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M1 TRANSITION STRENGTHS IN THE NEUTRON-PROTON INTERACTING BOSON MODEL

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We study the U(5), SU(3), SU^{*}(3) and O(6) limits of the neutron-proton interacting boson model. A group theoretical interpretation of the Majorana term is discussed. The $B(M1; 0_1^+ \rightarrow 1_1^+)$ values are calculated in the four limits and in different classes of translational nuclei.

The neutron-proton interacting boson model (IBM-2), the version of the IBM where a distinction is made between neutrons and protons [1], has been successful in explaining a great variety of nuclear collective properties. Whereas the IBM-1 has been thoroughly studied from a group theoretical point of view [2,3], studies in the framework of the IBM-2 usually were done numerically, with a particular form for the hamiltonian, which is dictated by microscopic arguments [4]. Only recently, the IBM-2 was studied using analytical methods [5-7]. Both the numerical and the analytical studies revealed an important aspect of the IBM-2, namely, the prediction of a magnetic dipole excitation mode, associated with the relative motion of the neutrons and the protons [8]. Investigations in terms of the geometrical model were carried out in refs. [9-12]. The recent confirmation of this prediction in ¹⁵⁶Gd [13] as well as in other deformed nuclei of the rare earth region [14], calls for a better under-

standing of the nature of this excitation mode.

In this letter, we give analytic expressions for the $B(M1; 0_1^+ \rightarrow 1_1^+)$ values in four dynamic symmetries of the IBM-2 [U(5), SU(3), SU^{*}(3) and O(6)]. These results are obtained without the use of classical approximations ($N_\nu, N_\pi \rightarrow \infty$) and thus extend, in particular, the result in the SU(3) limit [13] to finite boson number. We also study the M1 transition strength in different classes of transitional nuclei [SU(3) $\rightarrow$ U(5), O(6) $\rightarrow$ U(5) and SU(3) $\rightarrow$ O(6)].

The s, d-boson states which form a basis for the IBM-2, span the irreducible representation (irrep) $[N_\nu] \times [N_\pi]$ of $U_\nu(6) \times U_\pi(6)$, where N_ν (N_π) is the number of neutron (proton) bosons. The basic symmetry $U_\nu(6) \times U_\pi(6)$ can be broken dynamically in various ways, each corresponding to a certain group chain reduction. In the four symmetries mentioned above, the group chains are

$$\begin{aligned} \text{(I)} \quad U_\nu(6) \times U_\pi(6) &\supset U_{\nu+\pi}(6) \supset U_{\nu+\pi}(5) \\ &\supset O_{\nu+\pi}(5) \supset O_{\nu+\pi}(3) \supset O_{\nu+\pi}(2), \end{aligned} \quad (1a)$$

$$\begin{aligned} \text{(II)} \quad U_\nu(6) \times U_\pi(6) &\supset U_{\nu+\pi}(6) \supset SU_{\nu+\pi}(3) \\ &\supset O_{\nu+\pi}(3) \supset O_{\nu+\pi}(2), \end{aligned} \quad (1b)$$

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$$(II^*) \quad U_\nu(6) \times U_\pi(6) \supset SU_\nu(3) \times \overline{SU}_\pi(3) \\ \supset SU_{\nu+\pi}^*(3) \supset O_{\nu+\pi}(3) \supset O_{\nu+\pi}(2), \quad (1c)$$

$$(III) \quad U_\nu(6) \times U_\pi(6) \supset U_{\nu+\pi}(6) \supset O_{\nu+\pi}(6) \\ \supset O_{\nu+\pi}(5) \supset O_{\nu+\pi}(3) \supset O_{\nu+\pi}(2). \quad (1d)$$

In eq. (1c) appear the groups $SU(3)$ and $\overline{SU}(3)$, which have as a fundamental irrep $(2, 0)$ and $(0, 2)$, respectively, and are combined together to yield $SU^*(3)$. In this limit the neutrons are particle-like and the protons are hole-like (with respect to the nearest doubly-closed shell nucleus), or vice versa [5].

A general hamiltonian with a dynamic symmetry is written as sum of Casimir operators of the subgroups of the chain it corresponds to. We will not pursue such generality here, but rather consider a simplified hamiltonian, which contains the essential features of a particular symmetry and which enables us to study transitions in between the limits

$$H = \epsilon_d(n_{d_\nu} + n_{d_\pi}) + \kappa(Q_\nu + Q_\pi) \cdot (Q_\nu + Q_\pi) + aM, \quad (2)$$

where

$$Q_\rho = (s_\rho^+ \tilde{d}_\rho + d_\rho^+ s_\rho)^{(2)} + \chi_\rho (d_\rho^+ \tilde{d}_\rho)^{(2)}, \quad (3)$$

for $\rho = \nu$ or π , and n_{d_ρ} is the d_ρ -boson number operator. In eq. (2), we use the Majorana operator

$$M = (s_\nu^+ d_\pi^+ - s_\pi^+ d_\nu^+)^{(2)} \cdot (s_\nu \tilde{d}_\pi - s_\pi \tilde{d}_\nu)^{(2)} \\ - 2 \sum_{k=1,3} (d_\nu^+ d_\pi^+)^{(k)} \cdot (\tilde{d}_\nu \tilde{d}_\pi)^{(k)}, \quad (4)$$

instead of the quadratic Casimir operator of $U_{\nu+\pi}(6)$, C_{2U6} , which are related in the following way

$$M = \frac{1}{2}[N(N+5) - C_{2U6}]. \quad (5)$$

The symmetries of eqs. (1a–d) are realised with the following choice of the parameters

$$(I) \quad \kappa = 0, \quad (6a)$$

$$(II) \quad \epsilon_d = 0, \quad \chi_\nu = \chi_\pi = -\frac{1}{2}\sqrt{7}, \quad (6b)$$

$$(II^*) \quad \epsilon_d = 0, \quad \chi_\nu = -\chi_\pi = -\frac{1}{2}\sqrt{7}, \quad a = 0, \quad (6c)$$

$$(III) \quad \epsilon_d = 0, \quad \chi_\nu = \chi_\pi = 0. \quad (6d)$$

Note also that the Majorana term *cannot* be added to the hamiltonian of the $SU^*(3)$ limit, without breaking

the dynamic symmetry, because $U_{\nu+\pi}(6)$ does not appear in the group reduction (1c). Equivalently, one can say that the $SU^*(3)$ hamiltonian is *not* F-spin invariant. In relation to this, we remark that in the $U(5)$, $SU(3)$ and $O(6)$ limits, group chains can be chosen, different from those in eqs. (1a), (1b) and (1d), in which the neutron and proton degrees of freedom are taken together at the level of $U(5)$, $SU(3)$ or $O(6)$ [7]. The latter dynamic symmetries have the advantage that the neutron–neutron interaction can be chosen different from the proton–proton interaction, however at the cost of excluding a Majorana contribution from the hamiltonian for the dynamic symmetry to be preserved. Since for the description of mixed-symmetry states, the Majorana force plays an important role, we preferred the reduction scheme of eqs. (1a)–(1d). In more realistic calculations, situations in between the two dynamic symmetries have to be considered, that is, the hamiltonian should contain both F-spin breaking terms as well as a Majorana contribution.

Of crucial importance for the experimental observation of these states with mixed symmetry, is the M1 transition strength $B(M1; 0_1^+ \rightarrow 1_1^+)$. The M1 operator of the IBM-2 is

$$T(M1) = (3/4\pi)^{1/2} (g_\nu L_\nu + g_\pi L_\pi), \quad (7)$$

where $L_\rho = \sqrt{10}(d_\rho^+ \tilde{d}_\rho)^{(1)}$ is the angular momentum operator for the neutron ($\rho = \nu$) and proton ($\rho = \pi$) bosons and g_ν (g_π) is the neutron (proton) boson g -factor. Using the M1 operator (7), the $0_1^+ \rightarrow 1_1^+$ transition strength can be calculated in the four limits (1a)–(1d) and the result is

$$(I) \quad B(M1; 0_1^+ \rightarrow 1_1^+) = 0, \quad (8a)$$

$$(II) \quad B(M1; 0_1^+ \rightarrow 1_1^+) = \frac{3}{4\pi} \frac{8N_\nu N_\pi}{2N-1} (g_\nu - g_\pi)^2, \quad (8b)$$

$$(II^*) \quad B(M1; 0_1^+ \rightarrow 1_1^+) = \frac{3}{4\pi} \frac{2N_\nu N_\pi}{N} (g_\nu - g_\pi)^2, \quad (8c)$$

$$(III) \quad B(M1; 0_1^+ \rightarrow 1_1^+) = \frac{3}{4\pi} \frac{3N_\nu N_\pi}{N+1} (g_\nu - g_\pi)^2. \quad (8d)$$

The $B(M1)$ values to other $J^\pi = 1^+$ states vanish in the exact dynamic symmetries.

We stress that eqs. (8a)–(8d) are obtained without the use of classical approximations ($N_\nu, N_\pi \rightarrow \infty$). In particular, the expression (8b) for the $SU(3)$ limit is

Fig. 1. The $B(M1; 0_1^+ \rightarrow 1_1^+)$ values [in units $(g_\nu - g_\pi)^2$] as a function of the parameter ξ_i for the following transitions: (a) $SU(3) \rightarrow U(5)$ with $N_\nu = 5$ and $N_\pi = 3$; (b) $O(6) \rightarrow U(5)$ with $N_\nu = 4$ and $N_\pi = 2$; (c) $SU(3) \rightarrow O(6)$ with $N_\nu = 6$ and $N_\pi = 3$.

slightly different from the result given in ref. [8]. The difference is of the order of $1/N$, and not of the order $1/N_\nu$ and $1/N_\pi$, as it could have been, since in the classical approximation both N_ν and N_π are assumed to be large.

We also studied intermediate situations in between the limits. The transitions $SU(3) \rightarrow U(5)$, $O(6) \rightarrow U(5)$ and $SU(3) \rightarrow O(6)$, can be described by varying a transition parameter ξ_i in the hamiltonian (2)

$$SU(3) \rightarrow U(5): \quad \xi_1 = \eta_1 / (1 + \eta_1), \quad \eta_1 = \epsilon_d / \kappa N$$

$$\text{with } \chi_\nu = \chi_\pi = -\sqrt{7}/2, \quad (9a)$$

$$O(6) \rightarrow U(5): \quad \xi_2 = \eta_2 / (1 + \eta_2), \quad \eta_2 = \epsilon_d / \kappa N$$

$$\text{with } \chi_\nu = \chi_\pi = 0, \quad (9b)$$

$$SU(3) \rightarrow O(6): \quad \xi_3 = \chi_\nu = \chi_\pi \quad \text{with } \epsilon_d = 0. \quad (9c)$$

The three remaining transitional regions [$SU^*(3) \rightarrow U(5)$, $SU^*(3) \rightarrow SU(3)$ and $SU^*(3) \rightarrow O(6)$] can be studied similarly. The result for the $B(M1; 0_1^+ \rightarrow 1_1^+)$ value is shown in fig. 1. For the transitions (9a)–(9c), the $B(M1; 0_1^+ \rightarrow 1_1^+)$ values are independent of the strength of the Majorana force.

We also performed IBM-2 calculations studying the properties of mixed-symmetry states in various

Fig. 2. The $B(M1; 0_1^+ \rightarrow 1_1^+)$ values in $^{148-154}\text{Sm}$ for $g_\nu = 0 \mu_N$ and $g_\pi = 1 \mu_N$. The strength of the Majorana force is $a = 0.15 \text{ MeV}$ and all other parameters are taken from ref. [16].

chains of isotopes. As an example, we show in fig. 2 the $B(M1; 0_1^+ \rightarrow 1_1^+)$ value in $^{148-154}\text{Sm}$, using the estimate [8] $g_\nu \simeq 0 \mu_N$ and $g_\pi \simeq 1 \mu_N$. These calculations and similar ones for the PD and Pt isotopes, will be discussed in detail in a forthcoming paper [15].

In conclusion, we have calculated the $B(M1; 0_1^+ \rightarrow 1_1^+)$ values in the U(5), SU(3), SU*(3) and O(6) limits of the IBM-2 and studied the transitional regions in between the limits. The latter results are useful in obtaining a first estimate of the M1 transition strength. Of course, such simplified calculations should be complemented with more detailed, numerical ones [15]. For a better understanding of these states with mixed symmetry in neutrons and protons, more experiments are needed, especially for nuclei with an O(6) symmetry. From the theoretical side, a microscopic calculation of the strength of the Majorana force should be pursued.

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