The $\Delta$-isobar and proton-proton bremsstrahlung

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Abstract

We investigate the effects of the $\Delta$-isobar on proton-proton bremsstrahlung at a beam energy of 280 MeV. We find significant effects on both cross-section and analyzing power.

The nucleon-nucleon bremsstrahlung ($NN\gamma$) reaction as a tool for investigating the off-shell behaviour of the nucleon-nucleon ($NN$) interaction has recently regained increasing attention both experimentally and theoretically [1–8]. With the development of modern facilities to perform the necessary detailed experiments combined with more sophisticated theoretical models than were available in the past, we are now in a much better position to investigate off-shell effects. In addition to the intrinsic interest in the $NN\gamma$ reaction, the observation of high energy photons produced in intermediate energy heavy ion collisions has also motivated a considerable effort to better understand the elementary $NN\gamma$ process which is the main reaction mechanism for producing hard photons in these collisions [9,10]. The weakness of the electromagnetic interaction, combined with the fact that these photons are very energetic, makes them (the photons) a clean probe of the reaction dynamics. Also, the recent investigations of dilepton production in proton-nucleus collisions [11,12] have shown that the $NN\gamma$ and $\Delta$-decay processes are the dominant reaction mechanisms for producing these dileptons. In this work we investigate the influence of the $\Delta$-isobar degree of freedom ($\Delta$-decay) on the proton-proton bremsstrahlung ($pp\gamma$) reaction.

Most of the modern $NN$ interactions, although they may be based on quite different $NN$ potential models, differ only slightly in their predictions for observables in the bremsstrahlung reaction [4–8]. They are, however, based on purely phenomenological and/or meson exchange descriptions that include either nucleon or nucleon and meson degrees of freedom only. Moreover, the present status of the bremsstrahlung theory, within the framework of nucleon and meson degrees of freedom, leaves relatively little room for higher order corrections. Apart from the two-body current contribution beyond the Soft-Photon Approximation (SPA), most of the higher order corrections which are believed to be most important have been considered in recent calculations [4–7].

In the present work we enlarge our model space by incorporating an additional fermionic degree of freedom: in particular we include the $\Delta$ isobar as an intermediate off-shell state in the description of the $pp\gamma$ process. This problem has been studied
before by other authors [13–15]. Bohannon et al. studied the role of the $\Delta$ isobar in neutron-proton bremsstrahlung. Due to isospin factors the contributions of the $\Delta$ to $np\gamma$ are suppressed as compared to its role in $pp\gamma$. Other authors have addressed the proton-proton bremsstrahlung problem. However, they rely on rather crude approximations. Tiator et al. [14] evaluate the contributions of the radiative $\Delta$-decay in Born-approximation and add these incoherently to the ‘nucleonic’ contributions. The latter were calculated in the SPA. Although Szyjewicz and Kamal [15] evaluate all contributions in Born approximation, they do add these coherently. In the present paper we go beyond these approximations by using an off-shell $NN$ T-matrix for the nucleonic contributions and a $N\Delta$ T-matrix for the $\Delta$ decay diagrams. We show that, even at energies below the pion threshold, the influence of the $\Delta$ isobar on the $pp\gamma$ reaction is considerable and that it cannot be neglected in a quantitative comparison with the data.

Moreover, we point out that the importance of the $\Delta$ is due to strong interference between the dominant $NN\gamma$ current and the $N\Delta\gamma$ magnetic current when energetic photons are produced.

We consider two different $NN$ T-matrices. One is obtained using the Bonn OBEPQ interaction [16,17], the other one is the T-matrix including $\Delta$ degrees of freedom as calculated by ter Haar and Malfliet [18]. This T-matrix calculation elaborates on previous work by van Faassen and Tjon [19]. The $\Delta$ is introduced in the model by including $NA\pi$ and $N\Delta\rho$ vertices. All $\Delta\Delta$-meson vertices were neglected since little is known about their structure and strength. By ignoring these a reasonable fit to the data could be obtained, and it seems unnecessary to introduce additional free parameters by including these vertices. The finite decay width of the $\Delta$, crucial in the description of the inelasticities in the $NN$-channel above pion threshold, is included dynamically in the model by the $\Delta$ self-energy. The $\Delta$ self-energy is approximated by its lowest order contribution, the $\pi N$-loop diagram. This choice gives a good reproduction of the $P_{33}$ phase-shift [18]. All these ingredients are put into a coupled channel calculation, which involves solving a three-dimensional reduction of the coupled Bethe-Salpeter equations. In this procedure only the positive-energy parts of the nucleon and $\Delta$ propagators are retained. The solution of these equations gives simultaneously the $NN-NN$ and the $NN-N\Delta$ scattering matrices which are to be used in the bremsstrahlung calculation. Including $\Delta$ intermediate states the $NN$ T-matrix now describes the $NN$ phase-shifts and cross-sections up to 1 GeV reasonably satisfactorily.

While the $NN\gamma$ coupling is well known, there is some uncertainty about the $N\Delta\gamma$ vertex. Following Jones and Scadron [20] we write the gauge-invariant vertex for the excitation of a nucleon in a $\Delta$ and a photon:

$$\Gamma_{\mu}^{N\Delta\gamma} = K_{\mu}^{1} + K_{\mu}^{2} \quad \text{with}$$

$$K_{\mu}^{1} = ieG_{1}(\bar{\epsilon}_{\mu} - \bar{\epsilon}k_{\mu})\gamma^{5}T_{z}$$

$$K_{\mu}^{2} = ieG_{2}(\epsilon_{\mu}P \cdot k - \epsilon \cdot Pk_{\mu})\gamma^{5}T_{z}. \quad (1)$$

For the decay of a $\Delta$ in a nucleon and a photon we have:

$$\Gamma_{\mu}^{\Delta\gamma} = -K_{\mu}^{1} + K_{\mu}^{2}. \quad (2)$$

In these expressions the index $\mu$ is to be contracted with an index of the $\Delta$ propagator, $k_{\mu} = p_{\mu}^{\Delta} - p_{\mu}^{N}$ is the photon momentum and $P = p_{\mu}^{\Delta} + p_{\mu}^{N}$ is the total momentum. $T_{z}$ is third component of the isospin transition matrix for coupling an isospin $3/2$ to an isospin $1/2$ particle.

The coupling constants $G_{1}$ and $G_{2}$ are conventionally determined by fitting to the $M1^+$ and $E1^+$ multipole data on the photoproduction of pions from nucleons [20–24]. The values obtained depend on the treatment of the non-resonant background contributions. Although this leads to some uncertainty in the values, the parameters found in the literature are not too far apart. It has also been shown [23,24] that in order to accurately reproduce the $M1^+$ multipole data on the pion photoproduction around the resonance energy, one needs energy-dependent couplings $G_{1}$ and $G_{2}$. However, for the energies involved in the bremsstrahlung calculation considered in the present work we can safely ignore this dependence. Bearing in mind that the vertex $K_{\mu}^{1}$ gives the dominant contribution, we can classify the various sets of coupling constants by the magnitude of $G_{1}$. The lowest value is found by Nozawa et al. [21]: $G_{1} = 2.024$ (GeV$^{-1}$) and $G_{2} = -0.851$ (GeV$^{-2}$). Highest values are given by Jones and Scadron [20]: $G_{1} = 2.68$ (GeV$^{-1}$) and $G_{2} = -1.84$ (GeV$^{-2}$) and by Davidson et al. [22]: $G_{1} = 2.556$ (GeV$^{-1}$) and $G_{2} = -1.62$ (GeV$^{-2}$).
An alternative way to extract the \( N\Delta\gamma \) coupling parameters is to assume vector-meson dominance. On the \( N\Delta\gamma \) vertex only the isospin-1 vector meson contributes and the coupling strengths are determined by the ratio of \( g_{\rho NN} \) and \( g_{\rho N\Delta} \). This procedure gives \( G_1 = 2.0 \ (\text{GeV}^{-1}) \) and \( G_2 = 0 \ (\text{GeV}^{-2}) \), comparable with the values from pion photoproduction.

Having defined all interactions we now only have to specify the diagrams we include in our \( ppy \) calculation. In all diagrams we use the same propagator as is used in the calculation of the T-matrices. Following Ref. [5], with \( T_{NN-NN} \) the single scattering (Fig. 1a) as well as the rescattering contribution (Fig. 1b) are calculated. In these contributions the so-called relativistic spin correction is included. With \( T_{NN-N\Delta} \) we calculate the single scattering diagrams shown in Fig. 1c. For simplicity we do not include the rescattering diagrams with \( T_{NN-N\Delta} \). Their purely nucleonic counterparts have been shown to give significant contributions only in a limited sector of the available kinematical range, and we feel that one can estimate the dominant effect of the \( \Delta \)-isobar without these diagrams. Furthermore, exploratory calculations show that these diagrams are small compared to the single-scattering diagrams. All T-matrices are calculated in a plane-wave representation and we calculate the diagrams with \( N\Delta\gamma \) vertices in the same basis. This implies that the relativistic spin correction of these vertices is automatically included. Also we note that with the \( N\Delta\gamma \) vertices given in eqs. (1,2), the diagrams involving \( \Delta \)-isobars in Fig. 1c do not contribute to the SPA, thereby ensuring that the present model does not violate the low-energy theorem [25].

Upon inclusion of \( \Delta \) degrees of freedom we expect to see two types of effects. The most direct one will come from the single scattering diagrams with \( T_{NN-N\Delta} \). A more indirect effect will arise from the different off-shell behaviour of \( T_{NN-NN} \) due to the inclusion of the \( \Delta \)-isobar in the intermediate states. Because of the inclusion of the decay width of the \( \Delta \), \( T_{NN-NN} \) is now inelastic above the pion-threshold. This is essential for the reproduction of the phase-shifts beyond the pion-threshold. It also implies that the off-shell T-matrix will differ from those with only nucleon intermediate states. In inverse scattering theory one can show that T-matrices that are based on local (in each two-body partial wave state) and energy-independent NN potentials and that are on-shell equivalent for all energies will also have the same off-shell characteristics [26]. In spite of the fact that modern NN potentials are non-local, to some extent this property manifests itself in the existing bremsstrahlung calculations. Although off-shell effects are clearly seen, they tend to be rather similar for all ‘realistic’ interactions used, indicating that the off-shell behaviour of these T-matrices does not differ much, at least in the off-shell region sampled by the presently available \( ppy \) data. However, all of these interactions have only nucleon intermediate states and reproduce the (on-shell) phase-shifts up to pion threshold. Above pion threshold they will also be rather similar on-shell, but lacking an inelastic channel they do not reproduce the phase-shifts. However, the T-matrix including \( \Delta \) intermediate states reproduces the phase-shifts beyond pion threshold and is, even in principle, only up to pion threshold on-shell equivalent to the purely nucleonic T-matrices. (However, in practice there are on-shell differences in these type of T-matrices even below pion-threshold.) Using the argument of inverse scattering theory, we expect that the on-shell differences above pion-threshold will be reflected in the off-shell matrix elements below pion-threshold. These differences might show up at points where the off-shell matrix elements are probed by the \( ppy \) reaction.

In Fig. 2 we present a calculation of the cross-section and analyzing power for a selection of kine-
Upon inclusion of the \( \Delta \)-decay diagrams we see a significant increase, up to 35\%, in the cross-section at the intermediate photon angles. This considerable increase in the calculated cross-sections can be traced to a strong constructive interference of the single-scattering \( \Delta \) contributions with the nucleonic contributions. Calculating only the single-scattering diagrams with \( T_{NN-N\Delta} \) gives a cross-section on the order of 10\% of the cross-section arising from the nucleonic diagrams. The magnetic part of the \( NN\gamma \) vertex provides the dominant contribution to \( ppy \) [5]. The \( N\Delta\gamma \) vertex is almost exclusively magnetic and thus interferes strongly with the magnetic contributions of the diagrams with \( T_{NN-NN} \).

To be more specific; as can be seen from the expressions for the vertices Eqs. (1,2), the diagram where the photon is emitted before the strong interaction (pre-emission) has a minus-sign in the \( K^1_\mu \) vertex relative to the diagram where the photon is emitted after the strong interaction (post-emission). Since the \( K^1_\mu \) vertex provides the dominant contribution, the pre-emission and post-emission diagrams tend to have opposite effects; the pre-emission diagram reduces the cross-section, the post-emission diagram increases the cross-section. Although the cross-section calculated with only these diagrams is roughly constant as a function of the photon angle, the degree of the interference with the nucleonic contributions varies. The pre-emission diagrams interfere most strongly at the extreme photon-angles and have almost no effect on the cross-section at the intermediate angles. At intermediate angles the post-emission diagrams do interfere, leading to the observed increase of the cross-section. We note that the photon-energy is high even for geometries with proton angles of \( \theta_1, \theta_2 \, \sim \, 28^\circ \), varying from \( \sim 80 \) to 100 MeV (in the \( NN \) c.m. frame).

Our findings do not agree with those of Szyjewicz and Kamal [15]. We note that we could not reproduce all of their results: we were able to reach satisfactory agreement only for their results which included only pions. We also reproduce their value of the cross-section calculated with only the \( \Delta \)-decay di-
We present in Fig. 2 results for the analyzing power. Again we see substantial effects of the \( \Delta \)-decay diagrams, particularly at the intermediate photon-angles. This is not only due to the interference discussed above: at symmetrical proton-angles the analyzing power vanishes at photon-angles \( \theta = 0 \) and \( \theta = 180 \), consequently any difference will also vanish.

We also point out that the cross-sections calculated with the \( NN \) T-matrix of ter Haar (which contains \( \Delta \)-intermediate states) are lower than the ones calculated with the Bonn T-matrix (see also Fig. 3). Unfortunately it is difficult to discriminate in these results between genuine off-shell differences between the T-matrices (e.g. due to the inclusion of \( \Delta \) intermediate states) and on-shell differences due to differences in the quality of the fit of the phase-shifts up to pion threshold. This difference in the calculated \( ppy \) observables is at least partly due to on-shell differences in partial wave states to which the \( ppy \) process is very sensitive, like the \( ^3P_2-^3F_2 \) state. Note that, as mentioned earlier, differences in the on-shell behaviour of the \( NN \) interactions will be reflected in differences in their off-shell behaviour. In order to disentangle genuine off-shell differences, one needs to improve the quality of the fit of \( NN \) phase-shifts when \( \Delta \)-isobars are included as intermediate states. Note that this problem is not peculiar to the particular interaction we chose, but a rather general feature of \( NN \) interactions which include \( \Delta \) degrees of freedom, since these interactions are usually required to fit \( NN \) phase-shifts up to much higher energies than the pion threshold, while the \( NN \) interactions without the \( \Delta \) fit the phase-shifts only up to pion-threshold.

For a quantitative comparison with the TRIUMF data we want to use a \( NN \) T-matrix that provides the most detailed fit to the \( NN \) scattering data below pion-threshold. As discussed before the \( NN \) T-matrix of ter Haar is imperfect in this respect, and we resort to the T-matrix based on the Bonn OBEPQ potential. This choice also implies that we use the bare \( V_{NN-N\Delta} \) instead of the \( T_{NN-N\Delta} \) for the \( \Delta \)-decay diagrams (Fig. 1c). We expect that this does not lead to qualitative changes in the \( \Delta \)-decay diagrams, because, in contrast to the \( NN \) interaction, short range correlations are much less important for the \( N\Delta \) interactions [27]. We want to stress that in this calculation the only contributions of the \( \Delta \) isobar comes from the \( \Delta \)-decay diagrams. Results of this calculation are presented in Fig. 3. We observe the same trends upon including the \( \Delta \)-decay diagrams as in the previous calculation, although the details differ. At first glance it...
appears that the $\Delta$-contributions improve the description of the analyzing power. However, due to the experimental error-bars we cannot discriminate between the results with and without $\Delta$-decay diagrams. This again stresses the need for high precision data. At the largest proton scattering angle the agreement between calculated and measured cross-section also appears to be improved. Note however that the cross-section data suffer from an ambiguity in the absolute normalization [2].

In conclusion, we investigated the effects of $\Delta$-isobar degrees of freedom in $pp\gamma$ which are significant even at bombarding energies below pion-threshold and, consequently, cannot be ignored in a detailed comparison with the data. We showed that, depending on the choice of the $N\Delta\gamma$ coupling constants and the kinematics the $\Delta$-decay diagrams may enhance the cross-section considerably. We argued that this is due to a sizeable interference of the magnetic contributions from the diagrams with $N\Delta\gamma$ vertices and those with $NN\gamma$ vertices. We also pointed out that the $\Delta$ decay diagrams have a pronounced effect on the analyzing power. Although the results seem to agree better with the data, the error bars are too large to draw definitive conclusions. In addition, for a more stringent test of the role of the $\Delta$ isobar in $NN$ bremsstrahlung one needs to improve the quality of the fit of the $NN$ phase-shifts when the $\Delta$ isobar is taken into account. Work in this direction is in progress.

Future high-precision experiments are crucial in order to better test our model for the $pp\gamma$ reaction and, in particular, the off-shell behaviour of the $NN$ interaction when $\Delta$-isobars are taken into account. This is especially important at energies above pion-threshold, where the potential models are on less firm grounds than at lower energies.

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