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Novel soft-photon analysis of $pp\gamma$ below pion-production threshold

M.K. Liou, R. Timmermans, B.F. Gibson

Abstract

A novel soft-photon amplitude is proposed to replace the conventional Low soft-photon amplitude for nucleon-nucleon bremsstrahlung. Its derivation is guided by the standard meson-exchange model of the nucleon-nucleon interaction. The new amplitude provides a superior description of $pp\gamma$ cross section data for kinematic conditions where the Low amplitude disagrees with potential-model calculations.

Bremsstrahlung processes have been used as a tool to investigate electromagnetic properties of resonances, details of reaction mechanisms, and off-shell properties of scattering amplitudes. The most successful example in the first case is the determination of the magnetic moments of the $\Delta^{++}$ ($\Delta^{0}$) from $\pi^+ p\gamma$ ($\pi^- p\gamma$) data in the energy region of the $\Delta (1232)$ resonance [1]. In the study of reaction mechanisms, a well-known example is the extraction of nuclear time delays from the $p^112C\gamma$ data near the 1.7 MeV resonance [2]; time delay can distinguish between direct and compound nuclear reactions. The initial goal of nucleon-nucleon bremsstrahlung investigations was to distinguish among various phenomenological potential models of the fundamental two-nucleon interaction. Most existing $pp\gamma$ cross sections can, in fact, be reasonably described by potential-model calculations; however, the difference between predictions from any two realistic potentials appears to be too small to be distinguished by the data. Thus, $pp\gamma$ studies provide an ideal testing ground for calculation procedures to be used to investigate the $np\gamma$ process, where meson-exchange currents play a significant role. Hard-photon production in heavy-ion collisions is dominated by such $np\gamma$ processes.

For more than 30 years, the conventional Low soft-photon amplitude [3] was widely used for studying nuclear and particle bremsstrahlung processes. It provides a good description of the data for some processes. For instance, Nyman [4] and Fearing [5] used the Low amplitude to calculate $pp\gamma$ cross sections which were in reasonable agreement with several measurements and potential-model calculations. However, it was recently pointed out by Workman and Fearing [6] that the results from this conventional Low amplitude differ significantly from the potential-model calculations for the TRIUMF data at 280 MeV [7].

The primary purpose of this communication is to propose a new soft-photon amplitude to replace the conventional Low prescription. This novel amplitude is relativistic, manifestly gauge invariant, and consistent with the soft-photon theorem. It belongs to one of the two general classes of recently derived soft-photon amplitudes [8]. Its derivation is guided by the structure of the standard meson-exchange model of the two-nucleon interaction. We demonstrate that the...
ppy cross section data from low energies to energies near the pion-production threshold can be consistently described by this new amplitude. We compare results for the Low amplitude with those from our new soft-photon amplitude as a function of the incident proton energy and the proton scattering angles. Also, we explore why the conventional Low amplitude works for some cases but fails for others.

In order to elucidate these points, let us consider photon emission accompanying the scattering of two spin-1/2 particles A and B.

\[ A(q_f^\mu) + B(p_f^\mu) \rightarrow A(q_f^\mu) + B(p_f^\mu) + \gamma(K^\nu). \]  

Here, \( q_f^\mu (p_f^\mu) \) and \( p_f^\mu (p_f^\mu) \) are the initial (final) four-momenta for particles A and B, respectively, and \( K^\nu \) is the four-momentum for the emitted photon with polarization \( \varepsilon^\alpha \). Particle \( A (B) \) is assumed to have mass \( m_A (m_B) \), charge \( Q_A (Q_B) \), and anomalous magnetic moment \( \kappa_A (\kappa_B) \). For process (1), we can define the following Mandelstam variables:

\[ s_1 = (q_i + p_i)^2, \quad s_f = (q_f + p_f)^2, \quad t_q = (q_f - q_i)^2, \quad t_p = (p_f - p_i)^2, \quad u_1 = (p_f - q_i)^2, \quad \text{and} \quad u_2 = (q_f - p_i)^2. \]

Since a soft-photon amplitude depends only on either \((s, t)\) or \((u, t)\), chosen from the above set, we can derive two distinct classes of soft-photon amplitudes: \( M^{(1)}_{\mu} (s, t) \) and \( M^{(2)}_{\mu} (u, t) \) [8]. The general amplitude from the first class is the two-s-two-i special (TSTTs) amplitude \( M^{TSTTs}_{\mu} (s_1, s_f; t_q, t_p) \); that from the second class is the two-u-two-i special (TuTs) amplitude \( M^{TuTs}_{\mu} (u_1; u_2; t_q, t_p) \). The distinguishing characteristics of these amplitudes come from the fact that they are evaluated at different elastic-scattering or on-shell points (energy and angle). The soft-photon theorem does not specify how these on-shell points are to be selected.

A procedure for deriving these soft-photon amplitudes is described in detail in Ref. [8]. The fundamental tree diagrams of the underlying elastic-scattering process play an important role in this procedure for deriving the two general amplitudes. Therefore, we argue that \( M^{TSTTs}_{\mu} \) should be used to describe those processes which are resonance dominated [such as \( p^+ \gamma \) near 1.7 MeV and \( \pi^+ \gamma \) in the \((1232)\) region], whereas \( M^{TuTs}_{\mu} \) should be used to describe those which are exchange-current dominated (such as the \( np\gamma \) process). For the \( np\gamma \) process, which exhibits neither strong resonance characteristics nor significant enhancement due to \( u\)-channel exchange-current effects, either amplitude can be used in theory, although this has never been tested in conjunction with experimental data. We provide here the results of such an analysis. We emphasize that the general amplitude \( M^{TSTTs}_{\mu} \) (not \( M^{TuTs}_{\mu} \)) arises naturally for nucleon-nucleon bremsstrahlung if the derivation is guided by the standard meson-exchange model of the two-nucleon interaction.

The amplitude \( M^{TuTs}_{\mu} \) for the \( np\gamma \) process can be written in terms of five invariant amplitudes \( F^\alpha_\mu (\alpha = 1, \ldots, 5) \) as

\[ M^{TuTs}_{\mu} = \sum_{\alpha=1}^{5} [Q_{\alpha}(q_i) X_{\alpha \mu} u(q_i) \bar{u} (p_f) g^\nu u(p_i) ] + Q_{\beta} \bar{u} (q_f) g_{\alpha u} (q_i) \bar{u} (p_f) \gamma^\alpha \mu u(p_i). \]

where

\[ X_{\alpha \mu} = F^\alpha_\mu (u_1, t_q) \left[ \frac{q_f + R^\alpha_{\mu} q_i}{K} - \frac{(p_i - q_f) \mu}{(p_i - q_f) \cdot K} \right] g^\nu + F^\alpha_\mu (u_2, t_p) \left[ \frac{q_f + R^\alpha_{\mu} q_i}{q_i \cdot K} - \frac{(q_i - p_f) \mu}{(q_i - p_f) \cdot K} \right] g^\nu. \]

\[ Y^\nu_{\mu} = F^\nu_{\alpha} (u_1, t_q) \left[ \frac{p_i + R^\nu_{\alpha} p_f}{p_f \cdot K} - \frac{(q_f - p_f) \mu}{(q_f - p_f) \cdot K} \right] g^\nu - F^\nu_{\alpha} (u_2, t_p) \left[ \frac{p_i + R^\nu_{\alpha} p_f}{q_i \cdot K} - \frac{(p_i - q_f) \mu}{(p_i - q_f) \cdot K} \right] g^\nu. \]

In Eqs. (2-4), we have defined

\[ (g_1, g_2, g_3, g_4, g_5) \equiv (1, \sigma_{\mu\nu} / \sqrt{2}, i\gamma_5 \gamma_{\mu}, \gamma_{\mu}, \gamma_5), \]

\[ (g^1, g^2, g^3, g^4, g^5) \equiv (1, \sigma_{\mu\nu} / \sqrt{2}, i\gamma_5 \gamma_{\mu}, \gamma_{\mu}, \gamma_5), \]

and the magnetic factors \( R^Q_{\mu} \) (\( Q = q_f, q_i, p_f, p_i \)) can be expressed as

\[ R^Q_{\mu} = \frac{1}{\kappa} [\gamma_{\mu}, \vec{Q}] + \frac{\kappa}{8m} \{ [\gamma_{\mu}, \vec{Q}], \vec{Q} \}. \]

In Eq. (5), \( m (= m_A = m_B) \) and \( \kappa (= \kappa_A = \kappa_B) \) are the mass and the anomalous magnetic moment of the proton, \( \vec{Q} = Q^\mu \gamma_{\mu} \), and we have used \( [F,G] \equiv FG - GF \) and \( [F,G] \equiv FG + GF \). As one can see from Eqs. (3) and (4), the invariant amplitudes \( F^\alpha_{\mu} \) depend on \( u \) and \( t \). The same amplitudes but as functions of \( s \) and \( t \) can be obtained if we use the condition \( s + t + u = \)}
4m^2. For example, \( F^x(u_1, t_p) = F^x(s_1, t_p) \) where \( s_p + t_p + u_1 = -4m^2 \). Since \( F^x(s_1, t_p) \) (\( \alpha = 1, \ldots, 5 \)) are invariant amplitudes for the pp elastic process, the Feynman amplitude \( F(s_1, t_p) \) defined by Goldberger et al. [9] can be written in terms of the five Fermi covariants \( (S, T, A, V, P) \) as

\[
F(s_1, t_p) = F_S(s_1, t_p)S + F_T(s_1, t_p)T + F_A(s_1, t_p)A + F_V(s_1, t_p)V + F_P(s_1, t_p)P. \tag{6}
\]

The amplitude \( M_{\mu}^{TtH}(s_1, t_q, u, t_p) \) can be formally obtained from the amplitude \( M_{\mu}^{TtH}(u_1, u_2, t_q, t_p) \) given by Eqs. (2), (3), and (4) by making the following substitutions: (i) \( Q_u \rightarrow -Q_B \) and (ii) \( p^x \rightarrow -p^x \) and \( g^x R_\mu^x \rightarrow -R_\mu^x g^x \), keeping \( R_\mu^x \) and \( R_\mu^x \), and the spinors \( \bar{u} \) and \( \bar{u} \) unchanged. However, we emphasize that the two are not the same numerically.

If all \( F^x(s, t_i) \) (\( \alpha = 1, \ldots, 5 \), \( x = i, f \), and \( y = q, p \) in \( M_{\mu}^{TtH} \) are expanded about the average of \( s \) (3) and the average of \( t \) (7), then the first two terms of the expansion give the conventional Low amplitude \( M_{\mu}^{Low(s, t)}(\bar{s}, \bar{t}) \). We emphasize that this particular choice (3, 7) for the on-shell point at which the Low amplitude is evaluated is an ad hoc prescription, although it did provide a reasonable description of the available ppy data until the TRIUMF measurements at 280 MeV. Another Low amplitude \( M_{\mu}^{Low(s, t)}(\bar{s}, \bar{t}) \) was also derived in Ref. [8]. However, it can be shown that \( M_{\mu}^{Low(s, t)} \equiv M_{\mu}^{Low(s, t)} \), i.e., the Low amplitude is unique.

We have studied the amplitudes \( M_{\mu}^{TtH}, M_{\mu}^{TtH}, \) and \( M_{\mu}^{Low(s, t)} \) and have used them to calculate ppy cross sections at various energies, using state-of-the-art phase shifts from the latest Nijmegen pp partial-wave analysis [10]. Anecdotal results are shown in Figs. 1, 2, and 3. At 42 MeV for \( \theta_q = \theta_p = 26^\circ \) (see Fig. 1) the coplanar cross sections calculated from \( M_{\mu}^{TtH} \) are much larger than the Manitoba data [11]. The amplitudes \( M_{\mu}^{TtH} \) and \( M_{\mu}^{Low(s, t)} \), on the other hand, give similar results which agree well with both the data (within the experimental error) and the representative Hamada-Johnston-potential calculation [12]. (The results calculated using \( M_{\mu}^{Low(s, t)} \) are close to those obtained by Nyman and Fearing.) In the upper panel of Fig. 2, our coplanar cross sections calculated from \( M_{\mu}^{TtH} \) and \( M_{\mu}^{Low(s, t)} \) at 157 MeV for \( \theta_q = \theta_p = 35^\circ \) are compared with the Harvard data [13] and a Paris-potential calculation [14]. (Other potential-model calculations [6,15–17] yield similar results.) Cross sections calculated using the amplitude \( M_{\mu}^{TtH} \) are missing from Fig. 2 (and 3), because they are factors...
larger than those plotted. Again the amplitudes $M_{\mu}^{Tutis}$ and $M_{\mu}^{Low(s,t)}$ give very similar results at this energy and agree reasonably with both the potential-model curve and the Harvard data. (We return to the lower panels of Fig. 2 below.)

However, at an energy near the pion-production threshold and far from the on-shell point, the two amplitudes $M_{\mu}^{Tutis}$ and $M_{\mu}^{Low(s,t)}$ predict quite different results. This is demonstrated in Fig. 3. At 280 MeV for $\theta_q = 12.4^\circ$ and $\theta_p = 12^\circ$, the curve calculated from $M_{\mu}^{Tutis}$ agrees well with the published TRIUMF data [7], which include a normalization factor of 2/3, and with the curves calculated using the Paris potential and the Bonn potential [6,7]. (Ref. [6] takes into account the relativistic spin correction (RSC) but ignores the rescattering contribution (RC). The latter has been examined in Ref. [16], and both Refs. [14] and [15] include RSC and RC effects. The RC contribution may be 20% - 30% in some cases.) The amplitude $M_{\mu}^{Low(s,t)}$, on the other hand, predicts cross sections which are too small for forward $(\theta_p \leq 30^\circ)$ and backward $(\theta_p \geq 150^\circ)$ photon angles. That $M_{\mu}^{Low(s,t)}$ can describe most of the older ppy data but fails to fit the TRIUMF data has already been pointed out by Fearing. What is emphasized here is that the new amplitude $M_{\mu}^{Tutis}$ describes data where the conventional Low amplitude $M_{\mu}^{Low(s,t)}$ fails.

How can the failure of the conventional Low amplitude $M_{\mu}^{Low(s,t)}$ be understood? Consider the expressions given in Eqs. (2-4). If we impose the on-shell condition, $s + t + u = 4m^2$, we can write $F_{\mu}^r(t_{12}, t_{12}) = F_{\mu}^r(s_{12}, t_{12}) = F_{\mu}^r(s_{12}, t_{12})$, and $F_{\mu}^r(t_{12}, t_{12}) = F_{\mu}^r(s_{12}, t_{12})$, where $s_{12} = s_1 - 2q_f \cdot K, s_{2p} = s_2 - 2p_f \cdot K, s_{2q} = s_2 - 2q_f \cdot K$, and $s_{1q} = s_1 - 2q_i \cdot K$. This shows that $F_{\mu}^r$ will be evaluated at four different energies and four different angles in constructing $M_{\mu}^{Tutis}$. In contrast, $M_{\mu}^{Tutis}$ is evaluated at two energies and four angles, while $M_{\mu}^{Low(s,t)}$ is evaluated at just one energy and one angle. To be specific, at 100 MeV for $\theta_q = \theta_p = \theta_g = 30^\circ$, we have $s_{1p} = 3.648 \text{ GeV}^2, s_{2q} = 3.640 \text{ GeV}^2, s_{2p} = 3.632 \text{ GeV}^2$, and $s_{1q} = 3.655 \text{ GeV}^2$, whereas $s_1 = 3.709 \text{ GeV}^2$ and $s_f = 3.578 \text{ GeV}^2$, and finally $\bar{s} = 3.644 \text{ GeV}^2$. These quantities are the dominant factors determining the calculated cross sections. Since $s_{1p} \approx s_{2p} \approx s_{2q} \approx s_{1q} \approx \bar{s}$ (the differences in c.m. energy between $s_{1p}, s_{2q}, s_{1q}$ on one hand, and $\bar{s}$ on the other hand, are less than about 3 MeV), $M_{\mu}^{Low(s,t)}$ and $M_{\mu}^{Tutis}$ predict similar results at energies lower than 100 MeV and for large proton angles; each of the input amplitudes is calculated at approximately the same value of $s$. However, the value of $s_1$ is much larger than the
Fig. 3. Coplanar ppy cross section at 280 MeV for $\theta_\gamma = 12.4^\circ, \theta_p = 12^\circ$: --- result using $M_{TTKOP}$; --- result using $M_{\mu_{\text{L \text{ow} \text{e} \text{r} \text{e} \text{r} \text{a} \text{t}}} \text{e} \text{r}}$; --- result for Paris potential [7]; --- result for Bonn potential [7]. The data are from Ref. [7].
value of $s_f$ in the $M_{\text{TuTt}}^{\text{Tw}}$ evaluation. This is equivalent to a c.m. energy difference of some 34 MeV. This large difference between $s_i$ and $s_f$ is the primary reason for the huge cross sections predicted by $M_{\text{TuTt}}^{\text{Tw}}$. That is, for $s_f$ small, the corresponding amplitudes are much larger than those for $s_i$, etc., resulting in the $M_{\text{TuTt}}^{\text{Tw}}$ approximation being much too large. As the incident energy increases (or the proton angles decrease), the values of the four energies, $s_{1p}$, $s_{2p}$, $s_{1q}$, and $s_{2q}$, will no longer be close to one another, and they will differ significantly from $s_i$, as well as $s_i$ and $s_f$. Thus, the cross sections calculated using the amplitudes $M_{\mu}^{\text{TuTt}}$, $M_{\mu}^{\text{Low}(s_{1p})}$, and $M_{\mu}^{\text{TuTt}}$ will begin to differ even more.

We illustrate further the divergence of the cross section results of the Low amplitude from those of the TuTt amplitude in the lower panels of Fig. 2, where curves for $(20^\circ,20^\circ)$ and $(10^\circ,10^\circ)$ are plotted along with the results for $(35^\circ,35^\circ)$ that are compared with the data. As one departs further from the elastic scattering configuration $(45^\circ,45^\circ)$, the significance of using four energies and four angles in the TuTt prescription, compared to the one energy and one angle of the Low prescription, becomes more apparent.

A more complete analysis including polarization observables will be reported elsewhere. However, we try to provide here an intuitive understanding of why the amplitude $M_{\mu}^{\text{TuTt}}$ should be used to calculate $NN\gamma$ (pp$\gamma$ as well as np$\gamma$ and pn$\gamma$) cross sections. Based upon a meson-exchange theory of the NN interaction, one-boson-exchange (OBE) diagrams become the fundamental tree diagrams in the process. For example, a simple relationship between the individual Lorentz invariant amplitudes and the exchanged mesons has been derived by Horowitz [18]. The resulting $NN$ elastic amplitude depends only upon two momentum transfer variables “u” and “t”. Therefore, the $NN\gamma$ amplitude generated by such an OBE model will be an off-shell TuTt amplitude. For instance, if a photon is emitted from the $q_f$ leg, then we obtain an off-shell amplitude $M_{\mu}^{\text{TuTt}}(u_1, t_1)$. As was demonstrated in Ref. [8], $u_1$ and $t_1$ satisfy the following relation:

$$s_i + t_1 + u_1 = 2q_f \cdot K + 2m^2_A + 2m^2_B .$$  

If we impose the on-shell condition

$$s_{1p} + t_p + u_1 = 2m^2_A + 2m^2_B .$$  

then $M_{\mu}^{\text{TuTt}}(u_1, t_1)$ reduces to an on-shell amplitude

$$M_{\mu}^{\text{TuTt}}(s_{1p}, t_p),$$

which is similar to the one given by (2) and (3). Comparing (7) with (8), we observe that $s_{1p} = s_i - 2q_f \cdot K$. Because $u_1$ and $t_p$ are given by kinematics, other choices of for $s_{1p}$ will lead to inconsistencies. Finally, as noted in Ref. [8], the Low amplitude (as well as $M_{\mu}^{\text{TuTt}}$) does not include important meson-exchange effects; therefore, only $M_{\mu}^{\text{TuTt}}$ should be used to describe np$\gamma$ or pn$\gamma$ cross sections.

In conclusion, we have demonstrated that the amplitude $M_{\mu}^{\text{TuTt}}$, not the conventional Low amplitude $M_{\mu}^{\text{Low}(s_{1p})}$ nor the amplitude $M_{\mu}^{\text{TuTt}}$, is the correct soft-photon amplitude to be used in describing the nucleon-nucleon bremsstrahlung cross sections. Furthermore, below the pion-production threshold this new amplitude $M_{\mu}^{\text{TuTt}}$ provides a description of the $NN\gamma$ cross section data that is the equal of contemporary potential-model calculations.

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