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CORIOLIS COUPLING IN THE INTERACTING BOSON FERMION MODEL

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The properties of a boson core coupled to a single- j particle are examined in the framework of the SU(3) limit of the interacting boson fermion model. It is shown that the Coriolis interaction arises in a natural way in this model. Excitation energies are calculated in the large boson number approximation. The analogy to the particle plus rotor model is discussed.

The interacting boson fermion model (IBFM) [1] has been successfully applied to the description of odd- A nuclei [2–4], in particular in the transitional region. Such nuclei have often been studied by means of the rotor plus particle model [5]. The mixing of states with different projection quantum numbers K along the symmetry axis due to the Coriolis interaction has played the main role in the description of odd- A transitional nuclei. This has led to the concept of rotation alignment (RA) [6]. On the other hand, the IBFM has been able to describe features of excited nuclear states usually considered as typical for the RA. Therefore a legitimate question arises: does IBFM include anything similar to the Coriolis coupling, or we deal with different mechanisms producing the same effect?

The aim of this work is to answer this question by analysing the features of an equivalent of the rotor plus particle model, i.e. those of a particle coupled to an SU(3) boson core. For the sake of simplicity the discussion will be restricted to the single- j case. Further properties of this simple form of IBFM will be examined.

Let us briefly review the model [1,7]. The hamiltonian of an odd- A nucleus is

$$H = H_{\text{core}} + H_{\text{fermion}} + V_{\text{BF}}, \quad (1)$$

where the boson–fermion interaction energy is given in the single- j case by

$$V_{\text{BF}} = \Gamma \left[Q \times (a_j^\dagger \times \tilde{a}_j)^{(2)} \right]^{(0)} + \Lambda \left[(\tilde{d} \times a_j^\dagger)^{(j)} \times (d^\dagger \times \tilde{a}_j)^{(j)} \right]^{(0)}, \quad (2)$$

where

$$Q = (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \chi (d^\dagger \times \tilde{d})^{(2)}, \quad (3)$$

$s^\dagger$ (s), $d^\dagger$ (d) and $a_j^\dagger$ (a_j) are creation (annihilation) operators for s-bosons, d-bosons and fermions respectively, χ stands for normal ordering. In the SU(3) limit $\chi = -\sqrt{7}/2$. The coupling parameters are

$$\Gamma = \sqrt{5} \Gamma_0 (u^2 - v^2) \langle j \| Y_2 \| j \rangle, \quad \Lambda = -2\sqrt{5} \Lambda_0 (2uv \langle j \| Y_2 \| j \rangle)^2 / \sqrt{2j+1}, \quad (4,4')$$

and u , v have the same meaning as in the BCS theory [5], j is the particle angular momentum.

The first term in (2) represents a quadrupole interaction between the core and the single particle. The second, exchange, term has been interpreted as being caused by the effect of the Pauli principle acting

between identical nucleons on the quadrupole interaction between protons and neutrons [7,8] The general core hamiltonian of the dynamic symmetry $U(6) \supset SU(3) \supset O(3)$ has the form [9]

$$H = -2k \sum_{i,j} Q_i Q_j + k' \sum_{i,j} L_i L_j,$$

where Q_i and L_i are the boson quadrupole and angular momentum operator, respectively It is conventionally denoted

$$H = -2kQ Q + k'L L \quad (5)$$

Its eigenvalues are

$$E_c = \alpha L(L+1) + \beta C(\lambda, \mu), \quad \alpha = \frac{3}{4}k + k', \quad \beta = -k,$$

where $C(\lambda, \mu)$ is the Casimir operator of $SU(3)$ We will consider only the lowest $(2N,0)$ representation, N is the boson number Since we are interested only in the excitation energy and we consider only the coupling of the particle to the ground state band (gsb) of the core, we can write

$$E_c = \alpha L^2$$

Since the core angular momentum L is not a constant of motion of the odd- A nucleus, we must carry out the substitution

$$L = I - J, \quad (6)$$

where I is the total angular momentum This substitution has originally been used in the rotor plus particle model [5]

The eigenstates of the core plus particle system can be written in the asymptotic form valid for large N [7]

$$| [N], (2N, 0), K_c = 0, J, K, IKM \rangle \\ = \sqrt{2} \sum_L \sqrt{2L+1} \begin{pmatrix} J & L & I \\ K & 0 & -K \end{pmatrix} | [N], (2N, 0), K_c = 0, L, J, IM \rangle \quad (7)$$

where K_c and K are the projections on the intrinsic three-axis of L and J , respectively, and I is the total angular momentum

The matrix elements of V_{BF} in this basis have been calculated in ref [7]

$$\langle [N], (2N, 0), K_c = 0, J, K, IKM | V_{BF} | [N], (2N, 0), K_c = 0, J, K, IKM \rangle \\ = -2N \left\{ B_J (\Gamma/\sqrt{2}) [3K^2 - J(J+1)] + \frac{1}{3} \Lambda \sqrt{2J+1} B_J^2 [3K^2 - J(J+1)]^2 \right\}, \quad (8)$$

with

$$B_J = [(2J-1)J(2J+1)(J+1)(2J+3)]^{-1/2}$$

For an empty shell $v^2 = 0$ and consequently $\Lambda = 0$ The only term left is the quadrupole interaction, which splits particle levels with different K 's and is analogous to the deformed field (Nilsson splitting) A comparison with the Nilsson model shows that at small deformations the product $N\Gamma$ is proportional to the deformation parameter δ Both Γ_0 and Λ_0 are proportional to the strength of the effective proton-neutron interaction and are positive [7] These parameters are sensitive to the filling of the shell In particular, Γ changes sign at mid-shell, this corresponds to the transition from a particle to a hole state

The parameter Λ of the second, exchange, term does not change sign throughout the shell and since $\Lambda_0 > 0$, Λ is negative. In the case of a prolate nucleus with $u^2 > v^2$, Γ is also negative and the deformation term has its smallest value for $K = 1/2$, if Λ vanishes. On the contrary, if $\Lambda \neq 0$ the levels with small K are pushed up and the minimum will be reached for a value $K_{\min} > 1/2$. The effect of increasing $|\Lambda|$ is therefore equivalent to that of increasing the Fermi energy although there is no direct connection between BCS pairing and the exchange term. A further similarity arises from the action of the factor $(u^2 - v^2)$ on the quadrupole term. If the shell is neither empty nor full, this factor is smaller than unity and the quadrupole splitting is attenuated.

The hamiltonian of the $SU(3) \otimes$ particle system can be written

$$H = \alpha [I^2 + J^2 - 2K^2] - \alpha (I_+ J_- + I_- J_+) - 2N \left\{ (\Gamma/\sqrt{2}) B_j [3K^2 - J(J+1)] + \frac{1}{3} \Lambda \sqrt{2J+1} B_j^2 [3K^2 - J(J+1)]^2 \right\} + \epsilon_j, \quad (9)$$

where ϵ_j is the single particle energy.

The diagonal matrix element of this hamiltonian has the form

$$\begin{aligned} & \langle [N], (2N, 0), J, K, IK | H | [N], (2N, 0), J, K, IK \rangle \\ &= \alpha [I(I+1) + J(J+1) - 2K^2] + \delta_{K1/2} a (-1)^{I+1/2} (I+1/2) \\ & - 2N \left\{ (\Gamma/\sqrt{2}) B_j [3K^2 - J(J+1)] + \frac{1}{3} \Lambda \sqrt{2J+1} B_j^2 [3K^2 - J(J+1)]^2 \right\} + \epsilon_j \end{aligned} \quad (10)$$

The decoupling parameter a appears on the same ground as in the rotational model [5]. Its main effect is the signature splitting. In the single- j case it has the value

$$a = (-1)^{J-1/2} \alpha (J+1/2)$$

Since the single particle energy represents an additive constant we can discard it. The operators $I_{\pm}, J_{\pm}$ are defined as usual [5]. Obviously the Coriolis coupling is produced by the nondiagonal operator $I_+ J_- + I_- J_+$.

If the energy intervals of the core are small compared to those of levels with different K 's, the Coriolis term will be negligible. This is the strong coupling limit. We will assume prolate deformation and a nearly empty shell. Bands of rotational type will be based on each K -level, the one with $K = 1/2$ being the lowest.

If we now increase α , the nondiagonal matrix elements will be able to mix states with $\Delta K = 1$. At a certain stage the $I = J$ level will have the lowest energy.

This is the typical decoupled or rotation aligned situation. The lowest states with $I = J$ even form the favoured band (fig. 1a). The first unfavoured band has levels with $I = J$ odd and is higher in energy. In a different representation each band is characterized by the projection of J along the perpendicular (ox) axis, the most favoured one having the maximum projection [6].

If we now start filling the shell, i.e., $u^2 < 1$, $v^2 \neq 0$ the absolute value of Γ decreases while $|\Lambda|$ increases. As shown before, the energies of the intrinsic states with small K increase and consequently the corresponding amplitudes in the wave functions decrease. This can be best seen if we look at the wave functions in the basis where K is a good quantum number. Let us compare the distributions of the squared amplitudes of K -components (fig. 2) of the favoured $9/2$ and the unfavoured $11/2$ states. In the decoupled case $\Lambda = 0$ the favoured state has a strong $K = 1/2$ component as a result of alignment, while the unfavoured one has strong components with $K = 3/2$ and $5/2$. If we now increase $|\Lambda|$ the decrease of the $K = 1/2$ component strongly modifies the structure of the favoured states. At the same time the structure of the $11/2$ state is not drastically changed. The favoured band moves up compared to the unfavoured one (figs. 1b, 1c). The exchange term produces a decrease of alignment. This manifest itself through the

Fig 1 Theoretical energy levels $N=6$, $j=9/2$, $\alpha=0.1$ MeV, $\Gamma=-1.5$ MeV (a) $\Lambda=0$, (b) $\Lambda=-3$ MeV, (c) $\Lambda=-5$ MeV

lowering of the $11/2$ and $7/2$ states. Low-energy states with $I=j-1$ (in our case $I=7/2$) have often been seen in transitional nuclei [2] as well as in some shell-model nuclei [10]. Moreover, this phenomenon can also be explained by coupling the particle to a triaxial rotor [11].

In order to test the validity of the large- N approximation a numerical calculation has been carried out using the IBFM-code [12]. The parameter k of eq (5) has been chosen artificially large, so as to preclude any influence of the $(2N-4,2)$ representation on the low-energy states. The results of this calculation (fig 3) is quite similar to that obtained from the asymptotic calculation, especially for the favoured band. The decoupled features are again present.

Fig 2 K -amplitudes of the wave functions of states with $I=j$, and $I=j+1$ in cases (a) and (c) of fig 1

Fig 3 Effect of the large- N approximation on the favoured and first unfavoured bands (a) Eigenvalues of the exact IBFM hamiltonian [12] (b) Eigenvalues of the approximate hamiltonian (9) In both cases $J = 9/2$, $N = 6$, $\alpha = 0.1$ MeV, $\Gamma = -1.5$ MeV, $\Lambda = -3$ MeV

The question arises to what extent the $(2N-4,2)$ β - and γ -bands of the core can be ignored. The quadrupole term can not mix these bands with the gsb since Q is an $SU(3)$ generator. Therefore, in the $v=0$, RA limit there will be no mixing. Some of the multipole components of the exchange term can induce this mixing and the effect can become relevant towards the middle of the shell. However, this mixing is negligible in the large- N approximation, since the matrix elements of the exchange term connecting the lowest $SU(3)$ representations vanish [7]. It would be desirable to investigate this effect in

Fig 4 Positive parity, favoured and first unfavoured levels in ^{73}As (a) Experiment (from ref [12]) (b) Present calculation $J = 9/2$, $N = 7$, $\alpha = 0.1$ MeV, $\Gamma = -1.4$ MeV, $\Lambda = -3.1$ MeV (c) Particle plus rotor calculation (from ref [14])

the general case. The mixing of the gsb with states based on the two-boson 0^+ state of the $U(5)$ basis has been investigated in ref. [13].

We took ^{73}As as a specific example. This nucleus, which displays typical RA, has been previously described [14] by means of the rotor plus particle model. A comparison with a calculation using the hamiltonian (9) is shown in fig. 4, together with the result of a particle plus rotor calculation. As a consequence of using a constant inertial parameter α , the increase of excitation energy with angular momentum is too sharp. Although ^{73}As is certainly not an $SU(3)$ nucleus, it has been chosen considering its rotation aligned character.

A comparison of the Nilsson model with the $U(6|12)$ Bose-Fermi symmetry, including the Coriolis mixing has been carried out in refs. [15,16]. Exchange effects in the IBFM have recently been investigated in ref. [17].

In summary, we have shown that the Coriolis interaction appears explicitly in the IBFM in the same way as in the rotational model. The expression for the energies in the large- N limit of the IBFM for the coupling of a particle to an $SU(3)$ core has been compared with the results of an exact numerical calculation. In this case there is a strong resemblance to the particle plus rotor model. It has been shown that the exchange term causes a decrease of alignment.

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