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DOUBLE BETA DECAY IN THE INTERACTING BOSON APPROXIMATION

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In order to include the effects of nuclear deformation in the calculation of the nuclear many-body matrix element involved in the double beta decay process, we have performed a calculation in the interacting boson approximation. An explicit expression for the boson image of the 2β transition operator is given. Results for the $^{128,130}\text{Te} \rightarrow \text{Xe}$ decays are presented. The calculation indicates that the nuclear deformation strongly hinders the double beta decay.

Double beta decay is an interesting process since it probes the breakdown of lepton number conservation due to the finite mass of the Majorana neutrino or possible right-handed weak lepton current [1]. This can be determined from the ratio of the zero- and two-neutrino decay widths. However to extract evidence on the rates of the two possible decay processes from nuclear measurements, the nuclear many-body matrix element involved has to be known accurately. A major complication in the calculation of the nuclear double beta decay matrix element is that the g.s.-g.s. decay (which is the only allowed decay in most cases for Q value reasons) represents only a few percent of the total sum rule of strength. Extensive calculations of the nuclear matrix element have been done by Haxton et al. [2] in terms of the nuclear shell model.

For medium heavy nuclei there appears to be a large difference between the theoretically calculated nuclear matrix element and experiment [2]. Part of this discrepancy could arise from the fact that in the shell-model calculations, for technical reasons, it is assumed that the nuclei are very close to spherical. For medium heavy nuclei this need not to be a good approximation. In ref. [3] the effect of deformation has been studied using the Nilsson model. This calculation indicates that for well deformed nuclei the double beta decay rate vanishes. This indicates that the influence of deformation has a strong influence on the double

beta decay rate. To investigate the effects of nuclear deformation we have therefore repeated the calculation in the framework of the interacting boson approximation (IBA) [4].

In the treatment of double beta decay in the IBA framework the problem is divided into two. One is that of calculating the coefficients that appear in the boson image of the operator. This is solely related to the microscopic structure of the two-fermion states which are the equivalent of the bosons. This calculation can be done in many different ways. In this paper we present a calculation which is based on the spherical shell model. What remains is to calculate the boson wave functions of the nuclei in question. Since for most medium heavy nuclei IBA parameters are available, this part of the problem is already essentially solved. In making this division the problem is simplified considerably. Under the assumption, which is basic in the IBA, that the microscopic structure of the bosons is essentially independent of the structure of the nucleus, the coefficients in the boson image of the double beta decay operator are determined by the interaction between like fermions, and will change only gradually from nucleus to nucleus. The effects of the neutron-proton interaction, which is the main driving force towards deformation, is taken into account via the boson wave functions. This separation allows for the prediction of the double beta decay rates for nuclei that are vastly different in structure, without

having to go through complicated calculations.

In this letter we will limit ourselves to the calculation of two-neutrino double beta decay. Furthermore, since we want to compare the results with those of the shell-model calculations presented in ref. [2], we will make the same kind of approximations. In particular we will make use of the closure approximation to eliminate the explicit summation over intermediate states which occurs in the exact operator. In addition in the microscopic calculation of the coefficients in the boson image of the double beta decay operator will be limited to a single major shell only. In a forthcoming, longer paper the validity of some of these approximations will be addressed in more detail.

The operator describing the two-neutrino double beta decay process can to a good approximation be written as a double Gamow–Teller operator [1]. Using the closure approximation to carry out the summation over intermediate states, the operator can be written as [1]

$$O_{2\beta}^F = \frac{1}{2} \sum_{i,k} (\sigma(i)t_+(i)) \cdot (\sigma(k)t_+(k)), \quad (1)$$

where t_+ is the isospin raising operator (in our convention the proton is assigned $t_z = +\frac{1}{2}$). In eq. (1) we have limited ourselves to the description of $L = 0$ decay.

Introducing the shell model single-particle (s.p.) creation (annihilation) operators $a_j^\dagger$ (a_j), we may rewrite the operator $O_{2\beta}^F$ as

$$O_{2\beta}^F = -\frac{1}{2} \sum_{j_\pi j'_\pi j_\nu j'_\nu, I} (2I+1) \times \left((a_{j_\pi}^\dagger \times a_{j'_\pi}^\dagger)_M^{(I)} \times (\tilde{a}_{j_\nu} \times \tilde{a}_{j'_\nu})_M^{(I)} \right)^{(0)} \times G_{j_\pi j'_\pi j_\nu j'_\nu}^{(I)}, \quad (2)$$

where $\tilde{a}_{jm} = (-)^{j-m} a_{j-m}$, and

$$G_{j_1 j_2 j_3 j_4}^{(I)} = 6(-)^{j_1+j_3+l+l'+I} \hat{j}_1 \hat{j}_2 \hat{j}_3 \hat{j}_4 \times \left\{ \begin{matrix} j_1 & j_2 & I \\ j_4 & j_3 & 1 \end{matrix} \right\} \left\{ \begin{matrix} \frac{1}{2} & j_1 & l \\ j_3 & \frac{1}{2} & 1 \end{matrix} \right\} \left\{ \begin{matrix} \frac{1}{2} & j_2 & l' \\ j_4 & \frac{1}{2} & 1 \end{matrix} \right\}, \quad (3)$$

where $\hat{j} = (2j+1)^{1/2}$, and the curly brackets denote the usual $6j$ symbol.

In the IBA [4] model the structure of the low lying collective states in medium heavy nuclei are described in terms of a system of mutually interacting bosons. Each boson corresponds to a collective pair of neutrons or protons in either an $J = 0$ (s-boson) or an $J = 2$ (d-boson) state [5]. This equivalence relation insures for example, that in the description of a given nucleus, the numbers of neutron and proton bosons are strictly conserved.

In the nuclei for which there exists experimental data on the double beta decay process the protons are in the beginning of the valence shell while for the neutrons the valence shell is more than half-filled. This implies that in the conventional [5] IBA picture of the bosons the proton bosons correspond to particle pairs, while the neutron bosons correspond to hole pairs. In the double beta decay process a proton pair is created and two neutron particles are annihilated, or equivalently, a neutron–hole pair is created. In the boson space therefore the operator creates both a neutron and a proton boson, and in lowest order in the boson operators, the boson image of the $L = 0$ double beta operator can be written as

$$O_{2\beta}^B = A(s_\pi^\dagger \cdot s_\nu^\dagger) + B(d_\pi^\dagger \cdot d_\nu^\dagger) + C(s_\pi^\dagger s_\pi^\dagger s_\nu^\dagger s_\nu^\dagger \tilde{d}_\pi \cdot \tilde{d}_\nu) / Z_\pi Z_\nu + E_\pi(s_\pi^\dagger s_\pi^\dagger \tilde{d}_\pi \cdot d_\nu^\dagger) / Z_\pi + E_\nu(s_\nu^\dagger s_\nu^\dagger d_\pi^\dagger \cdot \tilde{d}_\nu) / Z_\nu, \quad (4)$$

where parameters $A-E$ have been introduced. The order of the operator in this context is defined by the number of d-boson creation and annihilation operators. The normalization factors Z are defined according to

$$Z_\rho = [N_\rho(N_\rho - 1)]^{1/2}, \quad \rho = \pi, \nu, \quad (5)$$

and serve only for convenience.

These parameters in eq. (4) can be determined from a microscopic model following the OAI [5] procedure, in which matrix elements between equivalent states in the boson and fermion spaces are equated. In the conventional microscopic picture behind the IBA model, mentioned above, a state with n_d d-bosons corresponds [5] to a state in the fermion space with (generalized) seniority

[6] $v = 2n_d$. The first term in eq. (4) thus corresponds to the part of the double beta operator which keeps the seniority fixed, for neutrons and protons separately, while the second term corresponds to the part in which the total seniority is increased by four units. In the fermion space the operator can also decrease the seniority by four units. In the image one therefore expects also a term which annihilates two d-bosons. This term must have the form of the third term in eq. (4), where the s-boson operators can be seen as to only take care of fermion number conservation. In spite of its complicated appearance it thus still is part of the lowest order boson operator. An equivalent argument also applies to the last two terms in eq. (4).

The microscopic calculation of the coefficients in the boson image of the double beta decay operator is based on the generalized seniority model [7], which in turn is based on the shell model. General details of the calculations involved are given in ref. [7]. As effective nucleon interaction we have chosen a surface delta force (strength A') with an enhanced quadrupole component (enhancement factor F2) [7] which should account for core polarization effects. The parameters were obtained from a best fit calculation to excitation energies in semi-closed shell nuclei and are given in table 1. The structure of the S-pair is determined by the structure of the ground state and that of the D-pair by the 2_1^+ in the semi-closed shell nuclei.

The coefficients which enter in the operator of the double β decay, can now be determined by equating [5] the matrix elements of the operator in the fermion space between states build up from S-pairs only and states containing at most one neutron and one proton D-pair, as calculated in

the generalized seniority model [7]. These matrix elements can be equated to those of the operator eq. (4) between states with equivalent numbers of s- and d-bosons. As an example of the calculations involved, we will discuss in some more detail the first coefficient in eq. (4). In the generalized seniority scheme, the matrix element can be calculated using eqs. (1)–(3) as

$$\begin{aligned} ME_A^F &= \langle S_{\pi}^{n_{\pi}-1} S_{\pi}^{n_{\pi}+1} | O_{2\beta}^F | S_{\nu}^{n_{\nu}} S_{\pi}^{n_{\pi}} \rangle \\ &= \sum_{j_{\nu} j_{\pi}} -\frac{1}{2} G_{j_{\nu} j_{\pi}}^{(0)} \langle S_{\nu}^{n_{\nu}-1} | (\tilde{a}_{j_{\nu}} \tilde{a}_{j_{\nu}})^{(0)} | S_{\nu}^{n_{\nu}} \rangle \\ &\quad \times \langle S_{\pi}^{n_{\pi}+1} | (a_{j_{\pi}}^{\dagger} a_{j_{\pi}}^{\dagger})^{(0)} | S_{\pi}^{n_{\pi}} \rangle, \end{aligned}$$

with

$$\begin{aligned} \langle S^n | S_- (a_j^{\dagger} a_j^{\dagger})^0 | S^n \rangle &= \alpha_j \hat{J}_{22}(j, j) \\ &\quad + \sum_{j'} \alpha_j \hat{J}_{21}(j, j') (j' + \frac{1}{2}), \end{aligned}$$

and

$$S_- = -\frac{1}{2} \sum \alpha_j \hat{J}(\tilde{a}_j \tilde{a}_j)^{(0)}.$$

The definition of the coefficients S_{ij} can be found in ref. [8]. This formula is exact, without any approximations, and can be calculated in a fraction of a second on a modern computer. The corresponding boson matrix element is given by

$$\langle s_{\nu}^{N_{\nu}-1} s_{\pi}^{N_{\pi}+1} | O_{2\beta}^B | s_{\nu}^{N_{\nu}} s_{\pi}^{N_{\pi}} \rangle = A [N_{\nu}(n_{\pi} + 1)]^{1/2},$$

where eq. (4) has been used for the boson operator. The value of the parameter A is now given by

$$A = ME_A^F / [N_{\nu}(N_{\pi} + 1)]^{1/2}.$$

For the other coefficients in (4) an exactly similar procedure has been used. The thus calculated

Table 1

Interaction parameters (see text), used to determine the microscopic structure of the s- and d-bosons in the generalized seniority calculation.

Nuclei	A'	F2	Single-particle energies				
			$0g_{7/2}$	$1d_{5/2}$	$1d_{3/2}$	$2s_{1/2}$	$0h_{11/2}$
$N = 82$	0.20	1.5	0.0	0.96	2.69	2.99	2.76
$Z = 50$	0.18	1.9	-2.3	-2.80	-0.30	-0.80	0.10

Table 2

The calculated values for the coefficients in the boson image of the double beta decay operator, eq. (4). Also the expectation value of the boson operators multiplying these coefficients is given.

Decays	A	B	C	E_π	E_ν
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	0.848	-0.402	-0.007	0.041	0.145
boson m.e.	2.20	1.18	1.19	0.19	0.33
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	0.900	-0.436	-0.005	0.039	0.117
boson m.e.	2.20	1.12	0.49	0.05	0.06

values for the nuclei in the question are given in table 2. In the calculation of the coefficients of the operator (4) the contributions from the different shell-model configurations that constitute the S and D pair state, almost all add up constructively. This makes that the calculated results are relatively insensitive to details of the shell-model interaction that is used in constructing the microscopic structure of the bosons.

The IBA model wave functions for the Te and Xe isotopes are calculated using the parameters given in refs. [9,10]. Using the thus obtained wave functions the matrix elements of the operator eq. (4) was calculated, and the results are given in table 3, where they are also compared with the results of Haxton [2]. From the values given in the table it can be seen that the decay rate is dominated by the first term in the operator (4), partially due to the fact that $|A| > |B|$ and partially to the fact that the boson matrix element for the first term in (4) is larger than that of the second. Since B has the opposite sign of A it does have the effect of decreasing the double beta decay matrix element. This cancellation is similar in nature to what has been calculated by Klapdor and Grotz [11] when they included the effect of phonon excitations in the RPA. The contribution coming from the last three terms of (4) is of only minor

importance (of the order of a few percent) and could have been safely neglected. When the number of particles (holes) is small, adding two particles (holes) has a higher probability of increasing the seniority than to decrease it. Since this is the case for the nuclei presently under investigation, where the valence protons are particle-like, while the valence neutrons are hole-like and the double beta decay process converts two neutrons into protons, one thus expects the B coefficient in eq. (4) to be larger than those of the last three terms.

The limiting case of a well deformed nucleus is described in the IBA model by the SU(3) limit. In this limit the number of d-bosons in the ground state is twice that of s-bosons. Taking the values for A and B as calculated for Te, this results in an essentially vanishing double beta decay rate. This is in agreement with the results obtained from Nilsson model calculations [3] in the absence of pairing. Also in the limit of an extreme shell-model nucleus, like ^{48}Ca , a link with the IBA model can be made. In this specific case the A and B coefficients are of the same order of magnitude (1.28 respectively -1.04). Using the Cohen and Kurath wave functions for ^{48}Ti the equivalent IBA wave function can be constructed and the matrix elements of the boson operators for the first two

Table 3

The calculated and experimental double Gamow-Teller matrix elements.

Decays	$ M_{\text{GT}} _{\text{IBA}}$	$ M_{\text{GT}} _{\text{Haxton}}^{\text{a)}}$	$ M_{\text{GT}} _{\text{exp}}^{\text{a)}}$
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	1.44	1.47	0.185-0.230
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	1.49	1.48	0.104-0.129

^{a)} Taken from ref. [2].

terms in eq. (4) can be calculated, giving values of 0.905 and 0.953, respectively. Also in this case one thus has an almost complete destructive interference, in agreement with the results given in ref. [2]. The picture now emerges that for near closed shell nuclei the nuclear matrix element vanishes. In going away from the closed shell the A term takes over in importance since the number of s -bosons in the ground state increases, while the number of d -bosons stays approximately constant in the vibrational regime. When reaching the point where deformation sets in, the trend is reversed, and the matrix element decreases again to zero, because of an increased cancellation between the A and B terms. As discussed before, it is assumed that A and B are independent of the structure of the nucleus, but are of course dependent on the number of neutrons and protons in the nucleus. The Xe and Te nuclei are just somewhere in the vibrational region where the number of s -bosons is appreciable, but the deformation still small. As a result the calculated double beta decay matrix element is relatively large.

It should be noted that the presently calculated values, although in agreement with those calculated by Haxton et al. [2], are much larger than the matrix elements obtained from experiment. The latter even constitute something like an upper limit since, in order to extract the matrix element, the neutrinoless decay is neglected. This discrepancy could have several different origins. In the calculation of the coefficients we have limited ourselves to a single major shell, in order not to have the complication of spurious center of mass motion. In the calculation of A in eq. (4) the contributions of the different valence orbits all have the same sign. One therefore might expect that extension of the model space will increase the value of A . The calculations presented in ref. [3] support this finding. For the other coefficients the picture is somewhat more complex, however still the contributions from the different components of the

D -pair state appear to add up mostly coherently. This would imply that extending the model space might not affect the double beta decay rate too much since the contribution related to the d -boson gives a destructive contribution. The effect of core polarization (or equivalently model space extension) on B is therefore not clear. Also degrees of freedom outside the IBA model could have an influence. The G -pair state in this respect will be most important. In a forthcoming longer paper we intend to investigate these points in some more detail.

In conclusion it has been shown that the IBA model provides a comprehensive framework for the calculation of double beta decay. Since it is relatively simple it can be used to calculate the effects of the corrections mentioned above.

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