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Dieperink, AEL; Deguerra, EM

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ON MAGNETIC DIPOLE FORM FACTORS IN DEFORMED NUCLEI

A.E.L. DIEPERINK

*Kernfysisch Versneller Instituut, 9747 AA Groningen, The Netherlands
and NIKHEF/K, P.O. Box 4395, 1009 AJ Amsterdam, The Netherlands*

and

E. MOYA DE GUERRA

Instituto de Estructura de la Materia, 119 Madrid 6, Spain

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Properties of magnetic dipole form factors for deformed nuclei are discussed in terms of the angular momentum projected Hartree-Fock-Bogoljubov approximation and the neutron-proton interacting boson model. It is pointed out that there exists a relation between the M1 form factor for the excitation of the orbital $K^\pi = 1^+$ band, the M1 form factor for the ground-state band, and the collective M1 form factor in odd- A nuclei.

1. Introduction. One of the most interesting recent developments in the study of nuclear collective properties has been the observation of a collective magnetic dipole mode with a predominantly orbital character in several deformed nuclei [1,2]. Clearly, measurements of M1 form factors and strength distributions are of interest since these provide information on the nature of the convection current in deformed nuclei, which are sensitive to details of nuclear collective models [3].

The aim of this letter is two-fold. First assuming an isovector orbital excitation we show that the M1 form factor from the ground state to the $K^\pi = 1^+$ band can be expressed in terms of M1 properties of the ground-state band. Secondly in deriving quantitative results we compare the predictions of two approaches with conceptually rather different foundations, namely the angular momentum projected Hartree-Fock-Bogoljubov method and the neutron-proton interacting boson model. While we find a remarkable similarity in the structure of the expressions for the M1 matrix elements, quantitatively the predictions appear to differ appreciably.

2. M1 form factors in the PHFB approach. In the

projected Hartree-Fock (PHF) approach one assumes that the various angular momentum eigenstates of a given rotational band $|IKM\rangle$ can be obtained from a single intrinsic Slater determinant Φ_K ,

$$|IKM\rangle = \frac{1}{2\mathcal{N}_{IK}} \int d\Omega [D_{KM}^\dagger(\Omega) \Phi_K + (-)^{I-K} D_{KM}^\dagger(\Omega) \Phi_K], \quad (1)$$

where Φ_K is the time-reversed state, and $\mathcal{N}_{IK}$ a normalization factor (the notation and conventions used here are the same as in ref. [3]). This approximation has been shown to provide a good description of form factor for transitions within the ground-state band of strongly deformed nuclei [4], where an expansion in powers of $1/\langle J_\perp^2 \rangle$ converges rapidly. In particular the M1 form factor for transitions $IK \rightarrow IK$ in the ground-state band of even-even nuclei ($K=0$) is given to first order in $1/\langle J_\perp^2 \rangle$, by

$$[F^{M1}(q)]_{I0 \rightarrow I0} = \langle I0 \| \hat{T}^{M1}(q) \| I0 \rangle / \sqrt{2I+1} \\ = \sqrt{2I(I+1)} \langle I11-1 | I0 \rangle F_R^{M1}(q), \quad (2)$$

where $F_R^{M1}(q)$ is the intrinsic M1 multipole [3],

$$F_R^{M\lambda}(q) = -\sqrt{2} \operatorname{Re} \langle \Phi_0 | \hat{T}_1^{M\lambda}(q) J_- | \Phi_0 \rangle / \langle J_-^2 \rangle \quad (3)$$

for $\lambda=1$. The form factors in eq. (2) of course cannot be measured directly by elastic (or inelastic) scattering from the nuclear ground state. The only information available are the static magnetic dipole moments of excited states, which are given by $\mu_{J0} = g_R I$, where g_R is related to the long-wavelength limit of $F_R^{M1}(q)$

$$g_R = -\sqrt{6\pi} \lim_{q \rightarrow 0} (2m/q) F_R^{M1}(q). \quad (4)$$

In general the spin magnetization contribution to eq. (3) is found to be quite unimportant for small q ($\sim 2\%$), and in that case

$$g_R \simeq \langle J_-^2 \rangle_\pi / \langle J_-^2 \rangle, \quad (5)$$

where $\langle J_-^2 \rangle = \langle J_-^2 \rangle_\pi + \langle J_-^2 \rangle_\nu$, $\langle J_-^2 \rangle_\rho = \langle \Phi_0 | J_\rho^2 | \Phi_0 \rangle$. This expression leads to g_R values in fair agreement with experiment for rare-earth nuclei [5]. Clearly, it is of interest to test the M1 form factors at finite q values which contain more detailed information about the current distribution. We note that the intrinsic collective M λ form factor (3) plays a role in two related experimentally accessible transitions:

(i) in odd- A nuclei where the magnetic form factors can be measured within the ground-state band. However, in this case the transition multipoles for larger q values are dominated by the contribution of the odd nucleon [6], and therefore no quantitative test of the $F_R^{M1}(q)$ is possible.

(ii) in collective isovector M1 transitions to $K^\pi = 1^+$ bands which have recently been observed in rare-earth nuclei. This is obviously the case if one assumes that the collective 1^+ state is the band head of a rotational band with angular momentum states constructed as in eq. (1) from an intrinsic $K^\pi = 1^+$ state, $|\Phi_1\rangle \sim (J_{\nu,+} - J_{\pi,+}) |\Phi_0\rangle$. Actually to avoid contributions from the spurious intrinsic state, $\Phi_1^S \equiv J_+ |\Phi_0\rangle$, the physical intrinsic state for the $K^\pi = 1^+$ band should be defined such that Φ_1 is orthogonal to Φ_1^S :

$$|\Phi_1\rangle = \mathcal{N}_1 (\langle J_-^2 \rangle_\pi J_\nu - \langle J_-^2 \rangle_\nu J_{\pi,+}) |\Phi_0\rangle, \quad (6)$$

where $\mathcal{N}_1$ is a normalization factor. Note that $|\Phi_1\rangle$, similarly to $|\Phi_0\rangle$, need not be an eigenstate of the

nuclear hamiltonian. It represents an intrinsic state in which the neutrons are rotated with respect to the protons. By construction it is non-spurious.

The matrix elements of the M λ operators for transitions from the ground state (0^+) to the $K^\pi = 1^+$, $I=\lambda$ state can be calculated with the wave functions in (1) using the techniques described in ref. [3]. To lowest order in $1/\langle J_-^2 \rangle$ one obtains

$$\begin{aligned} \langle I1 || \hat{T}^{M\lambda} || 00 \rangle &= \delta_{I,\lambda} \operatorname{Re} (\langle J_-^2 \rangle_\pi \langle \Phi_0 | \hat{T}_1^{M\lambda} J_{\nu,-} | \Phi_0 \rangle \\ &- \langle J_-^2 \rangle_\nu \langle \Phi_0 | \hat{T}_1^{M\lambda} J_{\pi,-} | \Phi_0 \rangle) \\ &\times (\langle J_-^2 \rangle_\nu \cdot \langle J_-^2 \rangle_\pi \cdot \langle J_-^2 \rangle / 2)^{-1/2}. \end{aligned} \quad (7)$$

(We note that the same result is obtained if the factorization approximation in the Bohr-Mottelson collective model is used.) The similarity of the matrix elements in eqs. (3) and (7) is apparent. If we write the M λ operator as

$$\hat{T}_\mu^{M\lambda}(q) = \sum_{\substack{\alpha=l,s \\ \rho=\pi,\nu}} g_{\rho\alpha} (\hat{T}_\mu^{M\lambda}(q))_{\rho,\alpha},$$

where l and s stand for convection and magnetization current, respectively, we see that the intrinsic M λ multipoles of the ground-state band have the structure

$$F_R^{M\lambda}(q) = \sum_{\alpha,\rho} g_{\rho\alpha} (F_R^{M\lambda}(q))_{\rho,\alpha}, \quad (8)$$

while the reduced matrix elements in eq. (7) have the form

$$\begin{aligned} \langle I1 || \hat{T}^{M\lambda}(q) || 00 \rangle &= \delta_{I,\lambda} [\langle J_-^2 \rangle / (\langle J_-^2 \rangle_\nu \langle J_-^2 \rangle_\pi)]^{1/2} \\ &\times \sum_{\alpha} [\langle J_-^2 \rangle_\nu g_{\pi,\alpha} (F_R^{M\lambda}(q))_{\pi,\alpha} \\ &- \langle J_-^2 \rangle_\pi g_{\nu,\alpha} (F_R^{M\lambda}(q))_{\nu,\alpha}]. \end{aligned} \quad (9)$$

If we furthermore assume that the $K^\pi = 1^+$ excitation is a pure orbital mode (as suggested by the experiments [1,2]) and use bare g_l values we can express the form factor (7) in terms of that for the ground-state band:

$$\begin{aligned} \langle I1 \| \hat{T}^{M\lambda}(q) \| 00 \rangle \\ = \delta_{I,\lambda} (\langle J_{\perp}^2 \rangle_{\pi} \langle J_{\perp}^2 \rangle_{\nu} / \langle J_{\perp}^2 \rangle_{\pi})^{1/2} (F_R^{M\lambda}(q))_{\pi I}. \end{aligned} \quad (10)$$

In particular one finds for the $B(M1)$ value

$$\begin{aligned} B(M1, 0^+ \rightarrow 1^+) \\ = \frac{3}{4\pi} \frac{\langle J_{\perp}^2 \rangle_{\pi} \langle J_{\perp}^2 \rangle_{\nu}}{\langle J_{\perp}^2 \rangle} [\mu_N^2]. \end{aligned} \quad (11)$$

3. Magnetic dipole properties in the IBA-2 model. The neutron-proton interacting boson (IBA-2) model has been shown to provide a simple description of many collective properties of even-even heavy nuclei in terms of a few parameters [7,8]. Moreover, using the underlying microscopic basis of the IBA model [9], namely the shell model, one is able to relate the phenomenological parameters with more fundamental ones.

Although the IBA model is formulated in the laboratory frame for axially symmetric deformed nuclei, it can be illuminating to work in the intrinsic frame, in which case the intrinsic ground-state wave function [10] can be regarded as the boson image of the number projected HFB wave function:

$$P_N \Phi_0^{\text{HFB}} = \mathcal{N}_F \prod_{\rho} (\Gamma_{\rho,0}^{\dagger})^{N_{\rho}} |0\rangle. \quad (12)$$

Here the pair creation operators $\Gamma_{\rho,0}^{\dagger}$ can be expanded into pairs with good angular momentum, $\Gamma_{\rho,0}^{\dagger} = \sum x_{\rho,\lambda} \Gamma_{\rho,0,\lambda}^{\dagger}$, where the multipole operators $\Gamma_{\rho,0,\lambda}^{\dagger}$ create correlated pairs of fermions coupled to given λ . Assuming that only pairs with $\lambda=0$ and 2 are important [11] ("SD dominance") the state (12) is mapped onto a corresponding boson condensate

$$\Phi_0^B \approx \mathcal{N}_B \prod_{\rho} (s_{\rho}^{\dagger} + \beta_{\rho} d_{\rho,0}^{\dagger})^{N_{\rho}} |0\rangle. \quad (13)$$

Boson images of fermion operators can be constructed by a simple mapping procedure [12]. In particular the magnetic dipole operator is mapped onto the one-boson operator $T^{(B)}(M1) = \sqrt{3/4\pi} (g_{\pi}^B I_{\pi}^{(1)} + g_{\nu}^B I_{\nu}^{(1)})$, where $I_{\rho,\mu}^{(1)} = \sqrt{10} (d_{\rho}^{\dagger} \tilde{d}_{\rho})_{\mu}^{(1)}$. An estimate for the boson g -factors, g_{ρ}^B , can be obtained by equating selected matrix elements in the fermion and boson spaces. For example, the simplest method is to put $n_d=1$, and require ($\rho=\pi, \nu$)

$$\begin{aligned} \langle S^{N-1} D \| T^{(F)}(M1) \| S^{N-1} D \rangle_{\rho} \\ \equiv \sqrt{3/4\pi} \sqrt{10} g_{\pi}^B \langle s^{n-1} d \| (d^{\dagger} \tilde{d})^{(1)} \| s^{n-1} d \rangle_{\rho}. \end{aligned} \quad (14)$$

Like in the PHF case it is found that due to the collective nature of the S and D pairs the spin magnetization contribution is small and thus $g_{\nu}^B \sim 0$.

Within the framework of IBA-2 simple analytic results can be derived for the M1 matrix elements [13]. First for low-lying states that are totally symmetric in the neutron-proton degree of freedom (possess maximum F -spin [9]) one has

$$g_R = (g_{\pi}^B N_{\pi} + g_{\nu}^B N_{\nu}) / N \sim g_{\pi}^B N_{\pi} / N. \quad (15)$$

It has been shown [14] that this simple expression describes magnetic moments of 2_1^+ in the rare-earth nuclei quite well with a constant N independent value of $g_{\pi}^B \sim 0.63 \mu_N$ and $g_{\nu}^B \sim 0$.

Comparing eqs. (15) and (5) we note that the PHFB result (5) is based upon a mean field approximation in which in principle all nucleons participate in the collective motion; shell effects and pairing correlations tend to reduce g_R from the extreme value Z/A to close to the experimental value. On the other hand the IBA-2 result, in which it is assumed that only the valence pairs contribute, agrees with experiment only if an appreciable overall reduction of g_{π}^B is introduced [14], indicating that in strongly deformed nuclei there is a non-negligible core contribution to g_R . (This is also suggested by the fact that for ^{156}Gd the PHF value for the ratio $\langle J_{\perp}^2 \rangle_{\pi} / \langle J_{\perp}^2 \rangle_{\pi+\nu} \sim 0.44$ is much smaller than the IBA value $N_{\pi}/N = 7/12$.)

Secondly in IBA-2 the $K^{\pi}=1^+$ band emerges naturally as an isovector orbital M1 excitation:

$$|\Phi_1\rangle = \mathcal{N}_1 (N_{\pi} J_{\nu,1}^{(1)} - N_{\nu} J_{\pi,1}^{(1)}) |\Phi_0\rangle_B. \quad (16)$$

In the $SU(3)$ limit $|\Phi_1\rangle$ is an eigenstate that exhausts all M1 strength. The $B(M1)$ sum rule [8]

$$B(M1, 0^+ \rightarrow 1^+) = \frac{3}{4\pi} \frac{4N_{\pi}N_{\nu}}{N} (g_{\pi} - g_{\nu})^2 \quad (17)$$

can be regarded as the analogue of eq. (11). The M1 form factor can be obtained by generalizing the mapping (14) to $q>0$. One then obtains the IBA-2 analogue of (10) with the important difference that in

Fig. 1 Comparison of the M1 form factor in PWBA for the $0^+ \rightarrow 1^+$ transition in ^{156}Gd obtained in the IBA-2 (---) and PHF (—) models

IBA only the valence protons in the 50–82 shell are assumed to participate. This difference shows up both in the total M1 sum rule strength and in the form factor which is shown in fig. 1 in PWBA. For example, in ^{156}Gd the $B(\text{M}1)$ strength in the PHFB approach eq. (11) amounts to $7 \mu_N^2$, which is similar to the results of the more schematic approach of ref. [15], whereas the IBA results is $2.8 \mu_N^2$ (for $g_\pi^B \sim 1, g_\nu^B \sim 0$), which is closer to the observed strength [1,2]. However, it should be noted that it is quite well possible that additional fragmented orbital M1 strength too weak to be detected is present at higher excitation energies (as predicted by QRPA calculations [16]).

In conclusion we would like to stress that the PHFB and the IBA predictions for the total $B(\text{M}1)$ strengths and the form factors are qualitatively similar in the

sense that effectively only neutrons and protons outside closed shells contribute. However, quantitatively the two model predictions are different because of the higher angular momentum pairs and shell mixing effects present in the intrinsic HFB wave function. As a result the PHF form factor (calculated using the DME effective hamiltonian as in refs. [5,6]) is seen to fall off faster with q than the IBA one, indicating a current distribution located more to the outside in coordinate space.

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