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ISOTOPE AND ISOMER SHIFTS OF SAMARIUM ISOTOPES IN THE INTERACTING BOSON MODEL

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The isotope and isomer shifts of the radii of proton and neutron matter distributions in the Sm isotopes are interpreted within the framework of the interacting boson model.

In the interacting boson model radii and changes in radii can directly be expressed in terms of the matrix elements of the monopole transition operator. If no distinction is made between proton and neutron bosons (IBA1), one obtains [1,2]

$$\langle r^2 \rangle = \langle r^2 \rangle^{(\text{core})} + \gamma N + \beta \langle n_d \rangle. \quad (1)$$

Here, $\langle r^2 \rangle$ and $\langle r^2 \rangle^{(\text{core})}$ are the mean-square radii of the distributions of all nucleons and of the core, respectively. All distributions considered in this work include the finite size of the nucleons. Furthermore, $N = \langle n_s \rangle + \langle n_d \rangle$ is the total number of fermion pairs outside the core, and $\langle n_s \rangle$ and $\langle n_d \rangle$ are respectively the number of s- and d-bosons. The expectation values for n_d follow from the same IBA calculations which also give excitation energies and electromagnetic transition rates. The second and third terms in eq. (1) describe the increases in radius with the number of particles and on account of nuclear deformation. The coefficients γ and β contain the nuclear structure information. Only the latter quantity enters into the calculation of other physical quantities such as monopole transitions. Eq. (1) can be used to derive expressions for the isomer, isotope and isotone shifts of nuclear radii. The isotope shift between even- A isotopes, for example, is given by

$$\Delta \langle r^2 \rangle \equiv \langle r^2 \rangle_{N+2} - \langle r^2 \rangle_N = \gamma + \beta (\langle n_d \rangle_{N+2} - \langle n_d \rangle_N). \quad (2)$$

Eq. (2) has been used [2] to describe nuclear charge radii but is strictly speaking applicable only to nuclear matter radii.

Using the version of the interacting boson model treats proton and neutron bosons separately (IBA2), eq. (1) has to be replaced by [3]

$$\langle r_\pi^2 \rangle \equiv \langle r_\pi^2 \rangle^{(\text{core})} + \gamma_{\pi\pi} N_\pi + \gamma_{\pi\nu} N_\nu + \beta_{\pi\pi} \langle n_{d_\pi} \rangle + \beta_{\pi\nu} \langle n_{d_\nu} \rangle \quad (3)$$

and

$$\langle r_\nu^2 \rangle \equiv \langle r_\nu^2 \rangle^{(\text{core})} + \gamma_{\nu\nu} N_\nu + \gamma_{\nu\pi} N_\pi + \beta_{\nu\nu} \langle n_{d_\nu} \rangle + \beta_{\nu\pi} \langle n_{d_\pi} \rangle. \quad (4)$$

The subscripts π and ν refer to proton and neutron bosons (or boson holes) in an obvious notation. The nuclear structure information is again contained in the coefficients γ_{ij} and β_{ij} , and the expectation values of the corresponding boson operators account for the dependence of the radii on particle numbers and deformation. Only the terms containing $\gamma_{\pi\pi}$, $\gamma_{\nu\nu}$, $\beta_{\pi\pi}$ and $\beta_{\nu\nu}$ are a direct consequence of the interacting boson model, whereas the mixed terms with $\gamma_{\pi\nu}$, $\gamma_{\nu\pi}$, $\beta_{\pi\nu}$ and $\beta_{\nu\pi}$ represent correction terms due to such effects as core polarization and the proton-neutron interaction in higher order. Using eqs. (3) and (4), the isotope shifts of proton and neutron matter radii between even- A isotopes can be written as

$$\begin{aligned}\Delta\langle r_\pi^2 \rangle &\equiv \langle r_\pi^2 \rangle_{N+2} - \langle r_\pi^2 \rangle_N \\ &= \gamma_{\pi\nu} + \beta_{\pi\pi}(\langle n_{d_\pi} \rangle_{N+2} - \langle n_{d_\pi} \rangle_N) \\ &\quad + \beta_{\pi\nu}(\langle n_{d_\nu} \rangle_{N+2} - \langle n_{d_\nu} \rangle_N)\end{aligned}\quad (5)$$

and

$$\begin{aligned}\Delta\langle r_\nu^2 \rangle &\equiv \langle r_\nu^2 \rangle_{N+2} - \langle r_\nu^2 \rangle_N \\ &= -\gamma_{\nu\nu} + \beta_{\nu\nu}(\langle n_{d_\nu} \rangle_{N+2} - \langle n_{d_\nu} \rangle_N) \\ &\quad + \beta_{\nu\pi}(\langle n_{d_\pi} \rangle_{N+2} - \langle n_{d_\pi} \rangle_N).\end{aligned}\quad (6)$$

Similar expressions are valid for the isotone shifts. The isomer shift of proton radii can be expressed as

$$\begin{aligned}\delta\langle r_\pi^2 \rangle &\equiv \langle r_\pi^2 \rangle_{2^+} - \langle r_\pi^2 \rangle_{0^+} \\ &= \beta_{\pi\pi}(\langle n_{d_\pi} \rangle_{2^+} - \langle n_{d_\pi} \rangle_{0^+}) + \beta_{\pi\nu}(\langle n_{d_\nu} \rangle_{2^+} - \langle n_{d_\nu} \rangle_{0^+}).\end{aligned}\quad (7)$$

While the terms with $\beta_{\pi\nu}$ and $\beta_{\nu\pi}$ are usually set equal to zero [4–6], the term with $\gamma_{\pi\nu}$ has been treated as an adjustable quantity [4,5] but has also been set to zero [6]. The latter assumption leads to a very small isotope shift for proton matter radii. It was done under the notion that core polarization and higher-order effects are not an inherent part of the basic IBA equations. In fact, it was then argued [6] that the resulting discrepancy is most likely the result of core polarization which plays an important role in the interpretation [7] of many aspects of isotope and isotone shifts.

Isotope shifts for the proton, neutron and nucleon matter radii of the samarium isotopes have been obtained and reviewed recently [6]. The data for the former two are displayed in fig. 1. The general characteristics of the isotope shifts including the large increase in the proton radius at the phase transition between $N = 88$ and 90 are essentially understood [6]. Data for the isomer shift between the first excited 2^+ and 0^+ ground states in ^{152}Sm and ^{154}Sm are displayed in fig. 2. They have been measured by the Mössbauer effect [8] and by observing nuclear excitations associated with muonic X-ray transitions [9]. Precise absolute mean-square radii have also been reported in the latter experiment [9].

The interpretation of the data in terms of IBA2 makes use of eqs. (5), (6) and (7). The solid lines in figs. 1 and 2 are obtained with calculated d-boson occupation numbers [10] and adjusted coefficients γ and

Fig. 1. Isotope shifts $\langle r^2 \rangle_{A+2} - \langle r^2 \rangle_A$ of proton matter (charge) and neutron matter radii for the even- A Sm isotopes. The data are plotted at the lower value of A , and the observed shifts between $A = 144$ and 148 are displayed at $A = 144-146$ and $A = 146-148$. The solid and dashed lines are the result of IBA2 calculations (see text).

Fig. 2. Isomer shift $\langle r^2 \rangle_{2^+} - \langle r^2 \rangle_{0^+}$ of proton matter radii (charge radii) for even- A Sm isotopes. The solid and dashed lines are the result of IBA2 calculations (see text). Other theoretical predictions are indicated by $\times$ [14] and $+$ [15].

β . The coefficients are $\gamma_{\pi\nu} = 0.26 \text{ fm}^2$, $\beta_{\pi\pi} = 0.09 \text{ fm}^2$ (protons) and $\gamma_{\nu\nu} = 0.38 \text{ fm}^2$, $\beta_{\nu\nu} = -0.11 \text{ fm}^2$ (neutrons) with $\beta_{\pi\nu} = \beta_{\nu\pi} = 0 \text{ fm}^2$. The agreement with the data and the values for the coefficients will be discussed below.

The positive and negative values for $\beta_{\pi\pi}$ and $\beta_{\nu\nu}$ can qualitatively be understood by considering S-pair and D-pair states, the microscopic equivalents of the corresponding bosons. In lowest order the structure of the D-pair state is obtained by operating with the quadrupole operator on S^N states. The $1d_{5/2}$ and $0g_{7/2}$ orbits are almost completely filled in the proton S^6 state, and the proton D-pair state therefore has a relatively large amplitude from the $0h_{11/2}$ single-particle intruder orbit. The latter orbit belongs to the next higher harmonic oscillator shell with its increased mean-square radius. As a result the proton d-boson will have a larger radius than the s-boson, and $\beta_{\pi\pi}$ should be positive. A reverse situation exists for the neutrons. The number of neutrons outside the core is small and both the $1f_{7/2}$ and $0g_{9/2}$ orbits and the $0i_{13/2}$ intruder orbit are essentially empty. Since pairing is stronger for high-spin orbits and since furthermore the normal-parity orbits can give rise to increased collectivity in the neutron D-pair state, the occupancy of the $0i_{13/2}$ orbit with its increased mean-square radius is expected to be larger in the S-pair than the D-pair state. Thus, $\beta_{\nu\nu}$ should be negative. This depends strongly on the number of neutron bosons, and $\beta_{\nu\nu}$ is expected to depend strongly on neutron number. (For large neutron numbers $\beta_{\nu\nu}$ should become positive similar to $\beta_{\pi\pi}$.) In the calculations displayed in fig. 1 the quantity $\beta_{\nu\nu}$ was kept constant for simplicity. The above arguments are confirmed by microscopic calculations in a generalized seniority scheme [11] which give $\beta_{\pi\pi} \approx 0.06 \text{ fm}^2$ almost independent of neutron number N while $\beta_{\nu\nu}$ varies from -0.51 to -0.03 fm^2 for $N = 84$ to 90 .

The isotope shift for the neutrons (fig. 1) is well reproduced by the calculations. The isomer shift is slightly overestimated (fig. 2, solid line). In order to understand the observed strong increase of the isotope shift for the protons at $A = 150$ – 152 and subsequent decrease at $A = 152$ – 154 (fig. 1, solid line), the origin of the coefficient $\gamma_{\pi\nu}$ in eq. (5) has to be considered in more detail. It may, as stated before, arise from the neutron–proton interaction. An energy gap in the proton single-particle levels is known to exist at $Z = 64$ when only a small number of neutrons with $N \geq 82$ is

present [12]. The gap is reduced gradually with increasing neutron number and disappears for $N > 90$ [13]. It is therefore reasonable to assume that the energy difference between the $0h_{11/2}$ and the $1d_{5/2}$ and $0g_{7/2}$ proton levels decreases approximately linearly with neutron number. The occupancy of the proton $0h_{11/2}$ level therefore increases linearly with the number of neutrons yielding an approximate constant and positive value of $\gamma_{\pi\nu}$ for neutron boson numbers $N_\nu \leq 4$. For $N_\nu > 4$ the nucleus is deformed and the $0h_{11/2}$ occupancy reaches a saturation value, hence $\gamma_{\pi\nu} \approx 0$. Using this extreme assumption, the dashed curve in fig. 1 is obtained with $\gamma_{\pi\nu} = 0.25 \text{ fm}^2$ ($\gamma_{\pi\nu} = 0 \text{ fm}^2$ for $N_\nu > 4$) and $\beta_{\pi\pi} = 0.16 \text{ fm}^2$. The agreement with the data is significantly improved giving support to the assumptions made although different explanations cannot be excluded. The neutron isotope shift (fig. 1) is not influenced by the above effect. However, the calculated isomer shift (fig. 2, dashed line) increases due to the increased value of $\beta_{\pi\pi}$. The agreement with the data becomes poorer. This is not considered too serious because for $^{152,154}\text{Sm}$ the calculated differences $\langle n_{d_\pi} \rangle_{2+} - \langle n_{d_\pi} \rangle_{0+}$ in eq. (7) amount to only about 10 to 15% of the values of $\langle n_{d_\pi} \rangle$. They are therefore very sensitive to the parameters used in the hamiltonian to calculate the wave functions and to second-order contributions to the operators. For the same reasons the earlier assumption that $\beta_{\pi\nu}$ is exactly zero may also not be fully justified. Data for ^{150}Sm and ^{148}Sm would permit a much more stringent comparison between experiment and theory. These isomer shifts are more difficult to measure and are apparently not available. Fig. 2 includes a few additional theoretical predictions [14,15] which also overpredict the observed isomer shifts.

While good agreement has been obtained in the present work for the samarium isotopes, further studies using the interacting boson model and other theoretical approaches are needed to achieve a comprehensive understanding of the isotope, isotone, and isomer shifts of nuclear radii.

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