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AN INTERPRETATION OF CASIMIR OPERATORS OF THE $U(6/12)$ GROUP

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An interpretation is presented of the $U^{B+F}(6)$ Casimir operator which appears in a group theoretical description of the $U(6/12)$ dynamical bosonfermion symmetry. The $U^{B+F}(6)$ term is shown to be equivalent to a very specific choice of the exchange term in the general hamiltonian of the interacting boson-fermion model. The analysis also suggests the possible importance of additional exchange terms in the hamiltonian which have so far been neglected.

The concept of a dynamical $U(6/12)$ boson-fermion symmetry [1] has recently been applied [2-6] with considerable success to the spectra of odd A, Pt and W nuclei. A dynamical symmetry emerges whenever the hamiltonian can be written as a sum of Casimir operators of a chain of subgroups. This allows one to obtain an analytic expression for the excitation energies and electromagnetic transition rates. For example, for deformed nuclei, the dynamical symmetry of interest is

$$\begin{aligned} U(6/12) \supset U^B(6) \times U^F(12) &\supset U^B(6) \times U^F(6) \times SU^F(2) \\ &\supset U^{B+F}(6) \times SU^F(2) \supset SU^{B+F}(3) \times SU^F(2) \\ &\supset SO^{B+F}(3) \times SU^F(2) \supset \text{spin}(3). \end{aligned} \quad (1)$$

The corresponding expression for excitation energies can be written as [4]

$$\begin{aligned} \alpha[N_1(N_1+5)+N_2(N_2+3)] + \beta[\lambda^2 + \mu^2 + \mu\lambda + 3(\mu+\lambda)] \\ + (\delta - 0.75\beta)L(L+1) + \gamma J(J+1). \end{aligned} \quad (2)$$

A similar expression can be constructed in cases for which the subgroups of $U^{B+F}(6)$ are $O^{B+F}(6)$ or

$U^{B+F}(5)$ instead of $SU^{B+F}(3)$. Note that in expression (1), the bosonic and fermionic degrees of freedom have been coupled at the level of $U^{B+F}(6)$. In the original formulation of the $U(6/12)$ symmetry [1], an alternative decomposition was used, which involved coupling bosons and fermions at a later stage so that, in expression (1), the subgroup $SU^B(3) \times SU^F(3) \times SU^F(2)$ was used in place of $U^{B+F}(6) \times SU^F(2)$. However, the application of the $O(6)$ chain in this framework to ^{195}Pt [5] revealed a significant discrepancy between the predicted and empirical energy spectra, which, it was later shown [7], could be remedied by adopting the $U^{B+F}(6)$ coupling. More recently, it has been shown [3] that this choice is also essential in the $SU(3)$ limit. Nevertheless, despite its evident empirical necessity, there has as yet been no physical justification suggested for the Casimir operator of $U^{B+F}(6)$, nor any physical understanding of the role it plays in the predicted energy spectrum. It is therefore the principal purpose of this Letter to offer a microscopic interpretation of this term, which in turn explains its crucial importance in reproducing the empirical data.

The parameters in an eigenvalue expression such as

eq. (2) can be freely adjusted in a phenomenological calculation to reproduce the experimental energies. However, in an alternative approach the spectra of odd mass nuclei can be calculated in the Interacting Boson–Fermion Model (IBFM) [8]. In this model the hamiltonian is constructed on the basis of a semi-microscopic theory. The boson–fermion interaction is written as [8]

$$V_{BF} = \sum_{jj'} \Gamma_{jj'} Q_B^{(2)} \cdot (a_j^\dagger \tilde{a}_{j'})^{(2)} + \sum_{jj'j''} \Lambda_{jj'}'' [(\tilde{d} a_j^\dagger)^{(j'')} \times (a_j^\dagger \tilde{a}_{j'})^{(j'')}]^{(0)} + \sum_j A_j (a_j^\dagger \tilde{d})^{(0)} (a_j^\dagger \tilde{a}_j)^{(0)}, \quad (3)$$

where

$$\Gamma_{jj'} = \Gamma_0 (u_j u_{j'} - v_j v_{j'}) Q_{jj'},$$

$$\Lambda_{jj'}'' = -2\sqrt{5} \Lambda_0 \beta_{jj'} \beta_{jj''} / j'',$$

$$\beta_{jj'} = (u_j v_{j'} + v_j u_{j'}) Q_{jj'}. \quad (4)$$

The coefficients $Q_{jj'}$ are the single-particle matrix elements of the quadrupole operator. Taking the radial integrals equal to unity these can be written as

$$Q_{jj'} = \langle l \frac{1}{2} j || Y^{(2)} || l \frac{1}{2} j' \rangle. \quad (5)$$

The coefficients $v_j^2, u_j^2 = 1 - v_j^2$ can be interpreted as the occupancies of the spherical shell model orbits. The quadrupole term in V_{BF} is related to the neutron–proton quadrupole force. The second term, the exchange force, can be derived from an interplay between the Pauli exclusion effect and the neutron–proton interaction [9]. It has been shown [10] that, with these two terms in the interaction, a changing v^2 reproduces for deformed nuclei similar effects on the spectrum as changing the Fermi surface in a Nilsson model calculation.

In the recent study [3] of the SU(3) limit of U(6/12), it was pointed out that effects on the energies, similar to those of changing the Fermi surface, are induced by varying the relative strength of the $U^{B+F}(6)$ and $SU^{B+F}(3)$ Casimir operators (C_{2U6} and C_{2SU3}). This situation is illustrated schematically in fig. 1. This earlier work established a one-to-one correspondence between the lowest rotational bands predicted by the

boson–fermion symmetry, and the Nilsson model orbits stemming from the $p_{1/2}, p_{3/2}$ and $f_{5/2}$ shell model states in the 82–126 shell. The upper part of fig. 1 shows the lowest three of these Nilsson orbits, while the lower portion shows the behavior of the corresponding bands in the U(6/12)–SU(3) scheme as the relative strength of the $U^{B+F}(6)$ and $SU^{B+F}(3)$ terms changes. It is clear that increasing the relative strength of C_{2U6} corresponds to a rising Fermi surface in the Nilsson scheme. Since the $SU^{B+F}(3)$ Casimir operator is obviously related to the boson–fermion quadrupole interaction, it would seem that C_{2U6} must be related to the exchange force in the IBFM.

The part of C_{2U6} that is second order in the d-boson operators can be written as [4]

$$2 \sum_{\lambda} (-1)^{\lambda} \tilde{\lambda} [(d^\dagger \tilde{d})^{(\lambda)} \times G_F^{(\lambda)}(2, 2)]^{(0)}, \quad (6)$$

where

$$G_F^{(\lambda)}(2, 2) = \sum_{jj'} (-1)^{\lambda+j'-1/2} \frac{1}{\sqrt{2}} j j' \times \left\{ \begin{matrix} j & j' & \lambda \\ 2 & 2 & 1/2 \end{matrix} \right\} (a_j^\dagger \tilde{a}_{j'})^{(\lambda)}.$$

The exchange force in IBFA reduces for the particular case in which $v_{1/2}^2 = v^2$ and $v_{3/2}^2 = v_{5/2}^2 = 0$ to

Fig. 1. Lower part: Low lying bandheads in the U(6/12) scheme as a function of α and β (eq. (2)); upper part: Corresponding Nilsson orbits (ref. [3]) and a schematic indication of the effective Fermi level.

$$\begin{aligned}
& -\sqrt{10} v^2 \Lambda_0 \sum_{jj'} Q_{1/2j} Q_{1/2j'} \\
& \times [(\tilde{d} a_j^\dagger)^{(1/2)} \times (d^\dagger \tilde{a}_{j'})^{(1/2)}]^{(0)} \\
& = \frac{\sqrt{5}}{2\pi} v^2 \Lambda_0 \sum_{jj'} (-1)^{j'+1/2} \hat{j} \hat{j}' \hat{\lambda} \begin{Bmatrix} 2 & 2 & \lambda \\ j' & j & 1/2 \end{Bmatrix} \\
& \times [(d^\dagger \tilde{d})^{(\lambda)} \times (a_j^\dagger \tilde{a}_{j'})^{(\lambda)}]^{(0)}, \quad (7)
\end{aligned}$$

where the single-particle matrix elements of the quadrupole operator have been evaluated using eq. (5). It can be seen that eqs. (6) and (7) are equal apart from an overall factor $v^2 \Lambda_0 \sqrt{10}/4\pi$, so that C_{2U6} indeed corresponds to a particular exchange force in the IBFM.

Thus the proposed similarity between the exchange and quadrupole forces of the IBFM on the one hand, and the $U^{B+F}(6)$ and $SU^{B+F}(3)$ Casimir operators in the SUSY model on the other hand, is based on the fact that, in certain limiting cases, an exact correspondence between the two can be shown to exist. Specifically, this correspondence demands fixed values of the particle occupation amplitudes in the *spherical* limit, namely, $v_{1/2}^2 = v^2$ and $v_{3/2}^2 = v_{5/2}^2 = 0$. The magnitude of the constant α of eq. (2) then reproduces the effect of a changing Fermi surface in the *deformed* scheme, while in the general IBFM hamiltonian, the same effect can be obtained by changing the magnitude of Λ_0 in eq. (4). Nevertheless, from a microscopic viewpoint, it would be preferable to regard the changing Fermi surface in the deformed potential as arising purely from changes in the spherical occupational amplitudes and the core structure. In fact, the $SU^{B+F}(3)$ Casimir operator corresponds to a pure quadrupole force, and this situation is realized in the IBFM for $v_{1/2}^2 = v_{3/2}^2 = v_{5/2}^2 = 0$, when the exchange term vanishes. Thus intermediate cases where both the $SU^{B+F}(3)$ and $U^{B+F}(6)$ Casimir operators are present in varying relative strengths can be regarded as cases in which the $j = 1/2$ orbit is only partially occupied.

Another important point that emerges from the present considerations is the evidence for the possible need to introduce an additional exchange force in the IBFM. The $U^{B+F}(6)$ operator also contains terms that are first order in the boson operators, which can be written as [4]

$$\sqrt{2} \sum_j \hat{j} [(s^\dagger \tilde{d})^{(2)} \times (a_j^\dagger \tilde{a}_{1/2})^{(2)}]^{(0)} + \text{h.c.} \quad (8)$$

These terms are not present in the standard boson–fermion interaction of the IBFM. The exchange force itself can be derived, as was done initially [8], from the effect of the Pauli principle on the interaction between like particles. This procedure however can also lead to an s – d boson exchange force which can be written as

$$V_{sd}^{\text{EX}}(s, d) = \sum_{jj'} \beta_{jj'} v_j u_{j'} \hat{j} \hat{j}' [(s^\dagger \tilde{d})^{(2)} \times (a_j^\dagger \tilde{a}_{j'})^{(2)}]^{(0)} + \text{h.c.} \quad (9)$$

For the case in which the odd particle occupies only a single j orbit or in the case in which the occupancies of the different orbits are equal, this term can be absorbed by renormalizing the quadrupole force. In the general case, however, this will give rise to a genuinely new term in the boson–fermion interaction which to date has not been considered. It can easily be checked that this new exchange force reduces to the expression (8) for the same values $v_{1/2}^2 = v^2$, and $v_{3/2}^2 = v_{5/2}^2 = 0$ which have been used to relate the usual IBFM exchange force to the $U^{B+F}(6)$ Casimir operator.

The remaining contribution of C_{2U6} to the boson–fermion interaction can be taken into consideration by the monopole term in eq. (3) and a renormalization of the quasi-particle energies in the IBFM.

In conclusion, we have provided a physical interpretation for some of the terms which enter in the hamiltonian in a group theoretical approach, which in turn makes the occurrence of boson–fermion symmetries in the spectra of odd-mass nuclei less ad hoc. Specifically, the interplay between the Casimir operators of $U^{B+F}(6)$ and $SU^{B+F}(3)$ has been shown to play the same role as that between the exchange and quadrupole interactions in the general IBFM treatment. Moreover, the empirically deduced necessity for the $U^{B+F}(6)$ decomposition can now be understood, in that it provides the ability to adjust the position of the effective Fermi surface in the deformed potential. Finally, evidence has been found for the importance of an additional term in the boson–fermion interaction which has hitherto escaped attention in the interacting boson–fermion model.

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