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DYNAMICAL MASS GENERATION IN SUPERSYMMETRIC QED$_3$

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We investigate the possibility of dynamical generation of a mass for the matter fields in supersymmetric QED$_3$ with N flavours. We conclude that mass generation occurs when the number of matter flavours is less than the critical value $N_c = 3.24$. The effect of supersymmetry is that a potentially dangerous contribution to the wave-function renormalization, which may spoil a similar analysis in QED$_3$, is cancelled.

Dynamical generation of masses for the matter fields in supersymmetric gauge theories is a subject of considerable interest. It has been shown [1] that in four dimensional supersymmetric QED (SQED$_4$) dynamical mass generation does not occur, due to the existence of a non-perturbative version of a mass renormalization theorem. In supersymmetric QED in three dimensions (SQED$_3$), however, such a non-renormalization theorem does not exist if the model contains only one two-component supersymmetry parameter ($n=1$). It is therefore interesting to study mass generation in this model and to compare the results with SQED$_4$ and non-supersymmetric QED$_3$. Our main result is that in $n=1$ massless SQED$_3$ the matter fields acquire a mass if the number of matter flavours does not exceed a certain critical value.

With respect to dynamical mass generation in non-supersymmetric massless QED$_3$ a controversy has arisen. A study by Appelquist et al. [2] revealed that the fermions remain massless if there are more than three four-component fermion flavours. The approach of ref. [2] was criticized by Pennington et al. [3]. There it was found that for any number N of flavours the fermions acquire a mass. The disagreement can be traced back to a non-uniformity in the $1/N$-expansion of the electron propagator wave-function renormalization. This observation is also important for the study of a supersymmetric version of QED$_3$. Before addressing the supersymmetric model explicitly, we first summarize the reasoning of refs. [2,3]. To stay in contact with our $n=1$ supersymmetric model, we use a two-dimensional representation for the γ-matrices instead of the four-dimensional one of refs. [2,3].

In ref. [2] the dynamical mass of the fermions is determined from the Dyson–Schwinger equation for the electron propagator. In this equation, the full electron–photon vertex is approximated by its bare value, while in the full photon propagator one-loop quantum corrections are taken into account. With these approximations, the Dyson–Schwinger equation is still exact to order $1/N$ in the $1/N$-expansion. In the Landau gauge the photon propagator then reads

$$D_{\mu\nu}(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+c/\sqrt{-k^2}}, \qquad (1)$$

where we have introduced $c=e^2N/16$, which is constant in the $1/N$-expansion. The full electron propagator can be expressed in terms of the two scalar functions $\alpha(p)$ and $\beta(p)$:

$$S(p) = \frac{\not p\beta(p)+\alpha(p)}{p^2\beta^2(p)-\alpha^2(p)}. \qquad (2)$$

The Dyson–Schwinger equation can be rewritten as a set of coupled equations for α and β. After angular integration the Wick-rotated equations read

$$\alpha(p) = \lambda c \int_0^\infty \mathrm{d}p'\ K(p,p')\,\frac{\alpha(p')}{p'^2\beta^2(p')+\alpha^2(p')}, \qquad (3a)$$

$$\beta(p)=1-\frac{\lambda c}{8p^2}\int_0^\infty dp'\, L(p,p')\,\frac{p'^2\beta(p')}{p'^2\beta^2(p')+\alpha^2(p')}, \tag{3b}$$

where $\lambda=16/\pi^2N$. The kernels K and L are

$$K(p,p')=\frac{p'}{p}\log\frac{c+p+p'}{c+|p-p'|}, \tag{4a}$$

$$L(p,p')=2\left(1-\frac{c^2+|p'^2-p^2|}{cp_>}\right)+\frac{c^2}{pp'}\log\frac{c+p+p'}{c+|p-p'|}-\frac{(p^2-p'^2)^2}{c^2pp'}\log\frac{1+c(p+p')^{-1}}{1+c|p-p'|^{-1}}, \tag{4b}$$

where $p_>=\max(p,p')$. To obtain the mass function $m(p)=\alpha(p)/\beta(p)$ to order λ, Appelquist et al. ignore wave-function renormalization, i.e. they set $\beta(p)\equiv 1$. Analysis of the remaining equation for $\alpha(p)$ [2,3] shows that there is a critical value $\lambda_c=1/4$ (i.e., $N_c=64/\pi^2\simeq 6.48$). For $N<N_c$ the fermions acquire a mass, while for $N>N_c$ they remain massless. Here N is the number of two-component fermion flavours, which equals twice the number of four-component flavours of ref. [2].

The criticism in ref. [3] is focussed on the approximation $\beta(p)\equiv 1$. This assumption may not be consistent, since the $1/N$-expansion of $\beta(p)$ is not uniform with respect to p. Numerical analysis of the combined equations for $\alpha(p)$ and $\beta(p)$ [3] reveals that β is of order λ instead of order 1 for small p^2. This is a consequence of the fact that part of the integral in (3b) is not of order 1 for small p^2 but of order $1/\lambda$.

To be consistent with the Ward identity for the electron–photon vertex, in ref. [3] the bare vertex is multiplied by a factor $\beta(p')$. With this ansatz, eq. (3b) takes the form

$$\beta(p)=1-\frac{\lambda c}{8p^2}\int_0^\infty dp'\, L(p,p')\,\frac{p'^2}{p'^2+m^2(p')}. \tag{5}$$

The integral in (5) can be done explicitly if $m(p')$ is replaced by $m(0)$. The regions $0<p'<p$ and $p<p'<\infty$ can be treated separately. Expanding the logarithms in terms of $p'/(p+c)$ (if $p'<p$) or $p/(p'+c)$ (if $p'>p$), we see that cancellations occur which remove almost all contributions that diverge for small p^2. Only one term remains that potentially can become of order $1/\lambda$ for small p^2. For $p\ll c$ one obtains

$$\beta(p)=1+\tfrac{1}{6}\lambda\log[p^2+m^2(0)]+\dots, \tag{6}$$

where the omitted terms are of order λ uniformly for all p^2. The numerical results [3] indicate that

$$m(0)=\exp(-3/\lambda). \tag{7}$$

Thus for small p^2 the term of order 1 in (6) is indeed cancelled. [For large p^2, the approximation $\beta(p)=1$ is still appropriate.] The behaviour (7) of $m(0)$ suggests that for any (finite) value of N a non-zero mass exists, in contradistinction to the result of Appelquist et al.

In this letter we will not attempt to resolve this controversy in QED_3. Instead, we concentrate on the supersymmetric extension of QED_3, in which we shall show that the $1/N$-expansion of β causes no problems.

In $n=1$ supersymmetric QED_3 ($SQED_3$), N matter multiplets, each consisting of a complex scalar field ϕ, a Dirac spinor ψ and a complex auxiliary field G, interact with a gauge multiplet that consists of a photon field A_μ and a photino field λ. The lagrangian density is

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}i\bar{\lambda}\not{\partial}\lambda+(D_\mu\phi)^*(D^\mu\phi)+G^*G+i\bar{\psi}\not{D}\psi-ie(\bar{\psi}\lambda\phi-\bar{\lambda}\psi\phi^*), \tag{8}$$

where $D_\mu=\partial_\mu+ieA_\mu$ is the gauge covariant derivative. We add to (8) a gauge fixing term $-(1/2a)(\partial_\mu A^\mu)^2$, in which we take $a\to 0$ (Landau gauge). the action corresponding to (8) is invariant under the supersymmetry transformations parametrized by the two-component Majorana spinor ϵ:

$$\delta_S\phi=\bar{\epsilon}\psi, \tag{9a}$$

$$\delta_S\psi=-(i\not{D}\phi+G)\epsilon, \tag{9b}$$

$$\delta_S G=i\bar{\epsilon}\not{D}\psi, \tag{9c}$$

$$\delta_S A_\mu=i\bar{\epsilon}\gamma_\mu\lambda, \tag{9d}$$

$$\delta_S\lambda=\tfrac{1}{2}\gamma_\mu\gamma_\nu\epsilon F^{\mu\nu}. \tag{9e}$$

the lagrangian density (8) is also invariant under the abelian gauge transformations

$$\delta_G A_\mu=\partial_\mu\Lambda, \tag{10a}$$

$$\delta_G\psi=-ie\Lambda\psi, \tag{10b}$$

$$\delta_G \phi = -ieA\phi\,. \tag{10c}$$

The photino field λ is gauge invariant.

To study dynamical mass generation for the matter fields, we follow the method of refs. [2,3]. In the supersymmetric model, in principle we have to determine both the electron propagator and the scalar propagator. Fortunately, because supersymmetry is not broken in this model [4], both propagators are related and it suffices to solve the Dyson–Schwinger equations for one of those propagators. The other propagator is then determined by supersymmetry. To be specific, consider the general form (2) of the electron propagator. From Lorentz invariance and supersymmetry of the vacuum we obtain the supersymmetry Ward identity

$$\langle 0|\delta_S(\phi(x_1)\bar{\psi}(x_2))|0\rangle = 0\,. \tag{11}$$

Using (2) and (9) we can derive from (11) the general form of the scalar propagator:

$$\Delta(p) = \frac{\beta(p)}{p^2\beta^2(p) - \alpha^2(p)}\,. \tag{12}$$

Both (2) and (12) have the wave-function renormalization factor $1/\beta(p)$, and a pole at $p^2 = m^2(p)$. We will use the form (12) in the derivation of the equations for $\alpha(p)$ and $\beta(p)$.

In SQED$_3$, the full electron self-energy (see fig. 1) consists of two terms: the usual one due to the electromagnetic interaction and one due to the Yukawa coupling. To find $\alpha(p)$ and $\beta(p)$ we approximate the electron Dyson–Schwinger equation in a way analogous to QED$_3$. Again, we replace the full vertices by $\beta(p')$ times their bare values, in accordance with the U(1) gauge Ward identities. Furthermore, as in refs. [2,3] we include one-loop quantum corrections to the inverse gauge propagators. The photon propagator

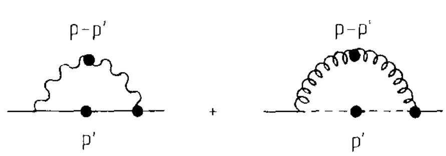

Fig. 1. the full electron self-energy in SQED$_3$. The solid line represents the electron propagator, the dashed line the scalar propagator, the wiggly line the photon propagator and the helical line the photino propagator. the dots represent full quantities.

receives corrections from both spinor and scalar fields, and the resulting vacuum polarization is precisely twice that of QED$_3$. Therefore the photon propagator now takes on the form

$$D_{\mu\nu}(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+2c/\sqrt{-k^2}}\,. \tag{13}$$

this implies that also in the kernel L, eq. (4b), the constant c must be replaced by $2c$ everywhere. Furthermore, the kernel L acquires an additional contribution due to the Yukawa coupling. This reads

$$\Delta L(p,p') = 2\left(\frac{2c}{p_>} - 1\right) - \frac{p^2 - p'^2 + 4c^2}{pp'}\log\frac{2c+p'+p}{2c+|p'-p|}\,. \tag{14}$$

To obtain (14) one requires the form of the photino propagator σ. By supersymmetry this propagator must be corrected in the same way as the photon propagator:

$$\sigma(k) = \frac{\not{k}}{k^2}\frac{1}{1+2c/\sqrt{-k^2}}\,. \tag{15}$$

Adding (14) to (4b) (with c replaced by $2c$), we obtain the complete, supersymmetric kernel L_S:

$$L_S(p,p') = \frac{-|p^2-p'^2|}{cp_>} - \frac{p^2-p'^2}{pp'}\log\frac{2c+p+p'}{2c+|p-p'|} - \frac{(p^2-p'^2)^2}{4c^2pp'}\log\frac{1+2c(p+p')^{-1}}{1+2c|p-p'|^{-1}}\,. \tag{16}$$

Analyzing eq. (5) for $\beta(p)$ with the kernel L_S in the same way as in the non-supersymmetric case, we see that the problematic log-term of (6) is now absent. Obviously it is cancelled by a similar contribution with opposite sign originating from the additional kernel ΔL, eq. (14). The conclusion is that the approximation $\beta(p) \equiv 1$ is a consistent choice in order to find the mass renormalization to order λ.

Thus in the supersymmetric case the problem is reduced to the analysis of eq. (3a) for $\alpha(p)$ with $\beta = 1$. Because the Yukawa term is proportional to γ_μ, the kernel K of this equation for $\alpha(p)$ is the same as in QED$_3$, except that the constant c is again replaced by $2c$ (note that the c in front of the integral does not change). By rescaling the momentum variable in the resulting integral equation one sees that the critical value of the coupling constant λ is twice as large as in QED_3. We conclude that the $SQED_3$ the critical value of the number of matter flavours, below which mass generation occurs, is $N_c = \frac{1}{2} \cdot 64/\pi^2 \approx 3.24$.

We have shown that dynamical mass generation can occur in $n=1$ supersymmetric QED_3. Our critical value of 3.24 two-component fermion flavours corresponds to 1.62 four-component ones. This equals half the critical value for ordinary QED_3, as obtained in ref. [2]. With a two-dimensional representation of the γ-matrices we do not distinguish between parity-conserving and parity-violating masses [5]. However, we do use this representation, because it is the natural choice for the $n=1$ supersymmetric theory. In $n=2$ supersymmetric QED_3 it is known that dynamical fermion mass generation does not occur [6]. In fact, the $n=2$ model considered in ref. [6] follows by dimensional reduction from $SQED_4$, and we already remarked that mass generation is absent in that model [1].

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References

[1] T.E. Clark and S.T. Love, Nucl. Phys. B 310 (1988) 371.
[2] T.W. Appelquist, D. Nash and L.C.R. Wijewardhana, Phys. Rev. Lett. 60 (1988) 2575;
T.W. Appelquist, M.J. Bowick, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. D 33 (1986) 3704.
[3] M.R. Pennington and S.P. Webb, preprint BNL-40886 (Janaury 1988);
D. Atkinson, P.W. Johnson and M.R. Pennington, preprint BNL-41615 (August 1988);
D. Atkinson, in: Proc. 1988 Intern. Workshop on New trends in strong coupling gauge theories (Nagoya, August 1988), eds. M. Bando, T. Muta and K. Yakawaki (World Scientific, Singapore) pp. 24–27.
[4] E. Witten, Nucl. Phys. B 202 (1982) 253;
I. Affleck, J. Harvey and E. Witten, Nucl. Phys. B 206 (1982) 413.
[5] T.W. Appelquist, M.J. Bowick, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. D 33 (1986) 3774.
[6] R.D. Pisarski, Phys. Rev. D 29 (1984) 2423.