief and incomplete discussion was given. Closed algebraic expressions for the eigenvalues are obtained if the hamiltonian can be expressed in terms of the Casimir operators of the groups in a given chain. As will be shown below, the group chains (4) are more useful for that purpose than those in (3).

To construct hamiltonians with dynamic symmetries (4b), (4b*), (4c) it is convenient to define quadrupole operators of proton and neutron bosons by

$$Q_{\pi}^{(2)} = d_{\pi}^{\dagger} s_{\pi} + s_{\pi}^{\dagger} \tilde{d}_{\pi} + \chi_{\pi} (d_{\pi}^{\dagger} \times \tilde{d}_{\pi})^{(2)}, \quad (5a)$$

$$Q_{\nu}^{(2)} = d_{\nu}^{\dagger} s_{\nu} + s_{\nu}^{\dagger} \tilde{d}_{\nu} + \chi_{\nu} (d_{\nu}^{\dagger} \times \tilde{d}_{\nu})^{(2)}. \quad (5b)$$

The rather schematic hamiltonian discussed in ref. [4] has the form

$$H = \kappa (Q_{\pi}^{(2)} + Q_{\nu}^{(2)}) \cdot (Q_{\pi}^{(2)} + Q_{\nu}^{(2)}) + \kappa' L^{(1)} \cdot L^{(1)}, \quad (6)$$

where $L^{(1)} = L_{\pi}^{(1)} + L_{\nu}^{(1)}$ is the total angular momentum [$L_{\rho}^{(1)} = (d_{\rho}^{\dagger} \times \tilde{d}_{\rho})^{(1)}$]. As discussed

below, if $\chi_{\pi} = \chi_{\nu} = \pm \frac{1}{2}\sqrt{7}$ the hamiltonian (6) has the $SU(3)$ symmetry (3b) [or (4b)], for $\chi_{\pi} = -\chi_{\nu} = \pm \frac{1}{2}\sqrt{7}$ the dynamic symmetry (4b*) is obtained, and for $\chi_{\pi} = \chi_{\nu} = 0$ the $O(6)$ symmetry (4c).

The hamiltonian (6) with $\chi_{\pi} = \chi_{\nu}$ is fully symmetric with respect to proton and neutron bosons. In the same state, the interactions between two proton bosons, between two neutron bosons and between a proton boson and a neutron boson are exactly equal. Eigenstates of such hamiltonians are characterized by the F -spin quantum number and the eigenvalues are independent of $M_F = \frac{1}{2}(N_{\pi} - N_{\nu})$.

It is well known that the effective interaction between identical nucleons has a very different character from the one between protons and neutrons in different orbits (the difference is between the $T = 1$ and $T = 0$ parts of the effective interaction). For the proton-proton and neutron-neutron interactions seniority (or generalized seniority) is a good quantum number [3,5]^{†2}. Such interactions cannot be approximated by a quadrupole-quadrupole term as is implied by the hamiltonian (6) either with $\chi_{\pi} = \chi_{\nu}$ (4b) or with $\chi_{\pi} = -\chi_{\nu}$ (4b*). On the other hand, the proton-neutron interaction (the $T = 0$ part) is dominated by a strong attractive quadrupole interaction which leads to deformations and rotational spectra [6]. The truncation to the space of S and D nucleon pairs (or s - and d -bosons) may lead to renormalization of the effective interaction, and this renormalization may introduce an effective quadrupole interaction between identical bosons [7]. Still it is interesting to see whether such effects are really necessary for dynamical symmetries.

We shall now see that the symmetries of type (4) lead naturally to a class of hamiltonians more general than (6). The dynamic symmetries may occur with interactions between identical bosons that are essentially different from the interactions between proton and neutron bosons. The groups in chain (4) supply more quantum numbers and Casimir operators than those in chain (3). For the $SU(3)$ cases, (4b) and

^{†2} A detailed discussion is given in ref. [6].

4b*), we can write the most general hamiltonian as

$$H = \frac{1}{2}\kappa C_3^{(\pi+\nu)} + \frac{1}{2}\kappa_\pi C_3^{(\pi)} + \frac{1}{2}\kappa_\nu C_3^{(\nu)} + \kappa' L^{(1)} \cdot L^{(1)}. \quad (7)$$

In (7) the various $C_3^{(p)}$ stand for the Casimir operator of the SU(3) group

$$C_3^{(p)} = 2Q_p^{(2)} \cdot Q_p^{(2)} + \frac{3}{4}L_p^{(1)} \cdot L_p^{(1)}, \quad (8)$$

with eigenvalues $\langle C_3^{(p)} \rangle = \lambda_p^2 + \lambda_p \mu_p + \mu_p^2 + 3(\lambda_p + \mu_p)$ in the states which belong to the (λ_p, μ_p) irreducible representation. The three Casimir operators $C_3^{(p)}$ in (7) commute and eigenstates of H are characterized by their eigenvalues and L . The eigenvalues of H are given by a linear combination of their eigenvalues as follows

$$\begin{aligned} E(\lambda, \mu, \lambda_\pi, \mu_\pi, \lambda_\nu, \mu_\nu, \lambda, \mu, L) \\ = \frac{1}{2}\kappa[\lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)] \\ + \frac{1}{2}\kappa_\pi[\lambda_\pi^2 + \lambda_\pi\mu_\pi + \mu_\pi^2 + 3(\lambda_\pi + \mu_\pi)] \\ + \frac{1}{2}\kappa_\nu[\lambda_\nu^2 + \lambda_\nu\mu_\nu + \mu_\nu^2 + 3(\lambda_\nu + \mu_\nu)] \\ + \kappa' L(L+1). \end{aligned} \quad (9)$$

The hamiltonian (6) is a special case of (7) obtained by putting $\kappa_\pi = \kappa_\nu = 0$. Other choices lead to very different interaction terms. If we put $\kappa_\pi = \kappa_\nu = -\kappa$ and $\kappa' = 0$ we obtain a hamiltonian with no interactions between identical bosons whereas proton boson–neutron boson interactions are given by

$$H' = 2\kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)} + \frac{3}{4}\kappa L_\pi^{(1)} \cdot L_\nu^{(1)}. \quad (10)$$

Another choice removes the dipole proton–neutron interaction form (7) and introduces a dipole interaction between identical nucleons. The latter interaction obviously does not break seniority. Putting $\kappa_\pi = \kappa_\nu = -\kappa$ and $\kappa' = -\frac{3}{8}\kappa$ in (7) yields the hamiltonian

$$H'' = 2\kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)} - \frac{3}{8}\kappa L_\pi^{(1)} \cdot L_\pi^{(1)} - \frac{3}{8}\kappa L_\nu^{(1)} \cdot L_\nu^{(1)}. \quad (11)$$

The eigenvalues (9) of the general hamiltonian (7) are determined by the values of the various (λ, μ) contained in the decomposition of $(\lambda_\pi, \mu_\pi)(\lambda_\nu, \mu_\nu)$ allowed for given numbers of proton and neutron bosons N_π, N_ν . In the case of SU(3), (4b), the lowest states of (7) for an attractive quadrupole interaction ($\kappa_p < 0$) are given by $(\lambda_\pi = 2N_\pi, \mu_\pi = 0)$ and $(\lambda_\nu = 2N_\nu, \mu_\nu = 0)$. Among all irreducible representations (λ, μ)

obtained by reduction of $(2N_\pi, 0) \otimes (2N_\nu, 0)$, the lowest is given by $(\lambda = 2N_\pi + 2N_\nu, \mu = 0)$. The eigenvalues of H' or H'' are then equal to $E(N_\pi, N_\nu, L) = 4\kappa N_\pi N_\nu + \kappa' L(L+1)$. Physically, in this ground state band the prolate shapes of proton bosons and neutron bosons are aligned and maximum overlap is obtained. These lowest states for different values of N_π and N_ν have good F -spin given by $\frac{1}{2}(N_\pi + N_\nu)$. Still, the corresponding eigenvalues of the hamiltonian (7) depend explicitly also on $M_F = \frac{1}{2}(N_\pi - N_\nu)$. This is in contrast to the lowest eigenvalues of (6), given by $\kappa[4(N_\pi + N_\nu)^2 + 6(N_\pi + N_\nu)] + \kappa' L(L+1)$, which depend *only* on $N_\pi + N_\nu$. If an IBA-1 hamiltonian equivalent to (7) is used for such fully symmetric states its coefficients must depend explicitly on N_π and N_ν as is actually done in applications to nuclei.

In the SU*(3) case (4b*) the situation is more complicated. For $\kappa_\pi = -\kappa_\nu = -\frac{1}{2}\sqrt{7}$, the SU*(3) irreducible representations are given by $(2N_\pi, 0)$, $(2N_\pi - 4, 2)$, ... whereas neutron boson states belong to irreducible representations of *another* SU(3) group realization. The irreducible representations of that group, denoted by SU^(v)(3), are the *conjugate* representations of SU^(v)(3). Hence, the allowed symmetric representations of neutron bosons are $(0, 2N_\nu)$, $(2, 2N_\nu - 4)$, ... In more physical terms the proton bosons are in a state with prolate deformation ("particles") while the neutron bosons behave like "holes" and have *oblate* deformation. The lowest states of (6) are obtained for $(\lambda, \mu) = (2N_\pi, 2N_\nu)$ obtained in the reduction of $(2N_\pi, 0)(0, 2N_\nu)$, and exhibit the angular momentum content of bands characteristic of a triaxial rotor in the ground state [8]: $L = 0, 2^2, 3, 4^3, 5^2, 6^4, \dots$. The maximum overlap between the prolate proton shape and neutron oblate shape is obtained when both axes are not aligned but form a right angle. The resulting mass distribution has a triaxial shape [4].

The lowest eigenvalues of (7) with $\kappa_\pi = \kappa_\nu = -\kappa$ also belong to the $(2N_\pi, 2N_\nu)$ irreducible representation and are given by $E(N_\pi, N_\nu, L) = 2\kappa N_\pi N_\nu + \kappa' L(L+1)$. The overlap between the oblate and prolate shapes is not as good as in case (4b), the average quadrupole interaction

being only one-half of the prolate-prolate case. Moreover, in this case the ground state becomes multiply degenerate. It has the same value for all $SU^*(3)$ states $(2N_\pi + 2k, 2N_\nu - 4k)$ obtained by coupling proton irreducible representations $(2N_\pi, 0)$ with the neutron ones $(2k, 2N_\nu - 4k)$ with integer values of $k \leq \frac{1}{2}N_\nu$. The same eigenvalues belong also to $SU^*(3)$ states $(2N_\pi - 4k, 2N_\nu + 2k)$ obtained by reducing $(2N_\pi - 4k, 2k)$ $(0, 2N_\nu)$ with integer values of $k \leq \frac{1}{2}N_\pi$. If either proton or neutron shape is maximally prolate (or oblate) the amount of quadrupole interaction does not depend on the shape of the other group.

This rather large degeneracy in the $SU^*(3)$ case which does not arise in actual cases can easily be removed. For example, a relatively weak $Q_\pi^{(2)} \cdot Q_\pi^{(2)} + Q_\nu^{(2)} \cdot Q_\nu^{(2)}$ term will split the abovementioned degeneracy of $SU^*(3)$ multiplets without breaking the $SU^*(3)$ symmetry. Also the introduction of terms $\epsilon_\pi n_{d_\pi}$ and $\epsilon_\nu n_{d_\nu}$, or the $SU^{(\pi+\nu)}(6)$ Casimir invariant into hamiltonian (7), which break the exact $SU^*(3)$ symmetry, are very effective in removing the angular momentum degeneracy within a given $SU^*(3)$ multiplet, without changing the main features of the $SU^*(3)$ symmetry (such as angular momentum structure and E2 selection rules). The states in such cases preserve some features of triaxiality and do not correspond to the $O(6)$ limit of IBA-1.

For completeness we write down the most general hamiltonian with $O(6)$ symmetry (4c):

$$H = AC_6^{(\pi+\nu)} + A_\pi C_6^{(\pi)} + A_\nu C_6^{(\nu)} + BC_5^{(\pi+\nu)} + CL^{(1)} \cdot L^{(1)}. \quad (12)$$

In (12) the various $C_6^{(\rho)}$ denote the Casimir operators of the $O^{(\rho)}(6)$ groups, they can be related to a quadrupole-quadrupole interaction by

$$C_6^{(\rho)} = \tilde{Q}_\rho^{(2)} \cdot \tilde{Q}_\rho^{(2)} + C_5^{(\rho)}, \quad (13)$$

where the quadrupole operators $\tilde{Q}_\rho^{(2)}$ are given by (5) with $\chi_\rho = 0$, and $C_5^{(\rho)}$, the Casimir operator of the $O(5)$ group, by

$$C_5 = 2 \sum_{k=1,3} (\vec{d}^+ \times \vec{d})^{(k)} \cdot (\vec{d}^+ \times \vec{d})^{(k)}. \quad (14)$$

The allowed irreducible representations of $O^{(\pi+\nu)}(6)$ are not necessarily fully symmetric and the $C_6^{(\pi+\nu)}$ eigenvalues are given by $\sigma_1(\sigma_1 + 4) + \sigma_2(\sigma_2 + 2)$. Similarly, the eigenvalues of C_5 are given by $\tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1)$. The lowest multiplets are characterized by $\sigma_1 = N_\pi + N_\nu$, $\sigma_2 = 0$. Such states are fully symmetric in proton and neutron bosons, have definite F -spin equal to $\frac{1}{2}(N_\pi + N_\nu)$, and have one-to-one correspondence to $O(6)$ states of IBA-1. Hence, in this limit the corresponding shape of the nucleus is that of a γ -unstable rotor. If we put $A_\pi = A_\nu = -A$ there are no quadrupole interactions between identical bosons but only seniority conserving interactions. The lowest eigenvalues are equal in that case to $2AN_\pi N_\nu$.

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References

- [1] A. Arima and F. Iachello, Phys. Rev. Lett. 35 (1975) 1069; 40 (1978) 385.
- [2] A. Arima and F. Iachello, Ann. Rev. Nucl. Part. Sc. 31 (1981) 75.
- [3] A. Arima, T. Otsuka, F. Iachello and I. Talmi, Phys. Lett. B66 (1977) 205; T. Otsuka, A. Arima, F. Iachello and I. Talmi, Phys. Lett. B76 (1978) 141.
- [4] A.E.L. Dieperink and R. Bijker, Phys. Lett. 116B (1982) 77.
- [5] I. Talmi, Nucl. Phys. A172 (1971) 1.
- [6] I. Talmi, in: From nuclei to particles, Proc. Intern. School of Physics "Enrico Fermi" (Varenna, 1980), ed. A. Molinari (North-Holland, Amsterdam, 1982) p. 172.
- [7] O. Scholten, Phys. Lett. B119 (1982) 5.
- [8] A.S. Davydov and G.F. Filippov, Nucl. Phys. 8 (1958) 237.